Vendor-Buyer Deteriorating Inventory Model with Progressive Interest

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Abstract: In competitive market, many vendors try to increase their market by offering delay in payment. However to keep their financial balance and reduce lost, vendor only give specific period without interest and then she charges the buyer with progressive interest. This scheme is interesting for the buyer since the buyer does not need to pay in advance. The problem is when the vendor set the grace period and the progressive time period. These decisions will affect the vendor's decision to set her order. This problem become more interesting for deteriorating items where the items are decay, evaporate, obsolescence, loss of quality or marginal value of a commodity. Deterioration decreases the usefulness of the good from its original condition. In this paper, we develop a mathematical model of vendor-buyer collaboration for deterioration item under progressive interest scheme. Since the model is too complex to be solved analytically, then we use Genetic Algorithm. A numerical example is used to illustrate the model and a sensitivity analysis is employed to verify the model. The solution of the model shows that collaboration model is more profitable for the vendor since the buyer will be forced to buy in large quantity.

Keywords: Inventory, deteriorating item, progressive interest, collaboration.

Introduction

In recent years business completion becomes tighter. Many ways are used by vendor to sell their product as much as possible and to get profit as higher as possible. One of the ways is offering delay of payment to buyers. However to reduce loss, vendor also charge some amount of interest to the buyer if she cannot pay at certain time period. This strategy interest many researchers to find the best strategy for vendor, buyer or both of them.

Goyal [2] is one of the first researchers who developed economic order quantity (EOQ) models by considering permissible delay in payment. Goyal [2] model was extended by Chung and Huang [1] by considering shortage. Huang [4] developed a production inventory model with permissible delay in payment. Later Liao [6] continued Huang [4] by considering deteriorating items. Some researchers tried to analyze the permissive delay of payment strategy for two players, vendor and the buyer. Teng *et al.* [9] developed vendor-buyer inventory model with permissible delay in payment for two conditions which are non-cooperative and integrated environments. They concluded that for integrated environments, vendor has important rule to reduce total cost of both parties. Vendor can provide simple permissible delay of payment without order of quantity restriction or a long permissible delay of payment linked order quantity. Jaber and Osman [5] developed an inventory model with permissible delay in payment for two-level supply chain. They introduced a profit-sharing scenario to generate net profit for both players.

All of research scenarios above are for single interest charge and delay of payment period. Soni and Shah [7] introduced a progressive payment scheme. In this scheme, supplier or vendor set two delay of payment period. If buyer pays before the first delay of payment deadline, then buyer is not charged by any interest. If buyer pays after the first delay of payment deadline and before the second payment deadline, then buyer have to pays some interest. If buyer pays after the second delay of payment deadline, buyer is charged by larger interest. Similar research was conducted by Goyal *et al.* [3]. Teng *et al.* [8] extended the work of Soni and Shah [7] by introducing non zero ending inventory, a profit maximization objective, limited inventory capacity and deteriorating items with constant deteriorating rate.

All of the inventory models with progressive payment above only consider buyer as the object of the research. Since vendor has importance rule in progressive payments, we develop a single vendor-buyer inventory model with progressive payment in this paper. We also introduce deteriorating items, since deteriorating items are more difficult to handle in progressive payment scheme. The model development is shown in section 2, and then a numerical analysis is provided in section 3 to shows how the model works. At the end some conclusions are derived in the last section.

Mathematical Model

In this model, we consider possibility cases. In the first case, the optimal replenishment time (T) less than the first delay payment period (M1). Case 2 occur if the replenishment time (T) greater time M1 and less than the second delay period (M2) and the third case occur if the replenishment time (T) bigger than the second delay payment period. The all cases have similar fitness function which is minimizing total supply chain cost (TC). The total supply chain cost consists of buyer inventory cost (TBUC) and the vendor inventory cost (TVUC).

Assumptions

The model in this paper follows some assumptions as below:

- Demand rate is constant during planning period.
- Shortages are not allowed
- Replenishment rate is continuous and instantaneous.
- Vendor allows the buyer to pay without interest if the buyer makes a payment before the first delay payment period (M1). When the buyer make a payment after M1 and before the second delay payment period (M2), the vendor charge interest Ic1 to the buyer. If the buyer has not until M1 time period, the buyer will be charged interest Ic2.
- The second interest rate Ic2 bigger than the first interest rate Ic1.
- The planning period is infinite.
- Production rate (P) bigger than the demand rate (D)

