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Stochastic Judgments in the AHP: Confidence Interval Construction Using Score Statistics Siana Halim† 1and **Indriati N.** **4**

Bisono2 Department of Industrial Engineering Petra Christian University, Surabaya 60236, INDONESIA Email: halim@petra.ac.id1 mlindri@petra.ac.id2 Abstract. In multicriteria decision-making

methods, such as the Analytic Hierarchy Process (AHP), **6**

single values are used to compare criteria and alternatives. Usually this single value given by decision makers follows the fundamental scale from 1 to 9. However, a decision maker often does not have complete support information for his or her making decisions. This lack of information causes the decision maker to become uncertain about his or her decisions. One of the options to overcome this problem is by using intervals instead of single-valued pairwise. This paper presents a methodology for analyzing the interval judgment using confidence intervals, constructed from score statistics. Moreover, inconsistency that can appear in the AHP will be restored using a consistency improving method (CIM). Data uniformly generated are used for implementing the method. The test showed that this interval judgment approach can be a representative method for covering the uncertainty in the decision-making process. Keywords: AHP, CIM, Score Statistics. 1. INTRODUCTION In classical Analytic Hierarchy Process (AHP), Saaty (1980) proposed a fundamental scale, i.e., 1-9 as a tool for helping a decision maker to make decision.

The decision maker provides a **single-valued pairwise preference judgments, yielding a** k×k **matrix A** = {aij} **of preference ratios with respect to a given criterion C, where k is the number of** evaluated **alternatives,** 2

aij represents the relative preference of alternative i over alternative j with respect to C. A is a reciprocal matrix in which aji = 1/aij for i = 1,…, k. However, a decision maker often does not have complete support information for his or her making decisions. This lack of information causes the decision maker to become uncertain about his or her decisions. In this case, there are several approaches to solve the problem. The two common methods are the hybrid of the AHP and Fuzzy Logic (Deng, 1999; Mikhailov, 2000, 2003; Xu, 2000) and the stochastic judgments (Stam, et al., 1997; Hahn, 2003; Halim, et al., 2007). The alternative solution offered in this paper is by letting the decision maker to have interval judgments about his or her preferences.

If the relative preference statements are represented by judgment intervals, rather than single values, then the rankings resulting from a classic **(deterministic) AHP analysis based on single judgment values may be reversed, and therefore incorrect.** 2

Stam and Silva (1997) developed

statistical techniques to obtain both point estimates and confidence intervals of the rank reversal probabilities. 2

They also simulated the realization of aij uniformly with allowing inconsistencies between the pairwise comparisons. In this paper we constructed confidence interval following Stam and Silva (1997). The simulation was modified to avoid inconsistencies in the pairwise comparisons using the consistency improving method (Xu, et al., 1997) and setting a margin such that the improved values will be out of range (Rahardjo, et al., 2001). In addition, instead of using the Clopper-Pearson statistics (1934), which was used by Stam, we proposed to use the score statistic. It is well known that the coverage probabilities of Clopper-Pearson is too high and the score statistics behaves well (Agresti, 2002). 2. METHODOLOGY In this section we will develop the confidence interval construction as well as the consistency improving method.

† : Corresponding Author 2.1 The Confidence Interval of the Probability Rank Reversal Construction. The construction of the confidence interval of probability of rank reversal Πij between alternatives i and j will follow Stam and Silva (1997) approached. The calculation of the rank reversal will be in the same lines as in the classical AHP methodology, therefore we need information

about the true principal right eigenvector w = (w1,..., wk **)T associated** 3

with interval judgments. Denote the pairwise comparison of alternatives i

and j (i, j = 1,…, **k) by** mij, **and let M** 3

= {mij}. We simulated the realization aij

for each entry of M above for **i < j,** 3

and set aij = 1/aij for i > j, completing the reciprocal matrix A. We checked the inconsistency and modified the simulation using consistency improving method when it occurred during the simulation. For each generated A, we calculated the principal right eigenvector w. Replicating this simulation n times, we obtain a sample w1, …,wn principal eigenvectors.

