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--Manuscript Draft-- Manuscript Number: IJCM-D-16-00084R2 Full Title:

Kriging-based Timoshenko Beam Elements with the Discrete Shear Gap Technique 4

Article Type: Research Paper Keywords: Timoshenko beam; Kriging-based finite element; shear locking; discrete shear gap. Corresponding Author: Foek Tjong Wong, Ph.D. Universitas Kristen Petra Surabaya, East Java INDONESIA

Corresponding Author Secondary Information: Corresponding Author's Institution: Universitas Kristen Petra **Corresponding Author's Secondary Institution: First Author: First Author Secondary Information:** 11

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Foek Tjong Wong, Ph.D. Foek Tjong Wong, Ph.D. Adam Sulistio, M.Sc. Hidayat Syamsoeyadi, B.Eng. Order of Authors Secondary Information: Abstract: Kriging-based

finite element method (K **-FEM) is** an enhancement **of the FEM** 20

through the use of Kriging interpolation in place of the conventional polynomial interpolation. In this paper,

the K-FEM is developed **for static, free vibration,** and buckling **analyses of Timoshenko beams. The** 14

discrete shear gap technique **is employed to eliminate shear locking. The** 12

numerical tests show that a Kriging-based beam element with cubic basis and three element-layer domain of influencing nodes is free from shear locking. Exceptionally accurate displacements, bending moments, natural frequencies, and buckling loads and reasonably accurate shear force can be achieved using a relatively course mesh. Response to Reviewers:

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Revision Notes 2 Here are the list of revision or rebuttal based upon the reviewers' comments: No. Reviewers' Comments Action Response 1 Thank you for revising the manuscript. I am still not in line with your opinion on the patch test. It has originally been introduced as a numerical consistency tests and correct modeling of pure bending in beam elements is not necessary for consistency. However, there must be space for different opinions in scientific exchange of ideas. Thank you for your insightful comments. Since for one-dimensional models (including the Timoshenko beam model) the finite element patch test is uncommonly used and it may seem excessive, I agree to remove the statements relating to 'patch test'. Thus, I have now revised the 1st paragraph on p. 13 right after subtitle 5.1.1, and the paragraph right before subtitle 5.1.2. In addition to that revision, I edited the figures to make them clearer for reading. Manuscript Click here to download Manuscript K-beam Elements with the DSG Technique (Revised2).docx

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Kriging-based

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through the use of Kriging interpolation in place of the conventional polynomial interpolation. In this paper,

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numerical tests show that a Kriging-based beam element with cubic basis and three element-layer domain of influencing nodes is free from shear locking. Exceptionally accurate displacements, bending moments, natural frequencies, and buckling loads and reasonably accurate shear force can be achieved using a relatively course mesh. Keywords: Timoshenko beam; Kriging-based finite element; shear locking; discrete shear gap. 1. Introduction

An enhancement of the finite element method, 6 referred to as the Kriging-based finite element method (K-FEM), was proposed by Plengkhom and Kanok-Nukulchai [2005]. In the 9

K-FEM Kriging interpolation (KI) is employed as the trial and test functions in place of a conventional polynomial function. This KI is constructed for each element

from a set of nodal values within a domain of influencing nodes (DOI) comprising the element itself and several layers of the surrounding elements. 2

The DOI is thus a polygon

in the 2D domain and a polyhedron in the 3D domain. The 2

advantages of this novel proposed method are as follows: (1)

a high degree of polynomial function can be easily included in the 9

trial function without adding any side or internal nodes to the element; (2) higher accuracy and better 1 2 F.T. Wong, A. Sulistio, H. Syamsoeyadi smoothness results for the field variables and their derivatives, in comparison to the conventional FEM,

can be obtained using even the simplest form of elements; (3) furthermore, the 2

computer implementation of the proposed method is very similar to that of conventional FEM, so an existing general FEM code can be extended to include the method without major changes. The