Notation

T: replenishment period
Q: ordering quantity
M: delivery quantity
K: delivery frequency during T period
W: delivery frequency during production up time
p: production rate (unit/year)
d: demand rate (unit/year)
A: Buyer ordering cost
A_v: Vendor production cost
C_t: Transportation cost
h_b: Buyer inventory cost/unit/period
h_o: Buyer opportunity cost
h_v: Vendor inventory cost/ unit/period
h_{vo}: Vendor opportunity cost
c: product unit cost
θ_v: *vendor deterioration rat*
θ_b: *buyer deterioration rate*
p_r: product price
IP: average vendor inventory
I_{c1}: Interest rate of the first delay of payment period
Ic2: Interest rate of the second delay of payment period
Ie: Buyer interest earned
M1 : First delay of payment period
M2: Second delay of payment period
TIev: Total vendor opportunity cost
TIeb: Total buyer opportunity cost
TIc1: Total vendor interest earned for the first delay of payment period
TIc2: Total vendor interest earned for the first delay of payment period
TIS: *Total Incremental Annual Cost*
TBUC: Total buyer cost
TVUC: Total vendor cost
TSC: Total supply chain cost

Case 1

In case 1, vendor allows the buyer to has delay of payment until time period M1, so the vendor has opportunity cost as follows:

$$TI_{ev} = \frac{h_{v0}pwT}{K}\ \ M_1 - \frac{wT}{2K} \tag{1}$$

At the other side, the buyer gets opportunity earn as follows;

$$TI_{eb} = I_e dT\ \ M_1 - \frac{T}{2} \tag{2}$$

The buyer total inventory cost consists of ordering cost, transportation cost, inventory cost and opportunity earn that can be modeled as:

$$TBUC = \frac{Ad}{mK} + \frac{C_t d}{m} + \frac{d\ \ 2+\theta_b{}^T\ \ {}_K}{\theta_b^2\ \ 2-\theta_b{}^T\ \ {}_K} - \frac{d+d\theta_b{}^T\ \ {}_K}{\theta_b^2}\ \ \ \frac{H_b K}{T} - I_e dT\ \ M_1 - \frac{T}{2} \tag{3}$$

The vendor total inventory cost consists of production setup cost, inventory cost and opportunity cost that can be modeled as follows:

$$TVUC = \frac{A_v d}{mK} + \frac{pw\frac{T}{K} - mK\ \ H_v}{\theta_v T} + \frac{h_{v0}pwT}{K}\ \ M1 - \frac{wT}{2K} \tag{4}$$

where

$$w = \frac{(d\ \ 1+\theta_v\ \ K)}{P}$$

The total supply chain cost is total of vendor inventory cost and the buyer inventory cost, one has:

$$TSC = \frac{Ad}{mK} + \frac{C_t d}{m} + \frac{d\ \ 2+\theta_b{}^T\ \ {}_K}{\theta_b^2\ \ 2-\theta_b{}^T\ \ {}_K} - \frac{d+d\theta_b{}^T\ \ {}_K}{\theta_b^2}\ \ \ \frac{H_b K}{T} - I_e DT\ \ M_1 - \frac{T}{2}\ \ + \frac{A_v d}{mK} + \frac{pw\frac{T}{K} - mK\ \ H_v}{\theta_v T} + \frac{h_{v0}pwT}{K}\ \ M1 - \frac{wT}{2K} \tag{5}$$

Case 2

In case 2, there are two possibilities where the first possibility is the buyer pays at the first delay period

(M1) and the second possibility, the buyer pays at the second delay payment period (M2)

Case 2.1

For the first case when the buyer pays at M1, there are two possibilities. First, possibility is the production period less than M1 and the second possibility is the production period bigger than M1. When the production period less than M1, the total supply cost can be modeled as:

$$TSC = \frac{Ad}{mK} + \frac{C_t d}{m} + \frac{d\ 2+\theta_b{}^T\ K}{\theta_b^2\ 2-\theta_b{}^T\ K} - \frac{d+d\theta_b{}^T\ K}{\theta_b^2}\ \frac{H_b K}{T} - I_e\ \frac{dM_1^2}{2}\ + \frac{A_v d}{mK} + \frac{pw\frac{T}{K}-mK\ H_v}{\theta_v T} + \frac{h_{v0}pwT}{K}\ M_1 - \frac{wT}{2K} \quad (6)$$

And for the second case, one has:

$$TSC = \frac{Ad}{mK} + \frac{C_t d}{m} + \frac{d\ 2+\theta_b{}^T\ K}{\theta_b^2\ 2-\theta_b{}^T\ K} - \frac{d+d\theta_b{}^T\ K}{\theta_b^2}\ \frac{H_b K}{T} - I_e\ \frac{dM_1^2}{2}\ + \frac{A_v d}{mK} + \frac{pw\frac{T}{K}-mK\ H_v}{\theta_v T} + \frac{h_{v0}pM_1^2}{2} \quad (7)$$

