Pricing decision model for new and remanufactured short-life cycle products with time-dependent demand

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A B S T R A C T

In this study we develop a model that optimizes the prices for new and remanufactured short life-cycle products where demands are time-dependent and price sensitive. While there has been very few published works that attempt to model remanufacturing decisions for products with short life cycle, we believe that there are many situations where remanufacturing short life cycle products is rewarding economically as well as environmentally. The system that we model consists of a retailer, a manufacturer, and a collector of used product from the end customers. Two different scenarios are evaluated for the system. The first is an independent situation where each party attempts to maximize his own total profit and the second is the Stackelberg leader in the independently optimized scenario, while in the other the intermediate prices are determined by coordinated pricing policy. The results suggest that (i) reducing the price of new products during the decline phase does not give better profit for the whole system, (ii) the total profit obtained from optimizing each player is lower than the total profit of the integrated model, and (iii) speed of change in demand influences the robustness of the prices as well as the total profit gained.

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1. Introduction

Technology-based product has shorter life cycle due to rapid innovation and development in science and technology, as well as customer behavior in pursuing latest innovation and style. Lebreton and Tannas [1] pointed out that technology-based commodities such as mobile phones and computers have shorter innovation cycle so that the previous generation becomes obsolete faster, either functionally and psychologically. Similarly, Hsueh [2] also argued that product life cycle in electronic industry is shorter than before, due to technology advances, and as a result, an outdated product could reach its end-of-use even it is still in a good condition. Shorter life-cycle has negative contribution toward sustainability, since there is an increase in product disposal. Customers want newer products and discard the old ones, and these preferences would exhaust landfill space in shorter time. In addition, there are more natural resources and energy used to create new products than actually needed, due to unnecessary increased obsolescence. To make it worse, electronic products are prominent as the ones with shorter and shorter life cycle, while the wastes are toxic and not environmentally friendly. There are many attempts made in developed countries to control electronic wastes such as Waste of Electric and Electronic Equipment (WEEE) directives, implemented in most European countries since 2003, RoHS in United States, 2003, and Extended Producer Responsibility (EPR) issued by OECD in 1984. However, these regulations pose as burdens to the industries when implemented only for conformity, because there are additional costs for handling e-wastes and increased material cost for avoiding or minimizing toxic materials.

Several strategies have been introduced to mitigate products disposal and wastes, such as life cycle approach, regulation and society approach. One aspect of life cycle approach is dealing with products at their end-of-use. According to de Brito & Beldicer [3], there are situations where customer has the opportunity to return a product at a certain life stage, which can be referred to leasing cases and returnable containers, and is called end-of-use return. Hsueh [2] considered a different kind of return, where a product may be returned because it has become outdated, and the customer...
wants to buy a new product. Herold [4] proposed alternatives to end-of-use products which are reprocessing, collect-and-sell, and collect-and-dispose. Remanufacturing is one option to manage products at their end-of-use which offers opportunity for complying with regulation while maintaining profitability [5-7]. Remanufacturing is a process of transforming used product into “like-new” condition, so there is a process of recapitalizing the value added to the material during manufacturing stage [8,9]. The idea of remanufacturing used products has gained much attention recently for both economic and environmental reasons. As suggested by Gray and Charter [9], remanufacturing can reduce production cost, the use of energy and materials.

There are numerous studies on remanufacturing. However, most of the published works on remanufacturing are focused on durable or semi-durable products. Very little attention has been made to study how remanufacturing may be applied to products with short life cycle. In some developing countries like Indonesia, there is a large segment of society that could become potential market for remanufactured short-life cycle products like mobile phones, computers and digital cameras.

In remanufacturing practice, there are three main activities, namely product return management, remanufacturing operations, and market development for remanufactured product [10]. In terms of marketing strategy, there are general concerns that remanufactured product would cannibalize the sales of new product. However, Acsu et al. [11] concluded that remanufacturing does not always cannibalize the sales of new products. He proposed that managers who understand the composition of their markets, and use the proper pricing strategy should be able to create additional profit. Therefore, pricing decision is an important task in an effort to gain economic benefit from remanufacturing practices.

There are several studies that focused on pricing of remanufactured products, but many of them have not considered the whole supply chain, and also only a very few concern about obsolescence of short life cycle products. Our study will be focused on pricing decisions in a closed loop supply chain involving manufacturer, retailer and collector of used products (cores), where customers have the option to purchase new or remanufactured short life cycle products in the same market channel. We consider a monopoly of a single item with no constraint on the quantity of remanufacturable cores throughout the selling horizon.

2. Literature review

Remanufacturing of mobile phones and electronic products has been recognized as an important practice in the United States, and as a potential in China and India. Higo [12] claimed that product life cycle has significantly shortened by rapid technological advancement, and coupled with fashionable design that attracts frequent purchases of new products, has generated pressure on opportunities for reverse logistics. Franke et al. [13] suggested that remanufacturing of durable high-value products such as automobile engine, aircraft equipment, and machine tools, has been extended to a large number of consumer goods with short life cycle and relatively low values, like mobile phones and computers. He also quoted market studies by Marcusen [14] and Directive 2002/96/EC which revealed that there is a significant potential for mobile phone remanufacturing due to the large supply market of the used mobile phones in Europe and the high demand market in Asia and Latin America.

Neto and Bloemhof-Ruwaard [15] found that remanufacturing significantly reduces the amount of energy used in the product life cycle, even though the effectiveness of remanufacturing is very sensitive to the life span of the second life of the product. They also proposed that the period of the life cycle in which the product is returned to recovery, the quality of the product (high-end versus low-end), the easiness to remanufacture and the recovery costs can affect whether or not remanufacturing is more eco-efficient than manufacturing. Rathore et al. [16] studied the case of remanufacturing mobile handsets in India. They found that used phone market is very important, even though with a lack of government regulation for e-wastes. It is also observed that there is a negative user perception of second hand goods and that the process of remanufacturing has not been able to capture much required attention from its stakeholders. Wang et al. [17] showed that the mobile phone market in China is growing rapidly. The number of mobile accounts is 565.22 million in February 2008 according to a report from Ministry of Information Industry of the People’s Republic of China. The above mentioned studies have affirmed the proposition that there is a high potential for remanufacturing short life cycle products.

Motives for deploying reverse chain can be for profitability (or cost minimization) or for sustainability (environmental impact mitigation), which either could be driven by regulation and/or morale. In our research, the underlying motive considered would be focused on profitability, which seems to be the suitable motive applied to industries in a situation with the absence of environment protection regulation, like in most of the developing countries. There are numerous studies that investigated the factors that influence decision to remanufacture as well as the factors for successful remanufacturing. We categorized the factors into four aspects, namely product characteristics, demand-related factors, process-related factors, and supply-related factors.

The first aspect, product characteristics of short life cycle products, consist of (1) innovation rate (fast vs. slow) as an extension to technology factor [8,18-20]; (2) residence time [21]; (3) product residual value [22]; (4) qualitative obsolescence, as an extension to product characteristics [23,24].