K-FEM [Plengkhom and Kanok-Nukulchai (2005)] 5

was subsequently

improved through the use of adaptive correlation parameters and 2

developed for analysis of

Reissner- Mindlin plates [Wong and Kanok-Nukulchai (2006a)]. 5

A shortcoming of the K-FEM is that the resulting approximate function is discontinuous across the 9

element boundaries, or in other words, nonconforming between interconnected elements. The issue of the nonconformity was studied in [Wong and Kanok-Nukulchai (2009b)] and it is concluded that the K-FEM with appropriate Kriging parameters always yield converging results. In

the development of the K-FEM for 6

analyses of shear deformable beams,

plates, and shells, as in the FEM, the difficulty of shear locking also 42

occurs. In attempts to overcome this difficulty, the field-matching technique of

Kanok-Nukulchai, et al. [2001], which works well in the element-free Galerkin method 33

[Belytschko et al. (1994)], has been employed [Wong (2009)]. Using this technique, the

shape functions for the rotational fields are taken as the derivatives of the shape functions for the 29

deflection. It is found that the K-FEM with the field- matching strategy yields erroneous results. Accordingly, the field-matching technique is not applicable in the framework of the K-FEM. Another attempt is the introduction of

assumed natural transverse shear strains in the K-FEM 8

[Wong and Kanok-Nukulchai (2006b)]. This study discovers that the assumed shear strain method

can relieve the locking but cannot eliminate it completely because the 8

locations of shear-strain sampling points in the K-FEM cannot be determined exactly. Henceforth, a basis function of sufficiently high degree (cubic or higher) in the K-FEM for analyses of plates and shell structures is employed as a provisional solution to relieve the shear locking [Wong and Kanok-Nukulchai, 2006a; 2008; 2009; Wong, 2009; 2013; Wong et al. (2015)]. The use of a high-degree polynomial basis, however, cannot completely eliminate the locking and makes the computational cost high. Thus, it is essential to have an effective

method to eliminate shear locking in the K-FEM. To 8

develop a locking-free formulation for the

K-FEM for analyses of plates **and** 6

shells,

it is instructive to study the K-FEM in the simpler problem **of the Timoshenko beam** 2

model in order to gain understanding and insight about a locking-free device. In this spirit, Wong and Syamsoeyadi [2011] developed the

K-FEM for static and free vibration analyses of Timoshenko beams. In this work, **the** 2

shear locking is eliminated using the well-known selective-reduced integration (SRI) technique. While the SRI technique can effectively eliminate shear locking, it deteriorates the accuracy in the case of thick beams and also worsens the shear force results [Wong and Syamsoeyadi (2011)]. Moreover, it has been proved

that the SRI technique is not **applicable** to a **Kriging-based** triangular **plate** 2

bending element [Wong (2009)]. Therefore, a new shear-locking-elimination technique that is extendable to a Kriging- based Reissner-Mindlin plate element needs to be explored.

Bletzinger et al. [2000] **presented a unified approach** to eliminate shear **locking** 1

in

shear deformable plates and shells, called the discrete shear gap (DSG) technique. In this approach, the shear | 1

gap

is defined as the difference between the total deflection and the | 37

bending

Kriging-based Timoshenko Beam Elements with the DSG Technique | 4

3 deflection, or in other words, it is the deflection corresponding to the shear strain. The shear gap is calculated

at the element nodes (called the discrete shear gaps) | 24

and interpolated across the element domain. The

substitute shear strain is then obtained from the derivatives of the interpolated discrete shear | 1

gaps. With this technique, a locking-free formulation of plate

elements, either triangular or rectangular of any polynomial | 24

degree, can be carried out in a simple way. The technique was subsequently generalized

to a more general concept applicable to other locking problems | 39

such as membrane locking

[Bischoff et al. (2003); Koschnick et al. (2005)]. | 67

The DSG technique has been applied not only in the conventional FEM but also in recent alternative computational methods such as the edge-based smoothed FEM [Nguyen-Xuan et al. (2010a)] and node -based smoothed FEM [Nguyen- Xuan et al. 26