Case 2.2

Similar as case 2.1, case 2.2 also have two cases. The total supply chain cost for the first case can be modeled as:

$$TSC = \frac{Ad}{mK} + \frac{C_t d}{m} + \frac{d\ 2+\theta_b{}^T\ K}{\theta_b^2\ 2-\theta_b{}^T\ K} - \frac{d+d\theta_b{}^T\ K}{\theta_b^2}\ \frac{H_b K}{T} + \frac{I_{c1}}{2p_r d}\ cdT - \ p_r dM_1 + \frac{p_r I_e dM_1^2}{2}{}^2 - I_e\ \frac{dM_1^2}{2}\ + \frac{A_v d}{mK} + \frac{pw\frac{T}{K}-mK\ H_v}{\theta_v T} + \frac{h_{v0}pwT}{K}\ M_1 - \frac{wT}{2K}\ - \frac{I_{c1}}{2p_r d}\ cdT - \ p_r dM_1 + \frac{p_r I_e dM_1^2}{2}{}^2 \quad (8)$$

and for the second case, one has:

$$TSC = \frac{Ad}{mK} + \frac{C_t d}{m} + \frac{d\ 2+\theta_b{}^T\ K}{\theta_b^2\ 2-\theta_b{}^T\ K} - \frac{d+d\theta_b{}^T\ K}{\theta_b^2}\ \frac{H_b K}{T} + \frac{I_{c1}}{2p_r d}\ cdT - \ p_r dM_1 + \frac{p_r I_e dM_1^2}{2}{}^2 - I_e\ \frac{dM_1^2}{2}\ + \frac{A_v d}{mK} + \frac{pw\frac{T}{K}-mK\ H_v}{\theta_v T} + \frac{h_{v0}pM_1^2}{2} - \frac{I_{c1}}{2p_r d}\ cdT - \ p_r dM_1 + \frac{p_r I_e dM_1^2}{2}{}^2 \quad (9)$$

Case 3

In case 3, the replenishment time (T) is bigger or equal than the second delay period (M2). For this case, there are three possibilities. In the first possibility, buyer pays full payment at the first delay period (M1). There are two conditions for this possibility. The first condition is the first delay period less than the production up time. The total cost can be modeled as follows:

$$TSC = \frac{Ad}{mK} + \frac{C_t d}{m} + \frac{d\ 2+\theta_b{}^T\ K}{\theta_b^2\ 2-\theta_b{}^T\ K} - \frac{d+d\theta_b{}^T\ K}{\theta_b^2}\ \frac{H_b K}{T} - I_e\ \frac{dM_1^2}{2}\ + \frac{A_v d}{mK} + \frac{pw\frac{T}{K}-mK\ H_v}{\theta_v T} + \frac{h_{v0}pwT}{K}\ M_1 - \frac{wT}{2K} \quad (10)$$

For the second condition, one has:

$$TSC = \frac{Ad}{mK} + \frac{C_t d}{m} + \frac{d\ 2+\theta_b{}^T\ K}{\theta_b^2\ 2-\theta_b{}^T\ K} - \frac{d+d\theta_b{}^T\ K}{\theta_b^2}\ \frac{H_b K}{T} - I_e\ \frac{dM_1^2}{2}\ + \frac{A_v d}{mK} + \frac{pw\frac{T}{K}-mK\ H_v}{\theta_v T} + \frac{h_{v0}pM_1^2}{2} \quad (11)$$

The second possibility, the buyer pays at the second delay period (M2). In this possibility there two conditions. The first condition is the first delay period less than the production up time. The condition can be modeled as:

$$TSC = \frac{Ad}{mK} + \frac{C_t d}{m} + \frac{d\ 2+\theta_b{}^T\ K}{\theta_b^2\ 2-\theta_b{}^T\ K} - \frac{d+d\theta_b{}^T\ K}{\theta_b^2}\ \frac{H_b K}{T} + \frac{I_{c1}}{2p_r d}\ cdT - \ p_r dM_1 + \frac{p_r I_e dM_1^2}{2}{}^2 - I_e\ \frac{dM_1^2}{2}\ + \frac{A_v d}{mK} + \frac{pw\frac{T}{K}-mK\ H_v}{\theta_v T} + \frac{h_{v0}pwT}{K}\ M_1 - \frac{wT}{2K}\ - \frac{I_{c1}}{2p_r d}\ cdT - \ p_r dM_1 + \frac{p_r I_e dM_1^2}{2}{}^2$$

(12)