Second, demand-related factors, consist of (1) market size or existence of the demand, [18,19,24]; (2) market channel, which is about selling remanufactured products using the same channel of the new product, or differentiated [8,18-21,24-27]; (3) pricing of new and remanufactured products, with demand as a function of price [28,32,33,30,18,19,26]; (4) existence of green segment, [23,31].

Supply-related factors can be described by (1) acquisition price and (2) source of return, whether it is limited and then pose as a constraint, or unlimited. These factors were studied in [8,18,19,23,24,32].

The last factors, which are process-related, consist of (1) remanufacturing technology availability [8,18,32]; (2) remanufacturing cost, [8,18,19,21,22,32]; (3) reverse flow structure readiness [8,20,24,32-34].

There are several studies that discuss pricing strategies involving remanufactured products, obsolescence, and nonlinear demand function. However, none has considered the situation that we address in this paper. Table 1 shows the review result and where our proposed model stands.

3. Problem description

A closed-loop supply chain consists of three members, which are manufacturer, retailer, and collector, as depicted in Fig. 1. The closed-loop is initiated by production of new product, which is sold at a wholesale price $P_m$ to the retailer. The new product is then released to the market at a retail price $P_r$, for the period when product life-cycle is within introduction-growth-maturity (IMG) phases, or during increasing and stable phases. When the new product has reached its decline phase, retailer starts to apply different pricing, $P_m$. In the model development, the price is differentiated between IMG phases and decline phase to study the impact of this differentiated pricing. Kotler & Armstrong [52] suggest that reduced price during decline phase could increase the quantity of demanded goods. An example of short life-cycle product where the new product reaches its decline phase in a short time is, Samsung Galaxy Tab 10.1 that was released on second quarter
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</tr>
<tr>
<td>Bakal and Akcali (2006) [36]</td>
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<td>Mitra (2007) [39]</td>
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<td>Wu (2012a) [47]</td>
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<td>Yes (Pr ≠ Pr)</td>
<td><em>Two period</em></td>
<td>Linear in price</td>
<td>Prices <em>New</em> Reman</td>
<td>Max profit</td>
<td>No</td>
</tr>
<tr>
<td>Chen &amp; Chang (2013) [49]</td>
<td>Manufacturer</td>
<td>Yes (Pr ≠ Pr)</td>
<td><em>Static</em> 2-period <em>Multi periods over life-cycle</em></td>
<td><em>Linear in price, with substitutable coefficient</em> Dynamic (over-time)</td>
<td>Price of <em>New</em> Reman for each period</td>
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<td>No</td>
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<td>Xiong et al. (2013) [51]</td>
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<td>Yes (Pr ≠ Pr)</td>
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<td>Function of time and price</td>
<td>Price of <em>Retail Wholesale</em> Collecting</td>
<td>Max profit</td>
<td>Yes</td>
</tr>
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Note: Pr = price of new product, Pr = price of remanufactured product.

Dn = demand of new product, Dr = demand of remanufactured product.

![Diagram](image)

**Fig. 1.** Framework of the closed-loop pricing model.

2011. According to trial documents of Apple Inc. vs. Samsung Electronics Co. Ltd [53], the sales of that product is entering decline phase starting on first quarter 2012.

After a certain period of time, some products reach their end-of-use and become the objects of used products collection. The used product would be acquired by collector under certain acquisition prices, P_{ac} and P_{se}, for product originated from IMG phases and decline phase, respectively. The collected product is then transferred to manufacturer at price P_{r}, as the input for remanufacturing process. The remanufactured product is sold to retailer at wholesale price P_{w}, and released to the market at retail price P_{r1}.

The product considered in this model is single item, short life-cycle, with obsolescence effect after a certain period. Demand functions are time-dependent functions which represent the short life-cycle pattern along the entire phases of product life-cycle, both for new and remanufactured products; and linear in price.

There are four periods considered in this model, as depicted in Fig. 2. In the first period [0, t1], only new product is offered to the market, while in second [t1, tμ] and third period [tμ, t3] both new and remanufactured products are offered. The difference between second and third period is on the segments of life-cycle phases for both types. During second period, both new and remanufactured products are at IMG phases. In the third period, the new product has entered the decline phase while remanufactured product has not. In the fourth period [t3, T], manufacturer has stopped producing new product and only offers remanufactured product which is assumed to be on the decline phase.

The market demand capacity is adopted from Wang and Tung [54] and extended to cover the obsolescence period, where demand decreases significantly. The demand patterns are constructed for both new and remanufactured product and the governing functions are formulated as follows:

\[
D_{n}(t) = \begin{cases} 
\frac{D_{n1}(t)}{U/ (1 + e^{-\lambda U})}; & 0 \leq t \leq \mu \\
\frac{D_{n2}(t)}{U/ (1 + e^{-\lambda U(t - \mu)})}; & \mu \leq t \leq t_{3} 
\end{cases}
\]

where \( \delta = 1 + e^{-\lambda U} \)

\[
D_{r}(t) = \begin{cases} 
\frac{D_{r1}(t)}{V/ (1 + he^{-\eta U - \xi})}; & 0 \leq t \leq t_{3} \\
\frac{D_{r2}(t)}{V/ (h e^{\eta U - \xi})}; & t_{3} \leq t \leq T 
\end{cases}
\]

where \( h = V/D_{r0} - 1 \)

\[
\delta = 1 + he^{-\eta U - \xi} \]

where \( D_{n}(t) \) and \( D_{r}(t) \) are demand pattern for new and remanufactured products, respectively, as seen in Fig. 2. U is a parameter representing the maximum possible demand for new product, \( \mu \) is the time when the demand reaches its peak, i.e., at U level. \( D_{n} \) is the demand at the beginning of the life-cycle (when \( t = 0 \)), and \( \lambda \) is the speed of change in the demand as a function of time. A parallel definition is applicable for \( V_{1}, V_{2}, D_{n0}, \) and \( h_{r} \) respectively for the remanufactured products. It is obvious that \( D_{n}(t) \) and \( D_{r}(t) \) are continuous at \( \mu \) and \( t_{3} \), respectively, as shown in Appendix A.

The new products are sold at retail price \( P_{r1} \) during [0, \( \mu \)], and \( P_{r2} \) during [\( \mu, t_{3} \)]. Since demand function is also linear in price, there is a maximum price \( P_{r0} \) (known and fixed) as the upper limit,
at which demand would be zero. Remanufactured products are sold at retail price $P_r$, during $[t_1, T]$, and the maximum price is $P_m$, since customer would choose to buy new product rather than remanufactured one when the remanufactured product price is as high as $P_m$.

Demand of new product during $[0, \mu]$:

\[ D_{n1}(t) = D_{n1}(t)(1 - P_{n1}/P_m) \]  

Demand of new product during $[\mu, t_2]$:

\[ D_{n2}(t) = D_{n2}(t)(1 - P_{n2}/P_m) \]  

Demand of remanufactured product during $[t_1, t_2]$:

\[ D_{r1}(t) = D_{r1}(t)(1 - P_r/P_{n1}) \]  

Demand of remanufactured product during $[t_3, T]$:

\[ D_{r2}(t) = D_{r2}(t)(1 - P_r/P_{n2}) \]  

Fig. 3 illustrates the demand of new product for the period of $[0, \mu]$.