(2010b)]. The most recent application of the DSG approach is to combine the DSG three-node triangular element with the cell-based smoothed FEM to produce

a three-node triangular plate element, which 28

is claimed to have some superior properties compared

to many existing plate elements of the 28

same class [Nguyen-Thoi et al., 2015; 2012]. These recent and successful applications of the DSG technique suggest that it may also be effective at eliminating

shear locking in Kriging- based shear deformable beams, plates, and 6

shells. The

aim of this paper is to present the development and testing of the K-FEM 2

with the DSG technique

for static, free vibration, and buckling analyses of Timoshenko beams. The 2

discretized equations are formulated using the standard displacement-based finite element procedure on the variational form. The same Kriging

shape functions are used for the deflection and 71

rotation variables. To apply the DSG technique, the discrete shear gaps at all nodes in a DOI under consideration are evaluated from the integration of the kinematic shear strain. The DSG shear strain is then calculated

from the derivative of the Kriging-based interpolated discrete shear gaps. 1

The

original

displacement-based transverse shear strain is replaced with the DSG shear strain 1

to eliminate the shear locking. A series of numerical tests in

static, free vibration, and buckling problems are carried out to evaluate the accuracy and convergence of the 31

Kriging-based Timoshenko beam element. Special attention is 6

given to numerical investigation of the shear locking. It is worth mentioning here that although some researchers have employed the DSG technique with a stabilization parameter α to improve the accuracy and stability against mesh distortion [Bischoff

et al. (2003); Nguyen-Thoi et al. (2012); Nguyen-Xuan et al., 43

2010a; 2010b], we do not resort to this approach because the parameter α is problem dependent and makes the role of the shear correction factor obscure.

2. Kriging Interpolation in the K-FEM Named after Danie G. Krige, a South African mining engineer, Kriging is a well-known geostatistical technique for spatial data interpolation in geology and mining (see, e.g., 3

geostatistics texts [Olea (1999); Wackernagel (2003)]. Reviews of KI in the framework of the

K-FEM have been published in several previous papers 6

[Wong and Kanok-Nukulchai, 2009a; 2009b; Wong and Syamsoeyadi (2011)]. In this paper only the key points of KI that are necessary for the subsequent development are addressed. 4 F.T. Wong, A. Sulistio, H. Syamsoeyadi 2.1. Kriging shape function We consider a one-dimensional problem domain Ω where a continuous field variable (a scalar function) u(x) is defined. The

domain is represented by a set of properly scattered nodes xI, I=1, 2, …, 2

N, where N is the total number of nodes in the whole domain. Given N field values u(x1), …, u(xN), the problem of interest is to obtain an estimated value of uh at a point x ? ?. In the Kriging method, the unknown value uh(x) is estimated from a linear combination of

the field

values at the neighboring nodes, that is, uh (x) 6

? ?ni?1?i(x)u(xi)

(1) where the λi(x)'s are the unknown Kriging weights and n is the number of nodes surrounding point x inside and on the boundary of a 2

subdomain ?x?????? n????. Here, small letters i and n are used instead of I and N to signify that the numbering is referred to the local (subdomain) numbering system.

In the context of the K-FEM [Wong and 5

Kanok-Nukulchai, 2009a; 2009b; Wong and Syamsoeyadi (2011)], the nodes are identical to finite element nodes and the weights are identical to shape functions. Furthermore, the subdomain ?x in the K-FEM is composed of the element of interest and several layers of surrounding elements and referred to as the DOI. The number of element layers of the DOI can be one, two, or more. In the case of one layer, the DOI is the finite element itself and the K-FEM becomes identical to the conventional FEM. Figure 1 illustrates a two-layer DOI encompassing local nodes 1, 2, 3, and 4. The Kriging weights λi

(x), i = 1, …, n, are obtained by solving the 82

Kriging equation system: R λ

(x) ? P μ (x) ? r (x) PTλ (x) ? p (x) 85

(2a) (2b)

in which ?C(h11) ... C(h1n)? R? ? ? ? ? ; P? ? ?p1(x1) ... pm(x1)? ... ... ??C(hn1) ... C(3 hnn)?? ??p1(xn) ... ? ? pm(xn)?? ? λ

(x)???1(x) ... ?n(x)? T ; μ (x)???1(x) ... ?m (x)? 46

T (2c) (2d) Fig. 1. One-dimensional domain and a two element-layer subdomain.