Rank reversal between two alternatives i and j occurs when **alternative i** is **preferred over j under perfect information (i.e. i j),** but it **is** calculated **to be less preferred based on** the **sample information on the interval judgments (i.e. wi < wj). Let** 3

πij = P(i j) and π i1j = P(wi < wj), then Π ij = π ij (1 − π i1j) + (1 − π ij)π i1j (1) If we assume that

in a given simulation trial the probability that (Wi **>Wj) is** 3

approximately equal to the probability of (i j) under complete information than equation (1) can be estimated as Πˆij = 2Pij(1− Pij) (2) It can be seen clearly, that (2) following the binomial distribution, Πˆij ~ bin(2,1) . The score confidence interval contains Π0 values for which zs < zα / 2 . Its endpoints are the Π0 solutions to the equations

Π 0 (1 − **Π 0) / n Πˆ − Π 0** 5

= ± zα / 2 (3) These are quadratic in Π0 . Firstly discussed by E.B. Wilson (1927), the interval is Πˆ n

n + zα / 2 2 n + zα2 / 2 + 1 zα2 / 2 ± n + zα2 / 2

Πˆ(1 − Πˆ)	**n** + zα2 / **2**	+	12	12	**n**	5
+zαz/α22/ **2**	**n**					

2 (4) zα / 2 1 The midpoint Π of the interval is a weighted average of Πˆ ~ and 1/2, where the weight n /(n + zα2 / 2) given Πˆ increases as n increases. Stam and Silva used the Cooper and Pearson Confidence interval (1934) as follows, PijL = pij pij +(1− pij +n−1)Fα/2,2n(1−pij+n−1);2npij piUj = 1 − pij 1− pij + (pij +n−1)Fα / 2,2n(pij +n−);2n(1− pij) (5) 2.2. Consistency Improving Method (CIM) Let Rk+ ={x = (x1, x2,..., xk)r | xi >

0,i =1,2,..., k} **Lemma** 1 **Let A=(aij)** is **an** k×k **positive matrix and λmax** is **the** maximum **eigenvalue of A. Then** 1

λmax = min max ∑ aij n x j x Rn+ i j = 1 x i (6)

Let A and λmax as in Lemma 1. **The positive** right **eigenvector** with respect **to λmax is called** as **the principal right eigenvector of A. Lemma 2. Let x > 0, y > 0, λ > 0 and μ > 0, and λ + μ = 1. Then** x **λ** y **μ** 1

≤ λx + μy . The equality is reached if and only if x = y Lemma

3 Let A is **an** k×k **positive reciprocal matrix, λmax** is **the** maximum **eigenvalue of A. Then λmax≥** k. The **equality** is reached **if and only A is consistent.** Theorem **1. Let A=(aij)** 1

is a k×k

positive reciprocal matrix, and **λmax** is **the** maximum **eigenvalue of A, w = (w1, w2,…,wn)T** is **the principal right eigenvector of A. Let B = (bij), where** 1

bij = (aij)λ wi 1−λ (7) w j

Let μmax is **the** maximum **eigenvalue of B then μmax ≤ λmax** , the **equality** is reached **if and only if A is consistent.** 1

Proofs of lemmas and theorem above can be seen at Xu and Wei (1999). Through Theorem 1, the inconsistent matrices can be transformed into consistent matrices by, ai(jk+1) =(ai(jk))λ. wwi((jkk)) 1−λ (8) In this transformed matrix, the consistency criteria are altered as follow δ = max{| aimj − aioj |},

i, j = 1,2,... k ij **∑i=** 1∑ **k k** j **=1(** (6)