The demand function information is shared to all members of the supply chain.

- Manufacturer decides the wholesales to new product ($P_{n1}$) and remanufactured product ($P_{n2}$) retailer determines the retail prices ($P_{r1}$, $P_{r2}$, $P_r$), while collector determines collecting prices ($P_{r1}$ and $P_{r2}$) cores collected from end-of-use product within the periods of $[0, \mu]$ and $[\mu, t_2]$, respectively. Since the product has short life cycle, remanufacturing process is only applied to cores originated from new products.

- Return rate ($r$) is an increasing function of the collecting price. We use the return rate function proposed by Qiao et al. [29], which was a result of their survey that employs a power function.

- The return rate $r$ is defined on $[t_1, T]$ and depends on $P_r$ as follows:

\[ r = \gamma P_r^\eta \]  

where $\gamma$ are positive constant coefficients, and $\eta \in [0, 1]$ are exponent of the power functions, which determine curve’s steepness. It is assumed that collector only accepts cores with a certain quality grade, and all collected cores will be remanufactured.

- Since our research is focusing on pricing decision, we do not make an attempt to show detailed derivation of production and operational costs, and instead treat these costs as given parameter, which consist of unit raw material cost for new product ($c_{rw}$), unit manufacturing cost ($c_m$), unit remanufacturing cost ($c_r$), and unit collecting cost ($c_c$). The objective of the proposed model is to find the optimal prices that maximize profit, and as we investigate two scenarios, (1) maximize profit independently, and (2) maximize joint profit along the supply chain.

4. Optimization

4.1. Independently optimized profit

4.1.1. Retailer’s optimization

In this scenario, manufacturer makes the first move by releasing initial wholesale prices ($P_{n1}$, $P_{n2}$). Retailer then optimizes the retail prices ($P_{r1}$, $P_{r2}$, and $P_r$). The profit function can be formulated as follows:

\[ \Pi_k = \int_0^\mu \left( \frac{U}{1 + k e^{-\alpha t}} \left( 1 - \frac{P_{n1}}{P_m} \right) \left( P_{r1} - P_{n1} \right) \right) dt 
+ \int_\mu^T \left( \frac{U}{1 + k e^{-\alpha t}} \left( 1 - \frac{P_{n2}}{P_m} \right) \left( P_{r2} - P_{n2} \right) \right) dt 
+ \int_{t_1}^T \frac{V}{1 + h e^{-\alpha (T-t)}} \left( 1 - \frac{P_{r1}}{P_{n1}} \right) \left( P_r - P_{r1} \right) dt 
+ \int_{t_3}^T \frac{V}{1 + h e^{-\alpha (T-t)}} \left( 1 - \frac{P_{r2}}{P_{n2}} \right) \left( P_r - P_{r2} \right) dt 
+ \left( \frac{d_3 + d_4}{4} \right) \left( 1 - \frac{P_{r1}}{P_{n1}} \right) (P_{r1} - P_{n1}) 
+ \left( \frac{d_3 + d_4}{4} \right) \left( 1 - \frac{P_{r2}}{P_{n2}} \right) (P_{r2} - P_{n2}) \]  

where

\[ d_1 = \frac{1}{k} \ln \left( \frac{\delta}{(1 + \kappa) e^{-\alpha \mu}} \right) \]  

\[ d_2 = \frac{1}{\alpha} \ln \left( \frac{\delta}{(1 + \kappa) e^{-\alpha \mu}} \right) \]  

\[ d_3 = \frac{1}{\eta} \ln \left( \frac{\zeta}{(1 + \zeta) e^{-\alpha (T-t_3)}} \right) \]  

\[ d_4 = \frac{1}{\eta} \ln \left( \frac{\zeta}{(1 + \zeta) e^{-\alpha (T-t_3)}} \right) \]  

The objective function is to maximize profit (4.1), and consequently it needs to satisfy the first derivative conditions $\partial \Pi_k / \partial P_{n1} = 0$, $\partial \Pi_k / \partial P_{n2} = 0$, and $\partial \Pi_k / \partial P_r = 0$. The profit function (4.1) is not always concave along the considered interval, because $P_{n1}$ took a hyperbolic form as a result of being the upper bound of $P_r$, so we need to establish the interval on which profit function is concave.

Property 1. The objective function (1) is concave when

\[ P_{n2} > \sqrt{\frac{(d_3 + d_4) P_{n1} P_{n2}}{4d_2}} \]  

Proof. See Appendix 14.14

The above result implies that the demand of remanufactured product ($d_3$ and $d_4$) influences the price of new product during the decline stage. Demand capacity during decline stage which is affected by the length of that period has also contributed in shifting the interval of the concave function.

The optimal retail prices ($P_{r1}^*$, $P_{r2}^*$, and $P_r^*$) are obtained by solving equations from first derivatives conditions:

\[ P_{r1}^* = (P_{n1} + P_{n2}) / 2 \]  

\[ P_{r2}^* = (P_{n2} + P_{n1}) / 2 \]  

\[ \frac{2}{P_{n1}} d_2 (P_{r2}^*)^3 + \left( d_2 \left( 1 + \frac{P_{n1}}{P_{n2}} \right) + \frac{d_3 + d_4}{4} \right) (P_{n2}^*)^2 
- \frac{(d_3 + d_4) (P_{n1})^2}{4} = 0 \]  

(4.5)
Following Property 1, Eq. (4.2) becomes the lower bound of $P_{R}$, It is expected that $P_{w}^{*}$ is lower than $P_{R}^{*}$, to increase demand rate at the decline stage, however, the model allows $P_{w}^{*}$ to attain higher value than $P_{w}^{*}$, which in turns is not attractive for customers. Our investigation showed that $P_{R}^{*}$ has a tendency to attain higher value than $P_{w}^{*}$, which is also consistent with Ferrer and Swaminathan’s finding [37]. However, this is not an expected result since higher price during decline stage is not attractive for customers and might not be able to improve the demand rate. Therefore, we set a common price for new product, $P_{R} = P_{R}^{*}$. Retailer’s optimization model becomes

$$
\max_{P_{R}} \Pi_{R} = (d_{1} + d_{2}) \left( 1 - \frac{P_{R}}{P_{w}} \right) \left( P_{R} - P_{w} \right) + (d_{3} + d_{4}) \left( 1 - \frac{P_{R}}{P_{w}} \right) \left( P_{R} - P_{r} \right).
$$

(4.6)

Decision variables:

- $P_{R}$: price of new product
- $P_{R}^{*}$: price of remanufactured product

Parameters:

- $P_{w}$: wholesale price of new product
- $P_{w}^{*}$: wholesale price of remanufactured product
- $P_{R}$: maximum price for new product
- $d_{1}$: total demand for new product within $[0, \mu]$
- $d_{2}$: total demand for new product within $[\mu, t_{2}]$
- $d_{3}$: total demand for remanufactured product within $[t_{1}, t_{2}]$
- $d_{4}$: total demand for remanufactured product within $[t_{2}, t_{1}]$

The existence of optimal prices $P_{R}$, $P_{R}^{*}$, is shown in Property 2, and the condition for obtaining prices that maximize retailer’s profit (4.8) is given in Property 2. The optimal retail prices are given in Proposition 1.