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5 r(x) ? ?C(h1x) C(h2x) ... C(hnx)?T ; p(x) ? ?p1(x) ... pm(x)?T (2e) Each entry in matrix R, C(hij),

is the covariance between U(xi) and U(xj), 2

which is a function of hij?? xj ? xi ; i = 1, … , n; j = 1, … , n. Thus,

R is the n ??n matrix of the covariance of U(x) at nodes in the DOI, x1, … , xn. The capital 'U(x)' 8

here signifies the corresponding random process of the deterministic function u(x). Matrix

P is the n ?? m matrix of monomial values at the nodes, where m is the number of 3

monomial terms. Vector λ(x) is the unknown

n ??1 vector of Kriging weights and vector μ(x) is the unknown m ??1 vector of Lagrange multipliers. 3

On the right hand side of Eqs. (2a) and (2b), vector

r(x) is the n ??1 vector of covariance between the nodes and the point of interest, x, and p(x) is the m ??1 vector of monomial values at x. Each entry in 3

r(x), C(hix),

is the covariance between U(xi) and U (x), 6 which is a function of hix ? x ? xi. A necessary condition to make the Kriging equation system solvable (non-singular) is

that the number of nodes in the DOI, n, should be 16

equal to or greater than the number of 70

monomial terms, m, that is, n ? m. Solving the Kriging equation system, the vector of Kriging weights is given as λT (

x) pT (x)A ? rT (x)B ? (3a) where 74

A ? (PTR?1P)?1PTR

?1 , B R?1(I ? PA) ? (3b, c) Here A is an m ? n matrix, B is an n ? n matrix, and I is the n ? n identity matrix. The expression for the estimated value 5

uh,

Eq. (1), can be restated in matrix form as 5

uh (x) λT (x)d N(x)d ? ? (4)

where d ? [u(x1) … u(xn)]T is an n? 1 vector of nodal values 2

and N (x)= λT(x) is the matrix of Kriging shape functions. 61

It is obvious that the Kriging weights are nothing but the shape functions.

2.2. Polynomial basis and correlation function 5

In order to construct Kriging shape functions, a polynomial basis function and a covariance function should be chosen. 2

In the present research, as in the previous research on the K- FEM for analysis of Timoshenko beams [Wong and Syamsoeyadi (2011)], polynomial bases of degree one to three are employed. The higher the polynomial degree, the more element layers are needed in the DOI because of the requirement that n ??m. The

minimum number of DOI **layers for different polynomial bases is listed in Table 1.** Table 1. Minimum **number of layers** 3

for different polynomial bases. Polynomial basis Monomial terms m Minimum number of layers Linear Quadratic Cubic

1 x 2 1 1 x x2 **3 2 1 x** 38

x2 x3 4 3 6 F.T. Wong, A. Sulistio, H. Syamsoeyadi The covariance C(h)

is more conveniently expressed in terms **of** a **correlation function,** 8

which is given as ?

