Pricing decision model for new and remanufactured short-life cycle products with time-dependent demand

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ARTICLE INFO

Article history:
Received 15 July 2014
Received in revised form 6 November 2014
Accepted 19 November 2014
Available online 3 December 2014

Keywords:
Short life cycle product
Remanufacturing
Closed loop supply chain
Pricing
Optimization

ABSTRACT

In this study we develop a model that optimizes the price for new and remanufactured short life-cycle products where demands are time-dependent and price sensitive. While there has been very few published works that attempt to model remanufacturing decisions for products with short life cycle, we believe that there are many situations where remanufacturing short life cycle products is rewarding economically as well as environmentally. The system that we model consists of a retailer, a manufacturer, and a collector of used product from the end customers. Two different scenarios are evaluated for the system. The first is the independent situation where each party attempts to maximize his/her own total profit and the second is the joint profit model where we optimize the combined total profit for all three members of the supply chain. Manufacturer acts as the Stackelberg leader in the independently optimized scenario, while in the other the intermediate prices are determined by coordinated pricing policy. The results suggest that (i) reducing the price of new products during the decline phase does not give better profit for the whole system, (ii) the total profit obtained from optimizing each player is lower than the total profit of the integrated model, and (iii) speed of change in demand influences the robustness of the prices as well as the total profit gained.

1. Introduction

Technology-based product has shorter life cycle due to rapid innovation and development in science and technology, as well as customer behavior in pursuing latest innovation and style. Lebreton and Tuma [1] pointed out that technology based commodities such as mobile phones and computers have shorter innovation cycle so that the previous generation becomes obsolete faster, either functionally and psychologically. Similarly, Hsueh [2] also argued that product life cycle in electronic industry is shorter than before, due to technology advances, and as a result, an outdated product could reach its end-of-use even it is still in a good condition. Shorter life-cycle has negative contribution toward sustainability, since there is an increase in product disposal. Customers want newer products and discard the old ones, and these preferences would exhaust landfill space in shorter time. In addition, there are more natural resources and energy used to create new products than actually needed, due to unnecessary increased obsolescence. To make it worse, electronic products are prominent as the ones with shorter and shorter life cycle, while the wastes are toxic and not environmentally friendly. There are many attempts made in developed countries to control electronic wastes such as Waste of Electric and Electronic Equipment (WEEE) directives, implemented in most European countries since 2003, RoHS in United States, 2003, and Extended Producer Responsibility (EPR) issued by OECD in 1984. However, these regulations pose as burdens to the industries when implemented only for conformity, because there are additional costs for handling e-wastes and increased material cost for avoiding or minimizing toxic materials.

Several strategies have been introduced to mitigate products disposal and wastes, such as life cycle approach, regulation and society approach. One aspect of life cycle approach is dealing with products at their end-of-use. According to de Brito & Dekker [3], there are situations where customer has the opportunity to return a product at a certain life stage, which can be referred to leasing cases and returnable containers, and is called end-of-use return. Hsueh [2] considered a different kind of return, where a product may be returned because it has become outdated, and the customer
wants to buy a new product. Herold [4] proposed alternatives to end-of-use products which are reprocessing, collect-and-sell, and collect-and-dispose. Remanufacturing is one option to manage products at their end-of-use which offers opportunity for complying with regulation while maintaining profitability [5–7].

Remanufacturing is a process of transforming used product into “like-new” condition, so there is a process of recapturing the value added to the material during manufacturing stage [8,9]. The idea of remanufacturing used products has gained much attention recently for both economic and environmental reasons. As suggested by Gray and Charter [9], remanufacturing can reduce production cost, the use of energy and materials.

There are numerous studies on remanufacturing. However, most of the published works on remanufacturing have considered durable or semi-durable products. Very little attempt has been made to study how remanufacturing maybe applied to products with short life cycle. In some developing countries like Indonesia, there is a large segment of society that could become potential market for remanufactured short-life cycle products like mobile phones, computers and digital cameras.

In remanufacturing practice, there are three main activities, namely product return management, remanufacturing operations, and market development for remanufactured product [10]. In terms of marketing strategy, there are general concerns that remanufactured product would cannibalize the sales of new product. However, Atasu et al. [11] concluded that remanufacturing does not always cannibalize the sales of new products. He proposed that managers who understand the composition of their markets, and use the proper pricing strategy should be able to create additional profit. Therefore, pricing decision is an important task in an effort to gain economic benefit from remanufacturing practices.

There are several studies that focused on pricing of remanufactured products, but many of them have not considered the whole supply chain, and also only a very few concern about obsolescence of short life cycle products. Our study will be focused on pricing decisions in a closed loop supply chain involving manufacturer, retailer and collector of used products (cores), where customers have the option to purchase new or remanufactured short life cycle products in the same market channel. We consider a monopolist of a single item with no constraint on the quantity of remanufacturable cores throughout the selling horizon.

2. Literature review

Remanufacturing of mobile phones and electronic products has been recognized as an important practice in the United States, and as a potential in China and India. Heho [12] claimed that product life cycle has significantly shortened by rapid technological advancement, and coupled with fashionable design that attracts frequent purchases of new products, has generated pressure on and opportunities for reverse logistics. Franke et al. [13] suggested that remanufacturing of durable high-value products such as automobile engine, aircraft equipment, and machine tools, has been extended to a large number of consumer goods with short life cycle and relatively low values, like mobile phones and computers. He also quoted market studies by Marcussen [14] and Directive 2002/96/EC which revealed that there is a significant potential for mobile phone remanufacturing due to the large supply market of the used mobile phones in Europe and the high market demand in Asia and Latin America.

Neto and Bloemhof-Ruwaard [15] found that remanufacturing significantly reduces the amount of energy used in the product life cycle, even though the effectiveness of remanufacturing is very sensitive to the life span of the second life of the product. They also proposed that the period of the life cycle in which the product is returned to recovery, the quality of the product (high-end versus low-end), the easiness to remanufacture and the recovery costs can affect whether or not remanufacturing is more eco-efficient than manufacturing. Rathore et al. [16] studied the case of remanufacturing mobile handsets in India. They found that used phone market is very important, even though with a lack of government regulation for e-wastes. It is also observed that there is a negative user-perception of second hand goods and that the process of remanufacturing has not been able to capture much required attention from its stakeholders. Wang et al. [17] showed that the mobile phone market in China is growing rapidly. The number of mobile accounts was 565.22 million in February 2008 according to a report from Ministry of Information Industry of the People’s Republic of China. The above mentioned studies have affirmed our intuitive proposition that there is a high potential for remanufacturing short life cycle products.