**Property 2.** There exists global extrema for profit maximization problem (4.8) in $(P_{R}, P_{R}^{*})$ such that $P_{R} \leq P_{R} \leq P_{w}$; $P_{R} \leq P_{R} \leq P_{w}$; $P_{w} \in R, P_{R} \in R$. 

**Proof.** See Appendix C.

**Property 3.** The objective function (4.6) is concave when

$$\frac{P_{w} - P_{R}}{2} + \frac{\sqrt{(d_{1} + d_{2}) P_{R}^{3}}}{d_{3} + d_{4}} > 0.
$$

(4.7)

**Proof.** See Appendix D.

**Proposition 1.** The optimal prices for optimization model (4.6) are $P_{R}^{*}$ and $P_{R}^{*}$ where

$$
\frac{-2(d_{1} + d_{2}) P_{R}^{3}}{P_{w}^{n}} \left( d_{1} + d_{2} \right) (P_{w} - P_{w}) + d_{3} + d_{4} \left( 1 - \frac{P_{R}}{P_{w}} \right) \left( P_{R} - P_{w} \right) - d_{3} + d_{4} \left( 1 - \frac{P_{R}}{P_{w}} \right) \left( P_{R} - P_{w} \right) = 0
$$

(4.8)

$$
\frac{P_{R}^{*}}{2} + P_{R}^{*} = 0.
$$

(4.9)

**Proof.** See Appendix E.

From Proposition 1, we can observe that the optimal prices are not only determined by wholesale prices given by the manufacturer, but also by the demand pattern imposed in $d_{1}$, $d_{2}$, $d_{3}$, and $d_{4}$, which confirms the influence of demand patterns to the optimal retail prices.

4.1.2. Collector's optimization

After retailer decides the optimal retail prices, collector then uses the resulting demand rate as the parameters in the profit optimization model, and the objective function is

$$
\max_{P_{C}} \Pi_{C} = \gamma P_{C}^{\alpha} \left( d_{1} + d_{2} \right) \left( 1 - \frac{P_{C}}{P_{w}} \right) \left( P_{C} - P_{C} - c \right).
$$

(4.10)

Decision variables:

- $P_{C}$: acquisition price for used product

Parameters:

- $P_{w}$, $P_{w}$, $d_{1}$, $d_{2}$ as mentioned earlier
- $P_{C}$: transfer price of remanufacturable core from collector to remanufacturer
- $\gamma$: constant coefficient of the linear rate
- $\theta$: exponent of the return rate power functions
- $c$: unit collecting cost

**Property 4** shows the existence of optimal price $P_{C}$ for collector’s profit function (4.10). First derivative condition is applied to obtain the optimal collecting price as shown in Proposition 2.

**Property 4.** There exists a global extrema for collector’s profit function (4.10) in $[P_{C}, 0 \leq P_{C} \leq P_{C}, P_{C} \in R]$. 

**Proof.** See Appendix F.

**Proposition 2.** The collector’s profit function (4.10) attains its maximum in $[P_{C}, 0 \leq P_{C} \leq P_{C}, P_{C} \in R]$ and the optimal collecting price is

$$
P_{C}^{*} = \frac{\theta (P_{C} - c)}{(\theta + 1)}.
$$

(4.11)

**Proof.** See Appendix G.

We assume balanced quantity throughout the supply chain, which is supported by Guide’s work [26]. Collector should only collect as much as the demand of the remanufactured product, which consequently determines transfer price.

**Proposition 3.** The optimal transfer price is

$$
P_{C} = \frac{\theta + 1}{\theta} \left( \frac{(d_{1} + d_{2}) (1 - P_{R}/P_{w})}{\gamma (d_{1} + d_{2})} \right)^{\frac{1}{2}}
$$

(4.12)

**Proof.** See Appendix H.

4.1.3. Manufacturer’s optimization

After observing retailer’s and collector’s prices, manufacturer determines the wholesale prices for both new $(P_{w})$ and remanufactured products $(P_{w})$ in order to maximize her profit which is expressed in the following objective function:

$$
\max_{P_{w}, P_{w}} \Pi_{M} = \left( d_{1} + d_{2} \right) \left( 1 - \frac{P_{w}}{P_{w}} \right) \left( P_{w} - P_{w} - c_{w} \right)
$$

$$
+ (d_{3} + d_{4}) \left( 1 - \frac{P_{w}}{P_{w}} \right) \left( P_{w} - P_{w} \right)
$$

$$
(4.8)
$$

$$(P_{w} - c_{w} - c) \left( \frac{P_{w} - c_{w} - c}{\theta + 1} \right) \left( \frac{d_{1} + d_{2}}{\gamma (d_{1} + d_{2})} \right)^{\frac{1}{2}}
$$

(4.13)

subject to the optimal prices of the retailer and collector. We apply Lagrange multiplier method, where we define Lagrangian function associated with (4.13) as given in (4.14)

$$
\mathbf{L} \left( P_{w}, P_{w}, P_{w}, P_{w}, \xi, \psi \right) = \Pi_{M} + \xi \left( -2(d_{1} + d_{2}) P_{w}^{3} \right)
$$

$$
+ \left( d_{1} + d_{2} \right) \left( P_{w} - P_{w} \right) \left( 1 - \frac{P_{w}}{P_{w}} \right) \left( d_{1} + d_{4} \right) \left( 1 - \frac{P_{w}}{P_{w}} \right) \left( P_{w} - P_{w} \right) \left( 1 - \frac{P_{w}}{P_{w}} \right)
$$

$$
+ \psi (2P_{w} - P_{w} - P_{w})
$$

(4.14)

where $\xi, \psi$ are the multipliers.
Decision variables:
- \( P_{nw} \): wholesale price of new product
- \( P_{rw} \): wholesale price of remanufactured product

Parameters:
- \( P_n, P_r, P_m, d_1, d_2, d_3, d_4, P_f, c, \gamma, \theta \) as mentioned earlier
- \( c_{M} \): unit raw material cost for producing new product
- \( c_{rm} \): unit manufacturing cost for producing new product
- \( c_{rf} \): unit remanufacturing cost for producing remanufactured product

The first order conditions of the Lagrangian are regarded as the necessary conditions for the constrained optimization, and yield a nonlinear system. We treat \( P_n \) and \( P_r \) as intermediary decision variables in this optimization problem since \( P_n \) and \( P_r \) from retailer's optimum pricing decisions are not expressed as explicit functions in \( P_{nw} \) and \( P_{rw} \), hence the relations are expressed in the constraint functions.