?h? ? C ?h? ? 2 (5) **where h is the** 62

distance between

points x and x + h, and σ2 **is the variance of the random function U(x). The** 2

variance σ2 has no effect on the resulting Kriging shape functions and is taken to be equal to 1 in this study. There are many possibilities for the correlation function model in the area of geostatistics [Olea (1999); Wackernagel (2003)], such as the Nugget-effect model, exponential model, and spherical model. In the present study, as in the previous research [Wong and Syamsoeyadi (2011)], the

Gaussian correlation function, that is, ?(h) ? exp(?(? r)2) **h d** 8

and the quartic spline (QS), that is,

?1 ? 6(?r h)2 ? 8(?r h)3 ? 3(?r h 4 for **0 ? ?r** 21

? 1 d d d) h ? (h) ? ?? d ? ? 0 for ?r ? 1 h ? d (6) (7) are chosen.

In these equations, θr >**0 is the correlation parameter** 2

and

d is a scale factor to normalize the distance h. Factor **d is taken to be the largest distance between any pair of nodes in the DOI. The** 3

parameter θr is an important parameter affecting the validity of Kriging shape functions. Too small a value of θr deteriorates the partition of unity property of the shape functions [Plengkhom and Kanok-Nukulchai (2005)], that is, ?ni?1Ni ?1 ?1 (8) Conversely, too large a value of θr may make the Kriging equation system singular. Based on these facts,

Plengkhom and Kanok-Nukulchai [2005] proposed **a rule of thumb for** 5

the lower and upper bounds of θr. Following this rule, the value of θr

should be selected so that it satisfies the lower bound criterion ? **n** i **?1** 2

Ni ?1 ? 1?10?10?a (9) **where a is the** degree **of the basis function, and the upper bound** criterion **det(R) ? 1?10?b** (10) **where b is the dimension of the problem. For** the 1D **problem** and quadratic **basis function, for example, a** 5

= 2 and b = 1.

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7 The range of appropriate values of θr varies with the

number of monomial terms m, **the number of** the **nodes in the** 44

DOI, n, and the correlation model used [Plengkhom and Kanok-Nukulchai (2005);

Wong and Kanok-Nukulchai (2006a)]. In 5

other words, it depends on the polynomial basis, the number of element layers, and the type of correlation function employed in the analysis. The range of θr for 1D problems satisfying the lower and upper bound criteria, Eqs. (9) and (10), has been numerically examined [Wong and Syamsoeyadi (2011)] and the results are presented in Table 2. It is recommended that the mid-value between the upper and lower bounds be taken to ensure good quality of the KI. Table 2. The lower and upper bounds of θr for 1D problems. Polynomial Number Gaussian ρ(h) Quartic spline ρ(h) basis of layers

Lower bound Upper bound Lower bound Upper bound 1 0 0. 2295 **0 0.** 27

098 Linear 2 10-4 1.0 10-5 0.44 3 10-4 1.9 10-5 0.86 Quadratic Cubic 2 3 3 10-4 1.0 10-5 0.44 10-4 1.9 10-6 0.86 10-4 1.9 10-8 0.86 3. Formulation of Kriging-based Timoshenko Beam Element We consider

a beam of length L with cross-sectional area and moment of inertia A and **I,** respectively. The beam **is** 1

made from **a homogeneous and isotropic material with modulus of elasticity E,** shear modulus G, **and** 32

mass density ρ (per unit volume).

A Cartesian coordinate system (x,y,z) is established, where **the x-axis** coincides **with the** neutral **axis and** the **y- and z-** axes coincide **with the principal axes of the cross- section** 7

(Fig. 2). **The beam is subjected to a** dynamic distributed transversal **load q** 57

= q

(x, t) and a distributed moment m = m(x, t), 35

0 ? x ? L, t ? 0.

In the Timoshenko beam theory, the motion **of** the **beam** 79 due to external loads is described using

two independent field variables, namely **the** transverse **displacement** (deflection) **of the neutral axis w = w(x,** t) **and the** rotation of the **cross-section** θ = θ **(x,** t). **The** 1

sign convention for these variables **is shown in Fig. 2.** 50

z w θ x Fig. 2.