Motives for deploying reverse chain can be for profitability (or cost minimization) or for sustainability (environmental impact mitigation), which either could be driven by regulation and/or morale. In our research, the underlying motive considered would be focused on profitability, which seems to be the suitable motive applied to industries in a situation with the absence of environment protection regulation, like in most of the developing countries. There are numerous studies that investigated the factors that influence decision to remanufacture as well as the factors for successful remanufacturing. We categorized the factors into four aspects, namely product characteristics, demand-related factors, process-related factors, and supply-related factors.

The first aspect, product characteristics of short life cycle products, consist of (1) innovation rate (fast vs. slow) as an extension to technology factor [8,18–20]; (2) residence time [21]; (3) product residual value [22]; (4) qualitative obsolescence, as an extension to product characteristics [3,23].

Second, demand-related factors, consist of (1) market size or existence of the demand, [18,19,24]; (2) market channel, which is about selling remanufactured products using the same channel as the new product, or differentiated [8,18–21,25–27]; (3) pricing of new and remanufactured products, with demand as a function of price [28,32,33,30,18,19,26]; (4) existence of green segment, [23,31].

Supply-related factors can be described by (1) acquisition price and (2) source of return, whether it is limited and then pose as a constraint, or unlimited. These factors were studied in [8,18,19,23,24,32].

The last factors, which are process-related, consist of (1) remanufacturing technology availability [8,18,32]; (2) remanufacturing cost, [8,18,19,21,22,32]; (3) reverse flow structure readiness [8,20,24,32–34].

There are several studies that discuss pricing strategies involving remanufactured products, obsolescence, and nonlinear demand function. However, none has considered the situation that we address in this paper. Table 1 shows the review result and where our proposed model stands.

3. Problem description

A closed-loop supply chain consists of three members, which are manufacturer, retailer, and collector, as depicted in Fig. 1. The closed-loop is initiated by production of new product, which is sold at a wholesale price \( p_{\text{wh}} \) to the retailer. The new product is then released to the market at a retail price \( p_{\text{st}} \) for the period when product life-cycle is within introduction–growth–maturity (IMG) phases, or during increasing and stable phases. When the new product has reached its decline phase, retailer starts to apply different pricing, \( p_{\text{so}} \). In the model development, the price is differentiated between IMG phases and decline phase to study the impact of this differentiated pricing, as Kotler & Armstrong [52] suggest that reduced price during decline phase could increase the quantity of demanded goods. An example of short life-cycle product where the new product reaches its decline phase in a short time is, Samsung Galaxy Tab 10.1 that was released on second quarter
<table>
<thead>
<tr>
<th>Supply Chain members involved</th>
<th>Differentiating New &amp; Reman</th>
<th>Planning Horizon</th>
<th>Demand function</th>
<th>Decision variables</th>
<th>Objective</th>
<th>Considering obsolescence</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guide et al. (2003) [35]</td>
<td>Remanufacturer</td>
<td>Only reman product</td>
<td>Single period</td>
<td>Dr known</td>
<td>Price of *Reman *Core</td>
<td>Max profit</td>
<td>No</td>
</tr>
<tr>
<td>Ferrer and Swaminathan (2006) [37]</td>
<td>Manufacturer</td>
<td>No (Pn = Pr)</td>
<td>*Infinite *Two period *Multi period</td>
<td>Linear in price</td>
<td>*Price *Quantity</td>
<td>Max profit</td>
<td>No</td>
</tr>
<tr>
<td>Vadde et al. (2006) [38]</td>
<td>Product recovery facility</td>
<td>Only reman product</td>
<td>Selling horizon</td>
<td>Function of price and obsolescence</td>
<td>*Price</td>
<td>Max profit</td>
<td>Yes</td>
</tr>
<tr>
<td>Atasu et al. (2008) [31]</td>
<td>Manufacturer</td>
<td>Yes (Pn ≠ Pr)</td>
<td>Two period</td>
<td>Linear in price</td>
<td>*Price *Quantity</td>
<td>Max profit</td>
<td>No</td>
</tr>
<tr>
<td>Qiaolun et al. (2008) [29]</td>
<td>*Manufacturer *Retailer *Collector</td>
<td>No (Pn = Pr)</td>
<td>Selling horizon</td>
<td>Linear in price</td>
<td>Price of *Retail *Wholesale *Collecting</td>
<td>Max profit</td>
<td>No</td>
</tr>
<tr>
<td>Li et al. (2009) [40]</td>
<td>Remanufacturer</td>
<td>Only reman product</td>
<td>Single period</td>
<td>Stochastic, function of price</td>
<td>Price of *Reman *Core</td>
<td>Max utilization</td>
<td>No</td>
</tr>
<tr>
<td>Liang et al. (2009) [41]</td>
<td>Remanufacturer</td>
<td>Only reman product</td>
<td>Single period</td>
<td>None</td>
<td>Price of core</td>
<td>High return on investment</td>
<td>No</td>
</tr>
<tr>
<td>Ferrer &amp; Swaminathan (2010) [42]</td>
<td>Manufacturer</td>
<td>Yes (Pn ≠ Pr)</td>
<td>*Infinite *Two period *Multi period</td>
<td>Linear in price</td>
<td>*Price *Quantity</td>
<td>Max profit</td>
<td>No</td>
</tr>
<tr>
<td>Ovchinnikov (2011) [28]</td>
<td>Manufacturer</td>
<td>Yes (Pn ≠ Pr)</td>
<td>Pn fixed</td>
<td>Selling horizon</td>
<td>Dn known &amp; constant Dr function of price</td>
<td>*Price *Quantity of reman</td>
<td>Max profit</td>
</tr>
<tr>
<td>Shi et al. (2011) [43]</td>
<td>Manufacturer</td>
<td>No (Pn = Pr)</td>
<td>Single period</td>
<td>Stochastic, linear in price</td>
<td>*Price *Quantity of new &amp; reman</td>
<td>Max profit</td>
<td>No</td>
</tr>
<tr>
<td>Vadde et al. (2011) [44]</td>
<td>Product recovery facility</td>
<td>No new products</td>
<td>Single period</td>
<td>Deterministic Prices</td>
<td>Max revenue Min cost</td>
<td>No</td>
<td>Consider several types of used products</td>
</tr>
<tr>
<td>Wei &amp; Zhao (2011) [45]</td>
<td>*Manufacturer *Retailer</td>
<td>No (Pn = Pr)</td>
<td>Single period</td>
<td>Linear in price</td>
<td>Price of *Retail *Wholesale *Collecting</td>
<td>Max profit</td>
<td>No</td>
</tr>
<tr>
<td>Pokharel &amp; Liang (2012) [46]</td>
<td>Consolidation center</td>
<td>Only cores</td>
<td>Single period</td>
<td>Dr is known</td>
<td>Core price *Quantity of cores</td>
<td>Min cost</td>
<td>No</td>
</tr>
<tr>
<td>Wu (2012a) [47]</td>
<td>*OEM *Remanufacturer</td>
<td>Yes (Pn ≠ Pr)</td>
<td>Two period</td>
<td>Linear in price</td>
<td>Prices *New *Reman</td>
<td>Max profit</td>
<td>No</td>
</tr>
</tbody>
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(continued on next page)
Tung and extended to cover the obsolescence period, in which is assumed to be on the decline phase. Producing new product and only offers remanufactured product when the product has entered the decline phase while remanufactured products are at the IMG phases. In the third period, the new product has entered the decline phase while remanufactured product has not. In the fourth period [t3, T], manufacturer has stopped producing new product and only offers remanufactured product which is assumed to be on the decline phase.