Following Lagrange multiplier theorem [54], if there exist optimal wholesale prices \( P_{nw}^*, P_{rw}^* \), \( P_r^* \), \( P_m^* \), then they are the solutions of first order conditions for (4.14), which are \( \frac{\partial L}{\partial P_n} = 0 \), \( \frac{\partial L}{\partial P_r} = 0 \), \( \frac{\partial L}{\partial P_m} = 0 \), \( \frac{\partial L}{\partial \theta} = 0 \), and \( \frac{\partial L}{\partial \gamma} = 0 \), that yields (4.15) to (4.20) respectively:

\[
\begin{align*}
\frac{1}{P_m} (d_1 + d_2) (P_{nw} - c_{uw} - c_m) - (d_1 + d_2) \left( 1 - \frac{P_r}{P_n} \right) \\
\times (P_{nw} - c_r - c) + \left( \theta + \frac{1}{\theta} \right) \left( d_1 + d_2 \right) \left( 1 - \frac{P_r}{P_n} \right)^{1/\theta} \\
\times \left( \frac{\theta + 1}{\theta^2} \right) \left( d_1 + d_2 \right) \left( 1 - \frac{P_r}{P_n} \right)^{1/\theta - 1} \\
\times \left( \theta - 1 \right) \left( 1 - \frac{P_r}{P_n} \right) \frac{P_f}{P_m} - (\theta - 3) \\
\times \left( \frac{P_n P_r}{P_m} + \frac{\theta P_n}{P_m} - \frac{P_n P_r}{P_m} \right) \\
+ 6P_f\frac{\theta}{\theta} \left( d_1 + d_2 + d_4 \right) \left( P_{nw} - c_{uw} - c_m \right) + (d_3 + d_4) \\
- \psi = 0
\end{align*}
\]

(4.15)

\[
\begin{align*}
\frac{d_1 + d_2}{P_n} \left[ P_{nw} - c_r - c - \frac{\theta + 1}{\theta} \left( d_1 + d_2 \right) \left( 1 - \frac{P_r}{P_n} \right) \right] \\
+ (d_1 + d_2) \left( 1 - \frac{P_r}{P_n} \right) \frac{\theta + 1}{\theta^2} \\
\times \left( \theta - 1 \right) \left( 1 - \frac{P_r}{P_n} \right) \frac{P_f}{P_m} - (\theta - 3) \\
\times \left( \frac{P_n P_r}{P_m} + \frac{\theta P_n}{P_m} - \frac{P_n P_r}{P_m} \right) \\
+ 6P_f\frac{\theta}{\theta} \left( d_1 + d_2 + d_4 \right) \left( P_{nw} - c_{uw} - c_m \right) + (d_3 + d_4) \\
- \psi = 0
\end{align*}
\]

(4.16)

\[
\begin{align*}
P_m - P_n - \epsilon P_n^2 = 0
\end{align*}
\]

(4.17)

\[
\begin{align*}
\left( 1 - \frac{P_n}{P_m} \right) - \frac{\psi P_{nw} - P_f}{2} - \psi = 0
\end{align*}
\]

(4.18)

\[
\begin{align*}
-2(d_1 + d_2)P_m + (d_1 + d_2) \left( P_{nw} - c_{uw} \right) + (d_3 + d_4) \\
- \frac{d_1 + d_4}{4} P_m^2 = 0
\end{align*}
\]

(4.19)

\[
\begin{align*}
2P_m - P_n - P_{nw} = 0.
\end{align*}
\]

(4.20)

The second order conditions for the Lagrangian function, which reflects the sufficient condition for a maxima, are not practical to be expressed analytically. Therefore, to ensure that the solution is a maxima we apply a numerical procedure to check the values of the function in a close neighborhood of the solution and maximize profit numerically under an optimization search procedure.

4.2. Joint profit optimization

Under the joint profit scenario, all parties aim at maximum total profit along the supply chain. The joint profit function is summation of retailer’s profit, collector’s profit, and manufacturer’s profit. Balanced quantity is also imposed in this model, and for remanufactured product, the quantity of demand is equal to the quantity of returns, which also means collector only collects as much as the demand for remanufactured products. The optimization problem is given by the following expressions:

\[
\begin{align*}
\text{Max } \Pi_J = \Pi_R + \Pi_C + \Pi_M \\
= (d_1 + d_2) \left( 1 - \frac{P_r}{P_n} \right) \left( P_n - P_{nw} \right) \\
+ (d_1 + d_4) \left( 1 - \frac{P_r}{P_n} \right) \left( P_r - P_{rw} \right) \\
+ \gamma P_f \left( d_1 + d_2 \right) \left( 1 - \frac{P_n}{P_m} \right) \left( P_f - P_c - c \right) \\
+ (d_1 + d_2) \left( 1 - \frac{P_n}{P_m} \right) \left( P_{nw} - c_{uw} - c_m \right) + (d_3 + d_4) \\
\times \left( 1 - \frac{P_r}{P_n} \right) \left( P_{rw} - P_f - c \right)
\end{align*}
\]

(4.21)

\[
\begin{align*}
\text{s.t. } (d_1 + d_4) \left( 1 - \frac{P_n}{P_m} \right) = \gamma P_f \left( d_1 + d_2 \right) \left( 1 - \frac{P_n}{P_m} \right)
\end{align*}
\]

(4.22)

The joint profit function is then simplified to a function of \( P_n \), \( P_r \), and \( P_m \) as presented in Eq. (4.22). Considering balanced quantity throughout the supply chain, then the optimization model for joint profit function becomes (4.23):

\[
\begin{align*}
\Pi_J = (d_1 + d_2) \left( 1 - \frac{P_r}{P_n} \right) \left( P_n - c_{uw} - c_m \right) + (d_3 + d_4) \\
\times \left( 1 - \frac{P_n}{P_m} \right) \left( P_r - c_r - P_c - c \right)
\end{align*}
\]

(4.23)

\[
\begin{align*}
\Pi_J = (d_1 + d_2) \left( 1 - \frac{P_n}{P_m} \right) \left( P_n - c_{uw} - c_m \right) \\
+ (d_1 + d_4) \left( 1 - \frac{P_r}{P_n} \right) \left( P_r - c_r - c \right) - \left( d_1 + d_4 \right) \\
\times \left( 1 - \frac{P_n}{P_m} \right)^{\frac{\theta + 1}{\theta}} \left( \gamma (d_1 + d_2) \left( 1 - \frac{P_n}{P_m} \right) \right)^{-\frac{1}{\theta}}.
\end{align*}
\]

(4.24)

In finding the optimal prices, we assign first derivatives to zero

\[
\begin{align*}
\frac{\partial \Pi_J}{\partial P_n} = (d_1 + d_2) \left( P_n - 2P_n + c_{uw} + c_m \right) \\
+ (d_1 + d_4) \left( P_r - c_r - c \right) - \left( d_3 + d_4 \right) \left( 1 - \frac{P_n}{P_m} \right)^{\frac{\theta + 1}{\theta}} \\
\times \left( \gamma (d_1 + d_2) \left( 1 - \frac{P_n}{P_m} \right) \right)^{-\frac{1}{\theta}} \\
\times \left( \frac{\theta + 1}{\theta} \left( 1 - \frac{P_r}{P_n} \right) - \frac{1}{\theta} \left( P_f - P_r \right) \left( P_f - P_r \right) \right) = 0
\end{align*}
\]