Coordinate system and positive directions for the deflection and rotation. 1

8 F.T. Wong, A. Sulistio, H. Syamsoeyadi To account for the

effect of axial **force on the deflection,** let **the beam is** also subjected **to** 36

an axial force P = P(t) at the ends of the beam. The sign of P is positive when it is a tensile force. This axial force is imposed at the outset, for example by a pre-stressed cable system. The deflection is assumed to be small so that the axial force P remains essentially constant. In addition, the beam is assumed to be initially, perfectly straight so that the coupling between bending and membrane deformations can be neglected. Thus, the present Timoshenko beam model is a linear-elastic model enhanced with a nonlinear Green strain term for the axial strain [Cook et al. (2002), Ch. 18]. This model can be reduced to the bifurcation buckling model by removing the transversal load q and external moment m and assuming that the axial force P is an unknown. The governing equation for the motion of the beam at time t, including the axial load effect,

can be expressed in a variational **form as** ?0 ? w ? **A w** 17

dx ? ?0 ?? ? I ? dx L L ? ?0 ?? , x EI? , x dx ? ?0 ?? GAs ? dx ? ?0 ? w, x P w, x dx L L L (11) ? ?0 ? w q dx ? ?0 ?? m dx L L In this equation, ? ? w,x ?? (12)

is the transverse **shear strain and** AS = kA **is the** effective **shear** 68

area, where

k is a shear correction factor that is dependent upon the cross-section 2 geometry.

The double dots signify the second partial

derivative of the corresponding variable with respect to the time variable t, while the 47

comma signifies the first partial derivative of it with respect to the variable next to it (i.e., x). The 1

operator δ signifies the variational operation on the corresponding variable. A detailed derivation of the variational equation, Eq. (11), using Hamilton's principle is given in references [Friedman and Kosmatka (1993); Kosmatka (1995)]. The

bending moment and shear force along the beam can be calculated from the deflection w and rotation θ as follows: M ? EI ? ,x (13) Q ? GAs ?w,x 2

?? ? ? P w,x (14)

To obtain an approximate solution using the concept of KI with a layered-element DOI, 5

the beam is subdivided into Nel elements and N nodes. We then consider

an element with a DOI that contains n nodes, 6

as illustrated in Fig. 3. The field variables w and θ

over the element are approximated using KI as follows: w ? Nw (x) d(2

t) (15a)

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9 Fig. 3. A typical beam element and its domain of influencing nodes (DOI). ? ? N? (x) d(t) (15b) where

Nw(x) ?

[N1(x) 0 N2(x) 0 N? (x) ? [0 N1 (x) 0 N2 (x) Nn (x) 0] (15c) 0 Nn(x)] 23

(15d)

are the matrices of Kriging shape functions for the deflection and 66

rotation, respectively, and d

(t) ? [w1(t) ?1(t) w2(t) ?2(t) wn (t) ?n (t)]T 30

(15e) is the vector of nodal displacement. The variable x here refers to the local (element) coordinate system. The

number of nodes, n, depends on the number of elements used in the 10

DOI and differs for the interior and exterior elements. For example, for an element with a two-layer DOI,

n = 3 for the exterior element and n = 4 for the interior element. 1

Substituting Eqs. (15a) and (15b) into Eq. (11) and

carrying out the standard finite element formulation yield the discretized system of equations md(t) ? (k ? P kg)d(t) ? f (t) In this equation, 1

m ? ? 0 Le NTw ? A Nw dx Le NT? ? I N? dx ? ? 0

is the element consistent mass matrix, 6

k ? ? Le 0 BT? EI B? dx ? ?0Le BT? GAs B?

dx is the element stiffness matrix, 6

kg ? ?0Le BTw Bw

dx is the element geometrical stiffness matrix, and f 2

(t) ? ?0Le NTw q dx ? ?0Le N?T m dx (16) (17) (18) (19) (20) 10 F.T. Wong, A. Sulistio, H. Syamsoeyadi

is the element equivalent nodal force vector. The order of all square matrices and 2

vectors is 2n. In Eqs. (18) and (19), matrices Bθ, Bw, and Bγ are defined as follows: B? ? dx N? , Bw ? d d Nw (21a) dx B? ? Bw ? N? (21b) The unknowns of Eq. (16) are the

element nodal acceleration vector () and the nodal displacement vector 9

d(t). The discretized

equations for static, free vibration, and buckling problems can be 55

obtained from Eq. (16) by simply reducing it, respectively, to kd ? f (22) md(t) ? kd(t) ? 0 (23) (k ? P kg)d ? 0 (24) The corresponding global discretized equations of these equations can be obtained using the finite element assembly procedure.