The market demand capacity is adopted from Wang and Tung [54] and extended to cover the obsolescence period, where demand decreases significantly. The demand patterns are constructed for both new and remanufactured product and the governing functions are formulated as follows:

\[
D_{n}(t) = \begin{cases} 
D_{n1}(t) = U / (1 + ke^{-\lambda U}) ; & 0 \leq t \leq \mu \\
D_{n2}(t) = U / (\lambda U (t - \mu) + \delta) ; & \mu \leq t \leq t3 
\end{cases} 
\]

where \( k = U/D_{0} - 1 \), \( \delta = 1 + ke^{-\lambda U} \) \( (3.1) \)

\[
D_{r}(t) = \begin{cases} 
D_{r1}(t) = V / (1 + he^{-\eta V (t-t3)}) ; & t1 \leq t \leq t3 \\
D_{r2}(t) = V / (\eta V (t - t3) + \varepsilon) ; & t3 \leq t \leq T 
\end{cases} 
\]

where \( h = V/D_{0} - 1 \), \( \varepsilon = 1 + he^{-\eta V (t3-t1)} \) \( (3.2) \)

where \( D_{n}(t) \) and \( D_{r}(t) \) are demand pattern for new and remanufactured products, respectively, as shown in Fig. 2. \( U \) is a parameter representing the maximum possible demand for new product, \( \mu \) is the time when the demand reaches its peak, i.e. at \( U \) level. \( D_{0} \) is the demand at the beginning of the life-cycle (when \( t = 0 \)), and \( \lambda \) is the speed of change in the demand as a function of time. A parallel definition is applicable for \( V, t1, D_{0}, \) and \( \eta \) respectively for the remanufactured products. It is obvious that \( D_{n1}(t) \) and \( D_{r1}(t) \) are continuous at \( \mu \) and \( t3 \), respectively, as shown in Appendix A.

The new products are sold at retail price \( P_{n1} \) during \([0, \mu]\), and \( P_{n2} \) during \([\mu, t3]\). Since demand function is also linear in price, there is a maximum price \( P_{n1} \) (known and fixed) as the upper limit,
at which demand would be zero. Remanufactured products are sold at retail price \( P_r \) during \([t_1, T]\), and the maximum price is \( P_{n2} \), since customer would choose to buy new product rather than remanufactured one when the remanufactured product price is as high as \( P_{n2} \). Therefore, 

\[
\text{Demand of new product during } [0, \mu] = D_{n1}(t)(1 - P_{n1}/P_m) \\
\text{Demand of new product during } [\mu, t_3] = D_{n2}(t)(1 - P_{n2}/P_m) \\
\text{Demand of reman product during } [t_1, t_3] = D_{r1}(t)(1 - P_r/P_{n2}) \\
\text{Demand of reman product during } [t_3, T] = D_{r2}(t)(1 - P_r/P_{n2}).
\]

(3.3) (3.4) (3.5) (3.6)

Fig. 3 illustrates the demand of new product for the period of \([0, \mu]\).

The demand function information is shared to all members of the supply chain. 

Manufacturer decides the wholesale prices for new product \( (P_{n1}, P_{n2}) \) and remanufactured product \( (P_{rw}) \), retailer determines the retail prices \( (P_{n1}, P_{n2}, P_r) \), while collector determines collecting prices \( P_{c1} \) and \( P_{c2} \) for cores collected from end-of-use product within the periods of \([0, \mu]\) and \([\mu, t_3]\) respectively. Since the product has short life-cycle, remanufacturing process is only applied to cores originated from new products.

Return rate \( \tau \) is an increasing function of the collecting price. We use the return rate function proposed by Qiaolun et al. [29], which was a result of their survey that employs a power function. The return rate \( \tau \) is defined on \([t_1, T]\) and depends on \( P_c \) as follows

\[
\tau = \gamma P_c^\theta
\]

(3.7)

where \( \gamma \) are positive constant coefficients, and \( \theta \in [0, 1] \) are exponents of the power functions, which determine curve’s steepness. It is assumed that collector only accepts cores with a certain quality grade, and all collected cores will be remanufactured.

Since our research is focusing on pricing decision, we do not make an attempt to show detailed derivation of production and operational costs, and instead treat those costs as given parameters, which consist of unit raw material cost for new product \( (c_{rw}) \), unit manufacturing cost \( (c_m) \), unit remanufacturing cost \( (c_c) \), and unit collecting cost \( (c) \). The objective of the proposed model is to find the optimal prices that maximize profit, and we investigate two scenarios, (1) maximize profit independently, and (2) maximize joint profit along the supply chain.