(4.25)
\[
\frac{\partial \Pi_R}{\partial P_r} = -\frac{1}{P_r} (d_1 + d_4) \\
\times \left( P_r - c_3 - c + \frac{\theta + 1}{\theta} \left[ \frac{(d_1 + d_4) \left(1 - \frac{P_r}{\gamma}\right)}{\gamma (d_1 + d_2) \left(1 - \frac{P_r}{\gamma}\right) + (d_1 + d_4) \left(1 - \frac{P_r}{\gamma}\right)} \right]^{1/\theta} \right) \\
+ (d_1 + d_4) \left(1 - \frac{P_r}{P_{w}}\right) = 0 \text{ or} \\
P_r - 2P_r + c_5 + c + \frac{\theta + 1}{\theta} \left[ \frac{(d_1 + d_4) \left(1 - \frac{P_r}{\gamma}\right)}{\gamma (d_1 + d_2) \left(1 - \frac{P_r}{\gamma}\right) + (d_1 + d_4) \left(1 - \frac{P_r}{\gamma}\right)} \right]^{1/\theta} = 0.
\]  

The optimal values of \( P_r^{**} \) and \( P_r^{***} \) that maximize joint profit are obtained by solving Eqs. (4.24) and (4.25). Since we assume balanced quantity (consistent with Guide’s work [26]), the optimization model is reduced to a problem with two decision variables, \( P_r \) and \( P_s \). These results only determine optimal retail prices, and leave the wholesale prices \( (P_{w}, P_{w}) \) and transfer price \( (P_t) \) to be determined under coordinated decision policy, which incorporate equal relative profit difference between independent and joint profit scenario between manufacturer and retailer. The integrated scenario accommodates coordinated pricing policy that ensures higher (or be lower) profit for each party, and makes this approach interesting for all members of the supply chain. The equal relative profit difference can be expressed as follows:

\[
\frac{\Delta \Pi_l}{\Pi_l} = \frac{\Delta \Pi_m}{\Pi_m} = \frac{\Delta \Pi_c}{\Pi_c}
\]

where \( \Delta \Pi_l = \Pi_{l,1} - \Pi_{l,3}; \Delta \Pi_m = \Pi_{m,1} - \Pi_{m,3}; \Delta \Pi_c = \Pi_{c,1} - \Pi_{c,3}. \)

Therefore, we get Eqs. (4.28)-(4.30) (see Box 1) where \( \delta = \Pi_{l} - (\Pi_{m} + \Pi_{c}) = \Delta \Pi_{l} + \Delta \Pi_{m} + \Delta \Pi_{c}. \)

This system of Eqs. (4.28)-(4.30) is solved for \( P_{w}, P_{w}, \) and \( P_t \) to obtain the wholesale prices and transfer price under joint profit scenario.

### 5. Numerical example and discussions

In this numerical example, the parameters in demand function are used from numerical example in [54], because it represents demand pattern of product with gradual obsolescence. However, that study does not consider used product’s return, therefore parameters in return function is taken from numerical example in [29]. As for the cost parameters, we developed the data based on case studies in a report for Ellen MacArthur Foundation, Towards The Circular Economy [55].

New product’s demand capacity parameters are \( U = 1000, \) \( D_0 = 90, \) \( \lambda = [0.01, 0.05, 0.1, 0.2]; \) and remanufactured product’s demand capacity parameters are \( V = 500, \) \( D_0 = 50, \) \( \eta = [0.01, 0.05, 0.1, 0.2]. \) Selling horizon is divided into four time periods where \( t_1 = 1, \mu = 2, t_3 = 3, \) and \( T = 3. \) The unit raw material cost for new product \( c_{w} = 1500, \) and unit collecting cost \( c_{c} = 900, \) and unit remanufacturing cost \( c_{r} = 1000.\)

\( \text{Maximum price is} P_{w} = 12000. \text{ Return rate parameters are} \gamma = 0.01, \text{ and} \theta = 0.7. \text{ The decision variables are} \) \( P_r, P_r, P_{w}, P_{w}, P_{t}, \) which represent the price of new product, price of remanufactured product, wholesale price of new product, wholesale price of remanufactured product, collection price and transfer price, respectively. Table 2 presents the results.

From the above results, we find that joint profit scenario gives a higher total profit rather than the optimized individual. It is also interesting to note that the joint profit model accommodates the retail price of new product, which ensures higher profit for each party, and makes this approach interesting for all members of the supply chain. Demand rate in the joint profit scenario is much higher than in the independent one, even from the same demand parameters. In the independent model, with the lack of integrated decision among the three players, the retail prices were set substantially higher than the true optimum. It is also observed that Collector profit is much lower than Retailer’s and Manufacturer’s, because Collector only gains from remanufactured product. This result is consistent with Guo’s [26].

The optimization modules also show that transfer price can be found by balancing the return rate with the demand of remanufactured product. Under this approach, we can determine transfer price that could benefit both manufacturer and collector, and it puts collector at the better position rather than the presumed condition that transfer price is negotiated between manufacturer and collector. Since manufacturer is the Stackelberg leader, it is possible that the collector would have been in a lower bargaining position. Even though this approach might not be interested for the manufacturer, as it puts limitation to manufacturer’s power, but it actually creates sustainability for the overall closed-loop supply chain.

The betterment in collector’s position would be a good motivation to continue collecting used products for remanufacturing, and support environment protection.

Different speeds of change in the demand of new and remanufactured product obviously result in different pricing decisions. However, faster penetration to the market, which is shown by the higher speed of change in demands, does not simply generate higher total profits. It can be seen from the demand function and optimization models that speed of change in demand will influence the sales volume in each period and subsequently has impacted the optimum pricing decision. As stated in Propositions 1 and 3, the optimal prices depend on the demand levels in respective periods. We can also find an interval of speed of change in demand where the total profit reaches its highest value. This could lead to a marketing decision where the players should control market penetration such that the speed of change is within the desirable interval.

### The impact of demand’s speed of change to the optimal prices

In this paper, demand of short life-cycle product is time-dependent and demand pattern is greatly influenced by several parameters, such as speed of change in demand. As shown in (3.1) and (3.2). The speed of change in the demands is determined by parameter \( \lambda \) and \( \eta \), for new and remanufactured product, respectively. The higher \( \lambda \) and \( \eta \), the faster demand increases
Table 2
Comparison between independent and joint profit scenarios.