The other condition can be modeled as follows:

$$TSC = \frac{Ad}{m} + \frac{C_t d}{m} + \frac{d\ 2+\theta_b{}^T\ K}{\theta_b^2\ 2-\theta_b{}^T\ K} - \frac{d+d\theta_b{}^T\ K}{\theta_b^2}\ \frac{H_b K}{T} + \frac{I_{c1}}{2p_r d}\ cdT - \ p_r dM_1 + \frac{p_r I_e dM_1^2}{2}{}^2 - I_e\ \frac{dM_1^2}{2}\ + \frac{A_v d}{mK} + \frac{pw\frac{T}{K}-mK\ H_v}{\theta_v T} + \frac{h_{v0}pM_1^2}{2} - \frac{I_{c1}}{2p_r d}\ cdT - \ p_r dM_1 + \frac{p_r I_e dM_1^2}{2}{}^2 \quad (13)$$

In the third possibility, the buyer pays after M2 period. For the first case, the first delay payment period (M1) is bigger than the production up time and one has:

$$TSC = \frac{Ad}{m} + \frac{C_t Kd}{m} + \frac{d\ 2+\theta_b{}^T\ K}{\theta_b^2\ 2-\theta_b{}^T\ K} - \frac{d+d\theta_b{}^T\ K}{\theta_b^2}\ \frac{H_b K}{T} + I_{c1}\ \frac{a+b}{2}\ M_2 - M_1\ + I_{c2}\ \frac{b^2}{2p_r d}\ - I_{eb}\ \frac{dM_1^2}{2}\ + \frac{A_v d}{mK} + \frac{pw\frac{T}{K}-mK\ H_v}{\theta_v T} + \frac{h_{v0}pwT}{K}\ M_1 - \frac{wT}{2K}\ - I_{c1}\ \frac{a+b}{2}\ M_2 - M_1\ + I_{c2}\ \frac{b^2}{2p_r d}\ , \quad (14)$$

For the second condition, the first delay payment period (M1) less than production up time. The problem can be modeled as:

$$TSC = \frac{Ad}{m} + \frac{C_t Kd}{m} + \frac{d\ 2+\theta_b{}^T\ K}{\theta_b^2\ 2-\theta_b{}^T\ K} - \frac{d+d\theta_b{}^T\ K}{\theta_b^2}\ \frac{H_b K}{T} + I_{c1}\ \frac{a+b}{2}\ M_2 - M_1\ + I_{c2}\ \frac{b^2}{2p_r d}\ - I_e\ \frac{dM_1^2}{2}\ +$$

$$\frac{A_v d}{mK} + \frac{pw\frac{T}{K} - mK \quad H_v}{\theta_v T} + \frac{h_{v0} p M_1^2}{2} - \quad I_{c1} \quad \frac{a+b}{2} \quad M_2 - M_1 \quad + I_{c2} \quad \frac{b^2}{2p_r d} \tag{15}$$

Numerical Example and Discussion

Mathematics

Since the model is an NP-hard, we used Genetic Algorithm to solve the problem. The operators of GA are:

- Number of population : 20
- Number of generation:1000
- *Selection*: *Roulette Wheel*
- *Crossover*: *Scattered*
- Mutation: *Constraint Dependent*
- *Elitism*: 2
- Decision variables: K, T, M_1, M_2

One set of data is used to show calculation of the model. The data set is shown in Table 1.

The GA method has lower bound solution for the replenishment time equla to 0 dan the upper bound is set to 1.99. The result of the GA solutions are the optimal delivery equal to 2, the optimal replenishment time equal to 1.99, the first delay of payment equal to 0, and the second delay of payment equal to 1.19. The decisions result in total buyer cost equla to \$ 3104.36, the optimal total vendor cost equal to 1192.1 and hte supply chain cost equal to 4296.46. The solution shows the vendor total cost is less than the buyer total cost This result is similar as the other collaboration models where the vendor has opportunity to get higher profit or less cost. Vendor can reduce his cost by force the buyer to buy as much as possible. So the optimal solution of the replenishment time is equal to upper bound of the GA method. The replenishment time is greater than the delay time (M1 and M2), so the vendor can get profit from delay of payment interest (I_{c1}, I_{c2}). Vendor can reduce cost by applying single delay payment. The vendor set the first delay payment equal to zero. It is meant that the vendor do not give the first delay od payment.

Conclusion

In this paper, a collaboration production inventory model for deteriorating items with progressive payment has been developed. Since there many decision variables that have been considered, so Genetic Algorithm method is used to solve the model. A numerical example is introduced to show calculation of the model. The result shows that vendor get higher benefit than the buyer for the collaboration model. Since the vendor tries to minimize the cost, then the progressive payment become single delay of payment. This research can be xtended by considering game model of vendor and the buyer.

Table 1. Parameters of numerical example

Parameter	Value
A	200
A_v	150
h_b	4
h_v	4
h_{vo}	110
I_e	4%
I_{c1}	2%
I_{c2}	6%
D	1000
P	4000
C_t	100
Pr	30
c	25
θ_v	9%
θ_b	9%

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