4. Optimization

4.1. Independently optimized profit

4.1.1. Retailer’s optimization

In this scenario, manufacturer makes the first move by releasing initial wholesale prices \( (P_{n1}, P_{rw}) \). Retailer then optimizes the retail prices \( P_{n1}, P_{n2}, \) and \( P_r \). The profit function can be formulated as follows:

\[
\Pi_R = \int_0^\mu \frac{U}{1 + ke^{-\mu t}} \left( 1 - \frac{P_{n1}}{P_m} \right) (P_{n1} - P_{nw}) \, dt \\
+ \int_0^{t_3} \frac{U}{\lambda U(t - \mu) + \delta} \left( 1 - \frac{P_{n2}}{P_m} \right) (P_{n2} - P_{nw}) \, dt \\
+ \int_{t_1}^{t_3} \frac{V}{1 + he^{-\varepsilon(t - t_3)}} \left( 1 - \frac{P_r}{P_{n2}} \right) (P_r - P_{rw}) \, dt \\
+ \int_{t_3}^T \frac{V}{P_{n2}} \left( 1 - \frac{P_r}{P_{n2}} \right) (P_r - P_{rw}) \, dt \\
= d_1 \left( 1 - \frac{P_{n1}}{P_m} \right) (P_{n1} - P_{nw}) + d_2 \left( 1 - \frac{P_{n2}}{P_m} \right) (P_{n2} - P_{nw}) \\
+ (d_3 + d_4) \left( 1 - \frac{P_r}{P_{n2}} \right) (P_r - P_{rw})
\]

(4.1)

where

\[
d_1 = \frac{1}{\lambda} \ln \left( \frac{\delta}{(1 + k) e^{-\theta t_1}} \right) \\
d_2 = \frac{1}{\lambda} \ln \left( \frac{\lambda U(t_3 - \mu) + \delta}{\delta} \right) \\
d_3 = \frac{1}{\varepsilon} \ln \left( \frac{e}{(1 + h) e^{-\varepsilon(t_3 - t_1)}} \right) \\
d_4 = \frac{1}{\eta} \ln \left( \frac{\eta V(T - t_3) + \varepsilon}{\varepsilon} \right).
\]

The objective function is to maximize profit (4.1), and consequently it needs to satisfy the first derivative conditions \( \partial \Pi_R/\partial P_{n1} = 0, \partial \Pi_R/\partial P_{n2} = 0, \) and \( \partial \Pi_R/\partial P_r = 0 \). The profit function (4.1) is not always concave along the considered interval, because \( P_{n2} \) took a hyperbolic form as a result of being the upper bound of \( P_r \), so we need to establish the interval on which profit function is concave.

**Property 1.** The objective function (1) is concave when

\[
P_{n2} \geq \sqrt{\frac{(d_1 + d_2) P_m P_{nw}^2}{4d_2}}.
\]

(4.2)

**Proof.** See Appendix B.

The above result implies that the demand of remanufactured product \( d_1 \) and \( d_2 \) influences the price of new product during the decline stage. Demand capacity during decline stage which is affected by the length of that period has also contributed in shifting the interval of the concave function.

The optimal retail prices \( P_{n1}^*, P_{n2}^*, \) and \( P_r^* \) are obtained by solving equations from first derivatives conditions:

\[
P_{n1}^* = \left( P_m + P_{nw} \right) / 2
\]

(4.3)

\[
P_r^* = \left( P_{n2} + P_{rw} \right) / 2
\]

(4.4)

\[
- \frac{2}{P_m} \left( d_2 (P_{n2}^*)^3 + d_2 \left( 1 + \frac{P_{nw}}{P_m} \right) + d_3 + d_4 \right) (P_{n2}^*)^2 \\
- \left( d_3 + d_4 \right) (P_{rw})^2 = 0.
\]

(4.5)
The objective function is concave when
\[
\frac{P_{nw}}{2} - P_r + \frac{\sqrt{d_1 + d_2 P_{n}^3 + (d_1 + d_2 P_{m}^3 + d_3 + d_4 P_{nw}^2 + d_2 + d_4)}}{d_1 + d_4} P_{m} > 0. \tag{4.7}
\]

Decision variables:
\( P_c \): price of new product;
\( P_r \): price of remanufactured product.

Parameters:
\( P_{nw} \): wholesale price of new product;
\( P_{rw} \): wholesale price of remanufactured product;
\( P_{n} \): maximum price for new product;
\( d_1 \): total demand for new product within \([0, \mu]\);
\( d_2 \): total demand for new product within \([\mu, t_2]\);
\( d_3 \): total demand for remanufactured product within \([t_2, t_3]\);
\( d_4 \): total demand for remanufactured product within \([t_3, T]\).

The existence of optimal prices \( P_c, P_r \) is shown in Proposition 2, and the condition for obtaining prices that maximize retailer’s profit (4.6) is given in Property 3. The optimal retail prices are given in Proposition 1.

**Proposition 2.** There exists a global extremum for profit maximization problem (4.6) in \( \{P_c, P_r\} | P_{nw} \leq P_n \leq P_m; P_{rw} \leq P_r \leq P_m; P_n \in R, P_r \in R \} \).

**Proof.** See Appendix F.

**Proposition 3.** The objective function (4.6) is concave when
\[
\frac{P_{nw}}{2} - P_r + \frac{\sqrt{d_1 + d_2 P_{n}^3 + (d_1 + d_2 P_{m}^3 + d_3 + d_4 P_{nw}^2 + d_2 + d_4)}}{d_1 + d_4} P_{m} > 0. \tag{4.7}
\]

**Proof.** See Appendix D.

**Proposition 1.** The optimal prices for optimization model (4.6) are \( P_{n}^* \) and \( P_r^* \) where
\[
-2(d_1 + d_2) P_{nw} + (d_1 + d_2 P_{m}^3 + d_2 + d_4 P_{nw}^2 + d_2) \left(1 - \frac{P_r}{P_m}\right) = 0.
\]

**Proof.** See Appendix E.

4.1.2. Collector’s optimization

After retailer decides the optimal retail prices, collector then uses the resulting demand rates as the parameters in the profit optimization model, and the objective function is
\[
\text{Max } \Pi_c = \gamma P^0_c \left( d_1 + d_2 \right) \left(1 - \frac{P_n}{P_m}\right) (P_r - P_c) - c. \tag{4.10}
\]

**Decision variables:**
\( P_c \): acquisition price for used product

**Parameters:**
\( P_r, P_m, d_1, d_2 \) as mentioned earlier,
\( P_{nw} \): transfer price of remanufacturable core from collector to remanufacturer,
\( \gamma \): exponent of the return rate power functions
\( c \): unit collecting cost

**Property 4** shows the existence of optimal price \( P_c \) for collector’s profit function (4.10). First derivative condition is applied to obtain the optimal collecting price as shown in Proposition 2.

**Proposition 4.** There exists a global extremum for collector’s profit function (4.10) in \( \{P_c | 0 \leq P_c \leq P_r; P_c \in R\} \).