It should be mentioned here that the assembly process involves all nodes in the DOI, not just the element nodes as in the 3

conventional FEM. 4. Application of the Discrete Shear Gap Concept

It is well known that the pure displacement-based 64

formulation of a Timoshenko beam (with exact integration of all integrals) leads to the shear locking phenomenon in the FEM [Bathe (1996); Cook el al. (2002); Hughes (1987); Liu and Quek (2003); Prathap 1993; 2001; Reddy (2006)]. The same is true for a Kriging-based Timoshenko beam element [Wong and Syamsoeyadi (2011)], even for an element with a cubic polynomial basis function. The primary cause of this locking is the inability of the approximate shear strain to vanish as the beam becomes extremely thin. The basic idea of the

discrete shear gap (DSG) concept [Bischoff et al. (2003); Bletzinger et al. (2000)] 4 is to replace the troublesome kinematic shear strain with a substitute shear strain field determined from the derivative of interpolated discrete shear | 1

gaps. This section presents a review of the DSG concept and its application to Timoshenko beam elements with KI. To apply the DSG concept, we begin with the definition of shear gap, that is, ?w? (x) ? ?x0? dx x (25) where $\Delta w_{\gamma}(x)$ is the

shear gap at point x, and x0 is the position of | 1

a chosen reference point. Inserting Eq. (12) into this equation results in ?w? (x) ? w xx0 ? ?xx0? dx

?w? (x) ? (w(x) ? w(x0)) ? ?x0? dx ? ?w (x) ? ?wb (x) x | 58

(26a) (26b)

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11 In these equations, Δw is the increase of the actual deflection between the positions x and x0, and Δw_b

is the increase of the deflection due to bending action. The shear | 49

gap Δw_{γ} thus represents the increase of the deflection due to shearing action. The discrete shear gap at a finite element node with position xi, $\Delta w_{\gamma i}$, is defined as ?w? i ? ?w? (xi) ? w xxi0 ? ?x0i ? dx x (27) A modified

shear gap field is defined as the interpolation of the nodal shear gaps, that is, n ?w? (x) ? ?Ni(x) ?w?i (28) i ?1 In the | 1

framework of the standard FEM,

n is the number of number of nodes in the element and Ni (x), i = 1, ..., n are the shape functions. In the | 25

present research, however,

n is the number of nodes in the DOI and Ni(x) are Kriging shape | 34 functions.

Differentiating Eq. (28) gives the substitute shear strain, that is, n ? (x)??Ni,x?w?i ?B?1w? (29a) i ?1 where B?1 ??N1,x N2,x Nn ,x ? (29b)

w? ? ???w?1 ?w? 2 ?w? n ?? T (29c) To avoid the shear locking problem, the 41

kinematic shear strain γ, Eq. (12), is replaced with ,̄ Eq. (29a). In order to implement this technique for the Kriging-based beam elements, we first need to express the nodal shear gaps in the DOI, wγ,