ai(jn) − ai(j0)) 2 σ = k (9) where δ < 2 and σ > 1. CIM is valid if the consistency ratio less than 0.1. However, Raharjo, et al. (2001) showed that CIM has two disadvantages. First, there is a possibility that the result of CIM lies outside the fundamental scale of AHP. Moreover, in one case study, 33.67% of resurvey results showed different result from the CIM. In the simulation these two disadvantages can be neglected. We only need to pay more attention to the first one, that is, by generated more random matrix until the consistency fulfilled and the range of each matrix elements is inside the fundamental scale. 3. NUMERICAL EXAMPLE Suppose a decision maker decides to use AHP for comparing four alternatives A1,…, A4. We simulated random uniform number between 1-9 and between 1/9-1 as the element of the comparison matrices. If these matrices are not consistent then we modified using the modified CIM until the inconsistencies in the matrices are solved. Then we normalized the matrices using geometric mean to get Pijs. We used this relationship for calculating Pij as follows, if A = 4B then Pij = A = 4B 4 A+b 4b+b = 5 = 0.8 We constructed the confidence interval of Pij using (4) Table 1. Comparing Confidence Interval of pairwise preferences using Fisher and Score statistics Pair Pij [Pij_L,Pij_U] Phi [Pij_l,Pij_u] (i,j) Fisher Fisher Scoring Scoring (1,1) 0.5 [0.0676,0.9324] 0.5 [0.1295,0.7123] (1,2) 0.5394 [0.0833,0.9465] 0.521 [0.1833,0.7565] (1,3) 0.5071 [0.0703,0.9351] 0.572 [0.1703,0.7351] (1,4) 0.4855 [0.0622,0.9268] 0.518 [0.1622,0.7268] (2,1) 0.4606 [0.0535,0.9167] 0.546 [0.1535,0.7167] (2,2) (2,3) (2,4) (3,1) (3,2) (3,3) (3,4) (4,1) (4,2) (4,3) (4,4) 0.5 [0.0676,0.9324] 0.500 [0.1676,0.7324] 0.5463 [0.0862,0.9488] 0.573 [0.1862,0.7488] 0.5148 [0.0733,0.9379] 0.582 [0.1733,0.7379] 0.4929 [0.0649,0.9297] 0.529 [0.1649,0.797] 0.4537 [0.0512,0.9138] 0.535 [0.1512,0.718] 0.5 [0.0676,0.9324] 0.5 [0.1676,0.724] 0.4526 [0.0508,0.9133] 0.526 [0.1508,0.713] 0.5145 [0.0732,0.9378] 0.515 [0.1732,0.778] 0.4852 [0.0621,0.9267] 0.522 [0.1621,0.727] 0.5474 [0.0867,0.9492] 0.511 [0.1867,0.742] 0.5 [0.0676,0.9324] 0.5 [0.1676,0.732] Table 1 shows that Pij lies in between 0.5, this is true since we generated the elements of the matrices from uniform distribution. Hence, the preferences probability are equal for every alternatives. Moreover, the confidence intervals constructed via score statistics show they are narrower than ones constructed via the Pearson-Copper Statistics. Therefore, we can conclude that the score statistics are more robust than the Pearson-Copper for this case. 4. CONCLUSION In this paper we constructed the confidence interval for preferences judgment using score statistics. We simulated the data by generated the element of the matrices uniformly and checked the consistency index using modified consistency index method. The result shows the nature of the uniformly data, that is, the equality of preferences in every alternatives. REFERENCES Agresti, A. (2002) Categorical Data Analysis, Wiley Interscience, New York. Cooper, C.J., Pearson, E.S. (1934) The use of confidence or fiducial limits illustrated in the case of the binomial, Biometrika, 26(III/IV), 404-413. Deng, H. (1999) Multicriteria analysis with fuzzy pairwise comparisons, International Journal of Approximate Reasoning, 21, 215-231. Mikhailov, L.A (2000) Fuzzy programming method for deriving priorities in the analytic hierarchy process, Journal of Operational Research Society, 51, 341-349. Mikhailov, L.A (2003) Deriving priorities from fuzzy pairwise comparison judgments, Fuzzy Sets and Systems, 134(3), 365-385. Hahn, E.D. (2003) Decision making with uncertain judgments: A stochastic formulation of the analytic hierarchy process, Decision Sciences, 34(3), 443-466. Halim, S., Rahardjo, J., Felecia, P. (2006) Stochastic Judgments dalam AHP: Menentukan selang kepercayaan dengan pendekatan simulasi – Studi kasus tentang pemilihan

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