**Proof.** See Appendix F.

**Proposition 2.** The collector’s profit function (4.10) attains its maximum in \( \{P_c | 0 \leq P_c \leq P_r; P_c \in R\} \) and the optimal collecting price is
\[
P_{r}^* = \frac{\theta (P_r - c)}{\theta + 1}. \tag{4.11}
\]

**Proof.** See Appendix G.

We assume balanced quantity throughout the supply chain, which is supported by Guide’s work [26]. Collector should only collect as much as the demand of the remanufactured product, which consequently determines transfer price.

**Proposition 3.** The optimal transfer price is
\[
P_{r}^* = c + \frac{\theta + 1}{\theta} \left( (d_1 + d_2 (1 - P_r)P_n) \frac{\theta}{\theta + 1} \right). \tag{4.12}
\]

**Proof.** See Appendix H.

4.1.3. Manufacturer’s optimization

After observing retailer’s and collector’s prices, manufacturer determines the wholesale prices for both new \( P_{nw} \) and remanufactured products \( P_{rw} \) in order to maximize her profit which is expressed in the following objective function:
\[
\text{Max } \Pi_m = (d_1 + d_2) \left(1 - \frac{P_n}{P_m}\right) (P_{nw} - c_{rw} - c_m) + (d_1 + d_4) \left(1 - \frac{P_r}{P_m}\right) (P_{rw} - c_r) \left( \gamma (d_1 + d_2) (1 - P_r/P_m) \right) \tag{4.13}
\]

subject to the optimal prices of the retailer and collector. We apply Lagrange multipliers method, where we define Lagrangian function associated with (4.13) as given in (4.14)
\[
L \left( \frac{P_{nw}}{P_m}, P_{rw}, P_n, P_r, \xi, \psi \right) = \Pi_m + \xi \left( -2(d_1 + d_2) P_{nw} + (d_1 + d_2 P_{m}^3 + d_2 + d_4 P_{nw}^2 + d_2) \left(1 - \frac{P_r}{P_m}\right) \right) + \psi \left( 2P_r - P_n - P_{rw} \right) \tag{4.14}
\]
Decision variables:

- $P_{nw}$: wholesale price of new product
- $P_{rw}$: wholesale price of remanufactured product

Parameters:
- $P_n, P_r, P_m, d_1, d_2, d_3, d_4, P_f, c, \gamma, \theta$ as mentioned earlier
- $c_{rw}$: unit raw material cost for producing new product
- $c_m$: unit manufacturing cost for producing new product
- $c_r$: unit remanufacturing cost for producing remanufactured product

The first order conditions of the Lagrangian are regarded as the necessary conditions for the constrained optimization, and yield a nonlinear system. We treat $P_r$ and $P_m$ as intermediary decision variables in this optimization problem since $P_n$ and $P_r$ from retailer’s optimum pricing decisions are not expressed as explicit functions in $P_{nw}$ and $P_{rw}$, hence the relations are expressed in the constraint functions.

Following Lagrange multiplier theorem [54], if there exist optimal wholesale prices $P_{nw}^*, P_{rw}^*, P_m^*$, $P_r^*$, then they are the solutions of first order conditions for (4.14), which are $\frac{\partial \Pi_m}{\partial P_n} = 0, \frac{\partial \Pi_m}{\partial P_r} = 0, \frac{\partial \Pi_m}{\partial P_m} = 0, \frac{\partial \Pi_m}{\partial P_r} = 0$, and $\frac{\partial \Pi_m}{\partial P_m} = 0$, that yields (4.15) to (4.20) respectively.

4.2. Joint profit optimization

Under the joint profit scenario, all parties aim at maximum total profit along the supply chain. The joint profit function is summation of retailer’s profit, collector’s profit, and manufacturer’s profit. Balanced quantity is also imposed in this model, and for remanufactured product, the quantity of demand is equal to the quantity of returns, which also means collector only collects as much as the demand for remanufactured products. The optimization problem is given by the following expressions:

$$\Pi_j = \Pi_R + \Pi_C + \Pi_M$$

$$= (d_1 + d_2) \left(1 - \frac{P_n}{P_m^*}\right) (P_n - P_{nw})$$

$$+ (d_3 + d_4) \left(1 - \frac{P_r}{P_m^*}\right) (P_r - P_{rw})$$

$$+ \gamma P_r \left(1 - \frac{P_r}{P_m^*}\right) (P_r - c)$$

$$+ (d_1 + d_2) \left(1 - \frac{P_n}{P_m^*}\right) (P_m - c_{nw} - c_m)$$

$$+ (d_3 + d_4) \left(1 - \frac{P_r}{P_m^*}\right) (P_r - c_r - c)$$

\begin{equation}
\times \left(1 - \frac{P_r}{P_m^*}\right) (P_{rw} - P_r - c_r)
\end{equation}

s.t. \((d_3 + d_4) \left(1 - \frac{P_r}{P_m^*}\right) (P_r - c_r - c) + (d_3 + d_4) \times \left(1 - \frac{P_r}{P_m^*}\right) (P_{rw} - P_r - c_r)\) (4.22)

The joint profit function is then simplified to a function of $P_n, P_r, P_m, P_f$, as presented in Eq. (4.22). Considering balanced quantity throughout the supply chain, then the optimization model for joint profit function becomes (4.23)

$$\Pi_j = (d_1 + d_2) \left(1 - \frac{P_n}{P_m^*}\right) (P_n - c_{nw} - c_m)$$

$$+ (d_3 + d_4) \left(1 - \frac{P_r}{P_m^*}\right) (P_r - c_r - c)$$

$$\times \left(1 - \frac{P_r}{P_m^*}\right) (P_{rw} - P_r - c_r)$$

\begin{equation}
\times \left(1 - \frac{P_r}{P_m^*}\right) (P_{rw} - P_r - c_r)
\end{equation}

(4.23)

In finding the optimal prices, we assign first derivatives to zero

$$\frac{\partial \Pi_j}{\partial P_n} = (d_1 + d_2) \left(\frac{P_m - 2P_r + c_{rw} + c_m}{P_m}\right)$$

$$+ (d_3 + d_4) \left(1 - \frac{P_r}{P_m}\right) \left(1 - \frac{P_n}{P_m}\right)^{\gamma - 1}.$$

\begin{equation}
\times \left(1 - \frac{P_r}{P_m}\right) \left(1 - \frac{P_n}{P_m}\right)^{\gamma - 1}.$$

(4.24)

The second order conditions for the Lagrangian function, which reflects the sufficient condition for a maxima, are not practical to be expressed analytically. Therefore, to ensure that the solution is a maxima we apply a numerical procedure to check the values of the function in a close neighborhood of the solution and maximize profit numerically under an optimization search procedure.