<table>
<thead>
<tr>
<th>λ, η</th>
<th>0.01</th>
<th>0.05</th>
<th>0.10</th>
<th>0.20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pn</td>
<td>9,889.78</td>
<td>9,996.60</td>
<td>9,989.68</td>
<td>8,936.88</td>
</tr>
<tr>
<td>Pr</td>
<td>8,318.83</td>
<td>8,336.32</td>
<td>8,347.02</td>
<td>8,346.56</td>
</tr>
<tr>
<td>Pnw</td>
<td>7,018.45</td>
<td>6,997.86</td>
<td>6,997.33</td>
<td>8,997.08</td>
</tr>
<tr>
<td>Pwo</td>
<td>6,747.90</td>
<td>6,795.85</td>
<td>6,797.06</td>
<td>6,796.26</td>
</tr>
<tr>
<td>Pw</td>
<td>422.04</td>
<td>408.28</td>
<td>409.59</td>
<td>408.78</td>
</tr>
<tr>
<td>Pw</td>
<td>1,124.56</td>
<td>1,237.25</td>
<td>1,246.21</td>
<td>1,238.27</td>
</tr>
<tr>
<td>Pw</td>
<td>2,971,233.07</td>
<td>2,453,196.12</td>
<td>2,443,651.42</td>
<td>2,434,076.08</td>
</tr>
<tr>
<td>Pw</td>
<td>1,246,142.45</td>
<td>1,279,206.64</td>
<td>1,274,245.01</td>
<td>1,269,241.66</td>
</tr>
<tr>
<td>Pw</td>
<td>145,869.03</td>
<td>176,221.10</td>
<td>176,253.30</td>
<td>175,094.88</td>
</tr>
<tr>
<td>Total profit</td>
<td>3,783,244.55</td>
<td>3,908,626.86</td>
<td>3,894,150.63</td>
<td>3,878,413.54</td>
</tr>
</tbody>
</table>

Joint profit scenario

| Pn   | 7,816.53  | 7,837.57  | 7,838.40  | 7,838.06  |
| Pn   | 4,720.08  | 4,758.91  | 4,759.25  | 4,759.25  |
| Pn   | 5,756.12  | 6,110.48  | 6,135.01  | 6,579.68  |
| Pn   | 4,133.21  | 3,566.64  | 3,560.95  | 2,742.55  |
| Pn   | 2,979.95  | 2,211.15  | 2,217.41  | 2,31.36  |
| Pn   | 915.79   | 1,004.01  | 1,006.38  | 1,004.03  |
| Pn   | 3,782,657.29 | 3,248,304.59 | 3,235,368.72 | 3,222,922.56 |
| Pn   | 1,656,492.54 | 1,693,841.07 | 1,687,088.16 | 1,680,582.67 |
| Pn   | 189,903.16 | 233,340.36 | 233,157.51 | 231,840.36 |
| Total profit | 5,029,052.98 | 5,175,145.93 | 5,155,814.39 | 5,135,343.58 |

Table 3
Optimal prices under various speeds of change in demand.

<table>
<thead>
<tr>
<th>λ, η</th>
<th>Pn</th>
<th>Pr</th>
<th>Pnw</th>
<th>Pwo</th>
<th>Pw</th>
<th>Pw</th>
<th>Pw</th>
<th>Pw</th>
</tr>
</thead>
<tbody>
<tr>
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<td>8319</td>
<td>7018</td>
<td>6748</td>
<td>1,246,142.45</td>
<td>2,391,233.07</td>
<td>145,869.03</td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>9897</td>
<td>8346</td>
<td>6988</td>
<td>6796</td>
<td>1,279,206.64</td>
<td>2,483,199.12</td>
<td>176,221.10</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>9897</td>
<td>8347</td>
<td>6998</td>
<td>6796</td>
<td>1,274,245.93</td>
<td>2,443,051.42</td>
<td>176,253.30</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>9897</td>
<td>8346</td>
<td>6988</td>
<td>6796</td>
<td>1,266,865.91</td>
<td>2,420,536.87</td>
<td>174,376.30</td>
<td></td>
</tr>
</tbody>
</table>

6. Conclusion and future research agenda

In this study, we have developed pricing decision models for remanufacturing of short life-cycle products. The study fills the gap of remanufacturing literature which to date has been mostly dominated by durable products. For some short-life cycle products, remanufacturing is a sensitive activity to do, but the speed of collecting and remanufacturing the used products should be quick as the demand for the product is diminishing fast. Here are some conclusions that we obtain from this study:

- Reducing the price of new products during the decline phase does not give better profit for the whole system.
- The total profit obtained from optimizing each player independently is lower than the total profit of the integrated model where we optimize the joint profit for three members in the supply chain, namely manufacturer, retailer, and collector. None of the player is worse off by moving from the independent model to the joint profit model, under coordinated pricing policy.
- The total demand is significantly higher under the integrated model. It is understandable because the retail prices are lower for both the new and remanufactured products. The lack of coordination in making the pricing decision has led the independent models to set high retail prices and hence the demand potential is not well exploited.
- Faster penetration to the market, which is shown by the higher speed of change in demands, does not simply generate higher total profits. There exists an interval of speed of change in demand where the total profit reaches its highest value. This could be a support for marketing decision by controlling market penetration such that the speed of change is within the desirable interval.
When demand penetration is low, small changes in the demand rate affect price settings substantially. However, when demand penetration is high, price decision is robust against the change in demand rate.

Future research may be directed toward development of models that consider different demand processes, multiple objective functions, and the case when balanced quantity is not the case. It may be possible that the collector is not able to collect at the quantity desired by the manufacturer. It is also possible that the manufacturer has a certain capacity constraint where not all demand can be satisfied. In such case it is important to take into account the service level.

Appendix A. \( D_r(t) \) and \( D_s(t) \) are continuous at \( \mu \) and \( t_1 \), respectively

\[
D_{h1}(\mu) = \frac{U}{1 + ke^{-\lambda U \mu}}
\]

\[
D_{h2}(\mu) = \frac{U}{\lambda U (\mu - \mu_t) + \delta} = \frac{U}{\delta} = \frac{U}{1 + ke^{-\lambda U \mu}}
\]

\[
\times \ D_{h1}(\mu) = D_{h2}(\mu)
\]

\[
\lim_{t \to \mu+} D_r(t) = \lim_{t \to \mu-} \frac{U}{1 + ke^{-\lambda U t}} = \frac{U}{1 + ke^{-\lambda U \mu}}
\]

\[
\lim_{t \to \mu+} D_s(t) = \lim_{t \to \mu-} D_s(t).
\]

Therefore \( D_r(\mu) = \lim_{t \to \mu} D_r(\mu) = \frac{U}{1 + ke^{-\lambda U \mu}} \to D_s(t) \) is continuous at \( t = \mu \).

Similarly,

\[
D_{r1}(t_2) = \frac{V}{1 + he^{-\eta V (t_2 - t_1)}}
\]

\[
D_{r2}(t_2) = \frac{V}{\eta V (t_2 - t_1) + \epsilon} = \frac{V}{\epsilon} = \frac{V}{1 + he^{-\eta V (t_2 - t_1)}}
\]

\[
\times \ D_{r1}(t_2) = D_{r2}(t_2)
\]

\[
\lim_{t_2 \to t_1+} D_r(t) = \lim_{t_2 \to t_1-} \frac{V}{1 + he^{-\eta V (t_2 - t_1)}} = \frac{V}{1 + he^{-\eta V (t_2 - t_1)}}
\]

\[
\lim_{t_2 \to t_1+} (\eta V (t_2 - t_1) + \epsilon) = \lim_{t_2 \to t_1-} (\eta V (t_2 - t_1) + \epsilon)
\]

\[
\times \lim_{t_2 \to t_1+} D_r(t) = \lim_{t_2 \to t_1-} D_r(t)
\]

Therefore \( D_r(\mu) = \lim_{t_2 \to t_1} D_r(\mu) = \frac{V}{1 + he^{-\eta V (t_2 - t_1)}} \to D_r(t) \) is continuous at \( t = t_1 \).