4.2. Joint profit optimization
The Circular Economy [55].

Scenario 1 presents demand pattern of product with gradual obsolescence. How using data from numericalexamplein [54], becausethejoint profit model accommodates coordinated pricing policy that ensures higher profit for each party, and making this approach interesting for all members of the supply chain. The impact of demand’s speed of change to the optimal prices depend on the demand volume in respective periods. We can also find an interval of speed of change in demand where the optimal prices are dependent and demand pattern over time is influenced by several parameters, such as speed of change in demand as shown in (3.1) and (3.2). The speed of n change in the demands is determined by parameter λ and η, for new and remanufactured product, respectively. The higher λ and η, the faster demand increases.

5. Numerical example and discussions

In this numerical example, the parameters in demand function are using data from numerical example in [54], because it represents demand pattern of product with gradual obsolescence. However, that study does not consider used product’s return, therefore parameters in return function is taken from numerical example in [29]. As for the cost parameters, we developed the data based on case studies in a report for Ellen MacArthur Foundation, Towards The Circular Economy [55].

New product’s demand capacity parameters are \( U = 1000, D_0 = 90, \lambda = [0.01, 0.05, 0.1, 0.2], \) and remanufactured product’s demand capacity parameters are \( V = 500, D_0 = 50, \eta = [0.01, 0.05, 0.1, 0.2], \) Selling horizon is divided into four time periods where \( t_1 = 1, \mu = 2, t_2 = 3, \) and \( T = 4. \) The unit raw material cost for new product \( c_{uw} = 1500, \) unit manufacturing cost \( c_\text{m} = 1000, \) unit remanufacturing cost \( c_\text{r} = 800, \) and unit collecting cost \( c = 100. \) Maximum price is \( P_m = 12000. \) Return rate parameters are \( \gamma = 0.01, \) and \( \theta = 0.7. \) The decision variables are \( P_n, P_r, P_m, P_w, P_f, \) which represent price of new product, price of remanufactured product, wholesale price of new product, wholesale price of remanufactured product, collection price and transfer price, respectively. Table 2 presents the results.

From the results above, we have shown that joint profit scenario gives a higher total profit rather than optimized individually. It is also interesting to note that the joint profit model accommodates coordinated pricing policy that ensures higher profit for each party, and making this approach interesting for all members of the supply chain. Demand rate in the joint profit scenario is much higher than in the independent one, even though it came from the same demand parameters. In the independent model, with the lack of integrated decision among the three players, the retail prices were set substantially higher than the true optimums. We also observed that Collector profit is much lower than Retailer’s and Manufacturer’s, because Collector only gains from remanufactured product. This result is consistent with Qiaojun’s [29].

The optimization models also show that transfer price can be found by balancing the return rate with the demand of remanufactured product. Under this approach we can determine transfer price that could benefit both manufacturer and collector, and it puts collector at better position rather than the presumed condition that transfer price is negotiated between manufacturer and collector. Since manufacturer is the Stackelberg leader, it is possible that collector would have been in lower bargaining position. Even though this approach might not be interesting for the manufacturer, as it puts limitation to manufacturer’s power, but it actually creates sustainability for the overall closed-loop supply chain. The betterment in collector’s position would be a good motivation to continue collecting used products for remanufacturing, and support environment protection.

Different speeds of change in the demand of new and remanufactured product obviously result in different pricing decisions. However, faster penetration to the market, which is shown by the higher speed of change in demands, does not simply generate higher total profits. It can be seen from the demand function and optimization models that speed of change in demand will influence the sales volumes in each period and subsequently has impacted the optimum pricing decision. As stated in Propositions 1 and 3, the optimal prices depend on the demand volumes in respective periods. We can also find an interval of speed of change in demand where the total profit reaches its highest value. This could lead to a marketing decision where the players should control market penetration such that the speed of change is within the desirable interval.

The impact of demand’s speed of change to the optimal prices

In this paper, demand of short life-cycle product is time-dependent and demand pattern over time is influenced by several parameters, such as speed of change in demand as shown in (3.1) and (3.2). The speed of n change in the demands is determined by parameter λ and η, for new and remanufactured product, respectively. The higher λ and η, the faster demand increases.
Table 2
Comparison between independent and joint profit scenarios.

<table>
<thead>
<tr>
<th>λ, η</th>
<th>Independent scenario</th>
<th>Joint profit scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pn</td>
<td>Pr</td>
</tr>
<tr>
<td>0.01</td>
<td>9,889.78</td>
<td>9,896.80</td>
</tr>
<tr>
<td>0.05</td>
<td>8,318.83</td>
<td>8,346.32</td>
</tr>
<tr>
<td>0.10</td>
<td>7,018.45</td>
<td>6,997.86</td>
</tr>
<tr>
<td>0.20</td>
<td>6,747.80</td>
<td>6,795.85</td>
</tr>
</tbody>
</table>

Table 3
Optimal prices under various speeds of change in demand.

<table>
<thead>
<tr>
<th>λ, η</th>
<th>Pn</th>
<th>Pr</th>
<th>Pnw</th>
<th>Prw</th>
<th>Π_M</th>
<th>Π_R</th>
<th>Π_C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>9270.08</td>
<td>4758.91</td>
<td>6110.48</td>
<td>6115.06</td>
<td>1,279,206.64</td>
<td>1,274,245.91</td>
<td>1,656,492.54</td>
</tr>
<tr>
<td>0.05</td>
<td>4,720.08</td>
<td>4,758.91</td>
<td>4,759.89</td>
<td>4,759.25</td>
<td>1,279,206.64</td>
<td>1,274,245.91</td>
<td>1,656,492.54</td>
</tr>
<tr>
<td>0.10</td>
<td>4,133.21</td>
<td>3,566.64</td>
<td>3,560.95</td>
<td>3,564.83</td>
<td>1,274,245.91</td>
<td>1,274,245.91</td>
<td>1,656,492.54</td>
</tr>
<tr>
<td>0.20</td>
<td>2,979.06</td>
<td>321.15</td>
<td>321.74</td>
<td>321.36</td>
<td>1,269,241.66</td>
<td>1,269,241.66</td>
<td>1,656,492.54</td>
</tr>
</tbody>
</table>

during IMG phases, and decreases during decline phase. Therefore, the accumulation of demand over time would influence the optimal prices, since we consider the whole product life cycle. We vary λ and η to study their impacts on decision variables and the objective functions. Table 3 and Fig. 4 show the results.

As shown in Fig. 4, optimal prices are sensitive to parameters that reflect the speed of change in demand for lower market penetration, but they are robust for higher speed of change in demand. This can be explained by the specific nature of demand pattern. For a short life cycle product with demand pattern as given in (3.1) and (3.2), higher speed of change in demand leads to a condition that is closer to constant demand. Therefore, optimal prices do not change significantly compared to the ones with lower speed of change in demand. This situation is applicable to a product with demand pattern such that in the beginning of product’s introduction phase, the increase in demand tends to be very steep, such as launching a new model of smartphone. After a certain time period, the demand decreases quite rapidly. We argue that the optimal prices are more robust in higher speed, because the effect of speed of change becomes less significant. This means, when speed of change in demand is high such that the sales reach maximum demand in a very short time, then the price setting does not need to be adjusted when there is a small change in the speed. However, when the speed is low, the sales climb up rather slowly to reach maximum demand, so it is important to adjust price setting when there is a change in the speed, to avoid sub-optimal prices.

The managerial insight for this matter can be explained as follows. Under lower speed of change in demand, decision makers need to carefully update the prices to avoid sub-optimality. But when it moves fast to the peak, price updates become less urgent, since the demand has reached its uniform pattern.

6. Conclusion and future research agenda

In this study we have developed pricing decision models for remanufacturing of short life-cycle product. The study fills the gap of remanufacturing literature which to date has been mostly dominated by durable products. For some short life cycle products, remanufacturing is a sensible activity to do, but the speed of collecting and remanufacturing the used products should be quick as the demand for the product is diminishing fast. Here are some conclusions that we obtain from this study:

• Reducing the price of new products during the decline phase does not give better profit for the whole system,
• The total profit obtained from optimizing each player independently is lower than the total profit of the integrated model where we optimize the joint profit for three members in the supply chain, namely manufacturer, retailer and collector. None of the player is worse off by moving from the independent model to the joint profit model, under coordinated pricing policy,
• The total demand is significantly higher under the integrated model. This is understandable because the retail prices are lower for both the new and remanufactured products. The lack of coordination in making the pricing decision has led the independent models to set high retail prices and hence the demand potential is not well exploited,
• Faster penetration to the market, which is shown by the higher speed of change in demands, does not simply generate higher total profits. There exists an interval of speed of change in demand where the total profit reaches its highest value. This could be a support for marketing decision by controlling market penetration such that the speed of change is within the desirable interval,
• When demand penetration is low, small changes in the demand rate affect price settings substantially. However, when demand penetration is high, price decision is robust against the change in demand rate.

Future research may be directed toward development of models that consider different demand processes, multiple objective functions, and the case when balanced quantity is not the case. It may be possible that the collector is not able to collect at the quantity desired by the manufacturer. It is also possible that the manufacturer has a certain capacity constraint where not all demand can be satisfied. In such case it is important to take into account the service level.

**Appendix A.** $D_n(t)$ and $D_r(t)$ are continuous at $\mu$ and $t_3$, respectively

\[
D_n(t) = \begin{cases} 
\frac{U}{1 + ke^{-\lambda t}} & \text{if } t < \mu \\
\frac{U}{\lambda (t - \mu)} & \text{if } t \geq \mu
\end{cases}
\]

\[
D_r(t) = \begin{cases} 
\frac{U}{1 + ke^{-\lambda t}} & \text{if } t < \mu \\
\frac{U}{\lambda (t - \mu)} & \text{if } t \geq \mu
\end{cases}
\]

Therefore $D_n(\mu) = \lim_{t \to \mu^-} D_n(t) = \frac{U}{1 + ke^{-\lambda \mu}} \rightarrow D_n(t)$ is continuous at $t = \mu$.

Similarly,

\[
D_1(t_3) = \left( 1 + \frac{he^{-\lambda(t_3 - t_1)}}{\lambda} \right) \frac{V}{\lambda} = \left( 1 + \frac{he^{-\lambda(t_3 - t_1)}}{\lambda} \right) \frac{V}{\lambda}
\]

\[
D_2(t_3) = \left( \eta V(t_3 - t_1) + \varepsilon \right) \frac{V}{\lambda} = \left( \eta V(t_3 - t_1) + \varepsilon \right) \frac{V}{\lambda}
\]

\[
D_3(t_3) = \left( 1 + \frac{he^{-\lambda(t_3 - t_1)}}{\lambda} \right) \frac{V}{\lambda} = \left( 1 + \frac{he^{-\lambda(t_3 - t_1)}}{\lambda} \right) \frac{V}{\lambda}
\]

Therefore $D_r(\mu) = \lim_{t \to t_3^-} D_r(t) = \frac{V}{1 + ke^{-\lambda(t_3 - t_1)}} \rightarrow D_r(t)$ is continuous at $t = t_3$.

**Appendix B. Proof of Property 1**

\[
\Pi_k = d_1 \left( 1 - \frac{P_{n1}}{P_m} \right) (P_{n1} - P_{nw}) + d_2 \left( 1 - \frac{P_{n2}}{P_m} \right) (P_{n2} - P_{nw})
\]

\[
= d_1 \left( d_3 + d_4 \right) (P_{r} - P_{rw})
\]

\[
\frac{\partial^2 \Pi_k}{\partial P_{n1}^2} = - \frac{2d_1}{P_m} < 0
\]

\[
\frac{\partial^2 \Pi_k}{\partial P_{n1} \partial P_{n2}} = \frac{\partial^2 \Pi_k}{\partial P_{n2} \partial P_{n1}} = \frac{\partial^2 \Pi_k}{\partial P_{n2}^2} = \frac{\partial^2 \Pi_k}{\partial P_{n1}^2} = \frac{2d_1}{P_m} + \frac{2 \left( d_3 + d_4 \right) P_r (P_r - P_{rw})}{P_{n2}^2} > 0; \quad \text{since } P_r \geq P_{rw}
\]
\[
|H| = \left| \begin{array}{ccc}
\frac{\partial^2 \Pi_R}{\partial n^2} & \frac{\partial^2 \Pi_R}{\partial P_n \partial n_1} & \frac{\partial^2 \Pi_R}{\partial P_1 \partial P_n} \\
\frac{\partial^2 \Pi_R}{\partial P_n \partial n_2} & \frac{\partial^2 \Pi_R}{\partial P_n^2} & \frac{\partial^2 \Pi_R}{\partial P_2 \partial P_n} \\
\frac{\partial^2 \Pi_R}{\partial P_1 \partial P_n} & \frac{\partial^2 \Pi_R}{\partial P_2 \partial P_n} & \frac{\partial^2 \Pi_R}{\partial P_1 \partial P_2} \\
\end{array} \right| = \frac{2d_1 (d_3 + d_4)}{P_m P_n^2} \left[ \frac{(d_3 + d_4) P_n}{P_m} - \frac{4d_2^4}{P_m^4} \right] \leq 0
\]