Appendix B. Proof of Property 1

\[
\Pi_k = d_1 (1 - \frac{P_{a1}}{P_m}) (P_{a1} - P_{bw}) + d_2 \left(1 - \frac{P_{a2}}{P_m}\right) (P_{a2} - P_{sw})
\]

\[
+ (d_3 + d_4) \left(1 - \frac{P_r}{P_{a2}}\right) (P_r - P_{sw})
\]

\[
\frac{\partial^2 \Pi_k}{\partial P_{a1}^2} = -\frac{2d_1}{P_m} < 0
\]

\[
\frac{\partial^2 \Pi_k}{\partial P_{a2}^2} = \frac{\partial^2 \Pi_k}{\partial P_{a1} \partial P_{a2}}
\]

\[
\frac{\partial^2 \Pi_k}{\partial P_r^2} = \frac{\partial^2 \Pi_k}{\partial P_{a1}^2} + \frac{\partial^2 \Pi_k}{\partial P_{a2}^2} + \frac{\partial^2 \Pi_k}{\partial P_r^2}
\]

\[
= \frac{2d_1}{P_m} \left(\frac{d_2}{P_m} + \frac{2(d_3 + d_4)P_r (P_r - P_{sw})}{P_{a2}}\right) > 0; \text{ since } P_r \geq P_{sw}
\]
\[ |H| = \begin{vmatrix} \frac{\partial^2 \Pi_k}{\partial P_k \partial P_h} & \frac{\partial^2 \Pi_k}{\partial P_m \partial P_h} & \frac{\partial^2 \Pi_k}{\partial P_a \partial P_h} \\ \frac{\partial^2 \Pi_k}{\partial P_m \partial P_n} & \frac{\partial^2 \Pi_k}{\partial P_n \partial P_h} & \frac{\partial^2 \Pi_k}{\partial P_a \partial P_n} \\ \frac{\partial^2 \Pi_k}{\partial P_a \partial P_m} & \frac{\partial^2 \Pi_k}{\partial P_a \partial P_n} & \frac{\partial^2 \Pi_k}{\partial P_a \partial P_n} \end{vmatrix} \leq 0 \\
\]

Since \( |H_1| = \frac{\partial^2 \Pi_k}{\partial P_m \partial P_a} \frac{\partial^2 \Pi_k}{\partial P_n \partial P_a} < 0 \) and \( |H_1| = (\frac{\partial^2 \Pi_k}{\partial P_m \partial P_a})(\frac{\partial^2 \Pi_k}{\partial P_n \partial P_a}) - (\frac{\partial^2 \Pi_k}{\partial P_m \partial P_a})^2 > 0 \), then for \( \Pi_k \) to be a concave function, \( |H| = |H_1| \) should be less than or equal to zero. Therefore

\[ \left[ \frac{(d_1 + d_2) P_{mn}^2}{P_{mn}^3} - \frac{4d_2}{P_m} \right] \leq 0 \]

or

\[ P_{mn} \geq \frac{\sqrt{(d_1 + d_2) P_{mn}^3}}{4d_2} \]

**Appendix C. Proof of Property 2**

Since \( P_k \) is a two-variable rational function with a rational parameter in the coefficients, then it is discontinuous when \( p_m = \) and \( p_n = \). However, \( P_k \) is a non-zero parameter, and \( P_{mn} \leq P_n \leq P_m \) with positive \( P_m \), therefore \( P_k \) is continuous in \( \{P_m, P_n\} \) and \( P_{mn} \leq P_m, P_n \leq P_n \leq P_m \), \( P_{mn} \in \mathcal{R}, P_m \in \mathcal{R} \). Since \( P_k \) is continuous in a closed interval then it attains global extremum there. \( \Box \)

**Appendix D. Proof of Property 3**

\[ \frac{\partial^2 \Pi_k}{\partial P_m^2} = \frac{1}{2} \frac{(d_1 + d_2) (P_{mn} + P_{mw} - 2P_m) + \left( \frac{d_3 + d_4}{P_m} \right) (P_m - P_n) P_{mn}} {P_m^3} \]

\[ \frac{\partial^2 \Pi_k}{\partial P_n^2} = -2 \frac{(d_1 + d_2)}{P_n} - 2 \frac{(d_3 + d_4)}{P_n} (P_n - P_m) \]

\[ \frac{\partial^2 \Pi_k}{\partial P_m \partial P_n} = \frac{d_3 + d_4}{P_n} (P_m - P_{nw}) \]

\[ D = \frac{\partial^2 \Pi_k}{\partial P_m^2} \frac{\partial^2 \Pi_k}{\partial P_n^2} - \left( \frac{\partial^2 \Pi_k}{\partial P_m \partial P_n} \right)^2 > 0 \]

\[ 4 \frac{(d_1 + d_2)}{P_m} - \frac{(d_3 + d_4)}{P_n} (P_m - P_{nw})^2 > 0 \]

\[ \frac{P_{mn}}{2} - P_m + \frac{P_n}{2} > 0. \]

\[ \Box \]

**Appendix E. Proof of Proposition 1**

The critical points for (4.6) are obtained by applying first derivatives condition,
Since $\theta (P_c - c) - (\theta + 1) P_c = 0$ at the critical point, then
\[ \frac{\partial^2 H_c}{\partial P_c^2} = - \left( \theta + 1 \right) \gamma (d_1 + d_2) \left( 1 - \frac{P_c}{P_m} \right) P_c^{\theta - 1} < 0. \]

Therefore $P^*_c$ is a local maximum point. Following Property 5, we check the boundary points and compare them with the local maximum. Since $H_c (0) = 0$, $H_c (P^*_c) < 0$, and $H_c (P^*_c) > 0$ then $P^*_c$ is the global maximum within interval $[P_c; 0 \leq P_c \leq P_f; P_c \in \mathbb{R}]$.

**Appendix H. Proof of Proposition 3**

Balanced quantity for demand of remanufactured product and acquired used product is satisfied when
\[ (d_1 + d_2) \left( 1 - \frac{P_c}{P_m} \right) = \gamma P_c^\theta (d_1 + d_2) \left( 1 - \frac{P_c}{P_m} \right). \]

Since optimal collecting price is given as (4.11), then the respective transfer price is
\[ (d_1 + d_2) \left( 1 - \frac{P_c}{P_m} \right) = \gamma \left( \frac{P_c - c}{\theta + 1} \right) (d_1 + d_2) \left( 1 - \frac{P_c}{P_m} \right) \]
\[ P^*_c = \frac{\theta + 1}{\theta} \left( 1 - \frac{d_1 + d_2}{d_1 + d_2} \left( 1 - \frac{P_c}{P_m} \right) \right). \]

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