Since \(|H_1| = \frac{\partial^2 \Pi_R}{\partial P_n^2} < 0\); and \(|H_2| = \left( \frac{\partial^2 \Pi_R}{\partial P_n \partial n_1} \right) \left( \frac{\partial^2 \Pi_R}{\partial P_1 \partial P_n} \right) - \left( \frac{\partial^2 \Pi_R}{\partial P_2 \partial P_n} \right)^2 > 0\), then for \(\Pi_R\) to be a concave function, \(|H_1| = |H|\) should be less than or equal to zero. Therefore

\[
\left[ \frac{(d_3 + d_4) P_n}{P_m} - \frac{4d_2^4}{P_m^4} \right] \leq 0
\]
or

\[
P_{n_2} \geq \sqrt{\frac{(d_3 + d_4) P_m P_n^2}{4d_2^4}}.
\]

**Appendix C. Proof of Property 2**

Since \(\Pi_R\) is a two-variable rational function with a rational parameter in the coefficients, then it is discontinuous when \(P_m = 0\) and \(P_n = 0\). However, \(P_m\) is a nonzero parameter, and \(P_n \leq P_m\) positive \(P_n\), therefore \(\Pi_R\) is continuous in \((P_n, P_m)\), \(P_n \leq P_m\), \(P_n \leq P_m, P_n \in \mathbb{R}, P_1 \in \mathbb{R}\). Since \(\Pi_R\) is continuous in a closed interval then it attains global extrema there. □

**Appendix D. Proof of Property 3**

\[
\frac{\partial \Pi_R}{\partial n} = \frac{d_1 + d_2}{P_m} (P_m + P_n - 2P_n) + \frac{d_3 + d_4}{P_m^2} (P_n^2 - P_n P_1 - P_2) < 0
\]

\[
\frac{\partial^2 \Pi_R}{\partial P_n^2} = -2 \frac{d_1 + d_2}{P_m} - \frac{2d_3 + d_4}{P_m^3} P_n P_1 < 0
\]

\[
\frac{\partial^2 \Pi_R}{\partial P_n \partial P_1} = d_3 + d_4 \frac{P_m}{P_m^2} P_n \left( \frac{P_m}{P_m} P_n - P_n + P_1 \right) < 0
\]

\[
\frac{\partial^2 \Pi_R}{\partial P_1^2} = -2 \frac{d_1 + d_2}{P_m} P_1 < 0
\]

\[
\frac{\partial^2 \Pi_R}{\partial P_1 \partial P_n} = \left( \frac{\partial^2 \Pi_R}{\partial P_n \partial P_1} \right)^2 = \left( \frac{\partial^2 \Pi_R}{\partial P_n \partial P_1} \right)^2 = \frac{4 (d_1 + d_2) (d_3 + d_4) - (d_3 + d_4)^2}{P_m P_n^2} (2P_n^2 - P_n P_1) > 0
\]

\[
2P_n - P_1 < \sqrt{\frac{d_1 + d_2}{d_3 + d_4} P_n^2} \frac{P_m}{P_n}.
\]

\[
P_{n_2} = \frac{P_n}{\sqrt{\frac{2}{2}}} > P_n > P_n > 0.
\]

**Appendix E. Proof of Proposition 1**

The critical points for (4.6) are obtained by applying first derivatives condition,

\[
\frac{\partial \Pi_c}{\partial P_c} = \gamma (d_1 + d_2) \left( 1 - \frac{P_n}{P_m} \right) P_c^{-1}
\]

\[
\left[ \theta (P_c^* - c) - (\theta + 1) P_c \right] = 0
\]

\[
\theta (P_c^* - c) - (\theta + 1) P_c = 0
\]

\[
P_c^* = \frac{\theta (P_c - c)}{\theta + 1}
\]

\[
\frac{\partial^2 \Pi_c}{\partial P_c^2} = \gamma (d_1 + d_2) \left( 1 - \frac{P_n}{P_m} \right) \left( \theta - 1 \right) P_c^{\theta - 2}
\]

\[
\times \left[ \theta (P_c^* - c) - (\theta + 1) P_c \right]
\]

\[
- (\theta + 1) \gamma (d_1 + d_2) \left( 1 - \frac{P_n}{P_m} \right) P_c^{\theta - 1}.
\]
Since $\theta \left( P_f - c \right) - \left( \theta + 1 \right) P_c = 0$ at the critical point, then
\[
\frac{\partial^2 \Pi_c}{\partial P_c^2} = -\left( \theta + 1 \right) \left( \gamma \left( d_1 + d_2 \right) - \frac{P_n}{P_m} \right) \frac{\left( d_1 + d_2 \right)}{\left( 1 - \frac{P_n}{P_m} \right)} < 0.
\]
Therefore $P_c^*$ is a local maximum point. Following Property 5, we check the boundary points and compare them with the local maximum. Since $\Pi_c\left(0\right) = 0$, $\Pi_c\left(P_c^*\right) < 0$, and $\Pi_c\left(P_c^*\right) > 0$ then $P_c^*$ is the global maximum within interval $\left[ P_f, P_c^* \right] \subset \mathbb{R}$. 

Appendix H. Proof of Proposition 3

Balanced quantity for demand of remanufactured product and acquired used product is satisfied when
\[
(1 - \frac{P_n}{P_m}) \left( d_1 + d_2 \right) = \gamma \left( \frac{P_n}{P_m} \right) \left( d_1 + d_2 \right) \left( 1 - \frac{P_n}{P_m} \right).
\]
Since optimal collecting price is given as (4.11), then the respective transfer price is
\[
(1 - \frac{P_n}{P_m}) \left( d_1 + d_2 \right) = \gamma \left( d_1 + d_2 \right) \left( 1 - \frac{P_n}{P_m} \right),
\]
and
\[
P_f^* = c + \frac{\theta + 1}{\theta} \left( (d_1 + d_2) \left( 1 - \frac{P_n}{P_m} \right) \gamma \left( 1 - \frac{P_n}{P_m} \right) \right). \quad \Box
\]

References