in terms of the degrees of freedom of the 48

Timoshenko beam. Choosing node 1 as the reference point (see Fig. 3) and inserting Eq. (15b) into Eq. (27), the discrete shear gaps are given as ?w? i ? wi ? w1 ? (?x1 N? (x) dx)d xi (30) Evaluating the discrete shear gaps for nodes 1 to n and writing them in matrix form, the discrete shear gaps for all nodes in the DOI can be expressed as w? ? B? 2d (31a) where 12 F.T. Wong, A. Sulistio, H. Syamsoeyadi ? 0 0 0 0 ? ??1 ?? x12 N1dx 1 x x ? ??x12 N2dx B? 2 ? ???1 ??xx13 N1dx 0 ??x1 N2dx x3 ? ? ????1 ??xx1n N1dx 0 ??x1 N2dx xn 0 0 0 ??x12 N3dx x 1 ??x1 N3dx x3 0 ? ?x1 N 3dx xn Substituting Eq. (31a) into Eq. (29a) yields ? (x) ? B? 1B? 2d ? B? d 0 0 ? 0 ??x12 Nndx? x ? 0 ??xx13 Nndx?? ? ? ??xx1n Nndx??? ? 1 (31b) (32) The implementation of the DSG concept is accomplished by replacing matrix Bγ

in the expression for the element stiffness matrix, Eq. (18), with 81

matrix ̄ as defined in Eq. (32). The

Kriging-based Timoshenko beam elements with the DSG shear 4

strains are hereafter referred to as the K-beam-DSG elements. 5. Numerical Results

A series of numerical tests were carried out to evaluate: (1) the effectiveness of the DSG technique in eliminating the 1

shear locking and (2) the

accuracy and convergence characteristics of the K -beam- DSG elements 1 with different K-FEM options in static, free vibration, and buckling analyses. The K-FEM options used comprised linear to cubic basis functions with one to three element-layer DOIs, and the Gaussian or QS correlation functions with the mid-value correlation parameters between the lower and upper bound values (Table 2). Abbreviations of the form P*-*-G or P*-*-QS were used to denote different K-FEM options. The first asterisk represents the polynomial basis, while the second represents the number of DOI element layers. The last letter or letters represent the Gaussian

(G) or quartic spline (QS) correlation function. 84

For example, the abbreviation P2-3-QS means the K-FEM options of a

quadratic basis function, three element layers, and a quartic spline correlation function 16

(with a mid-value correlation parameter). In all tests, three Gaussian sampling points were employed to evaluate the integrations in the element stiffness matrix equations, Eq. (18), while two sampling points were used to evaluate the integration in the element consistent nodal load vector, Eq. (20), and ̄2, Eq. (31b). The abovementioned numbers of sampling points were chosen because we found by trial and error that they could provide accurate results with the minimum computational cost. In addition, in all tests the shear correctio n factor employed was given as [Cowper (1966) as cited in Friedman and Kosmatka (1993); Kosmatka (1995)]:

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13 M

L/4 L/4 L/4 L/4 (a) 1

Regular node distribution M 0.1L 0.1L 0.6L 0.2L (b) Irregular node distribution Fig. 4. Cantilever beam modeled with (a) regular and (b) irregular node distributions.

5.1. Static analysis 5.1.1. Pure bending test 54

k ? 10(1 ? ?) 12 ? 11? (33)

A cantilever beam of rectangular cross-section b ??h 1

modeled with meshes of regular and irregular node distributions is shown in Fig. 4. The beam is in pure bending state with constant bending moment, M, and zero shear force along the beam. This problem 1

is a simple test involving linear rotation and quadratic displacement fields. The beam was analyzed using the K-beam-DSG with different polynomial bases, numbers of layers, and correlation functions. Numerical values used in the analyses were

L = 10 m, b = 2 m, E = 2000 kN/ m2, v = 0.3, and M = 1 kN-m. Two different length-to- thickness ratios were considered. 1

One was moderately thick, that is, L/h = 5, and the other was extremely thin, that is, L/h = 10000. The resulting deflections and rotations at the free end, wL and θL,

were observed and then normalized to the corresponding exact solutions, 1

that is, wL exact ? M2ELI2 ML , ?Lexact ? EI (34a, b) The resulting

bending moments and shear forces at the clamped end, 2

calculated using Eqs. (13) and (14), were observed as well. The bending moments were then normalized to the exact bending moment, M. The shear forces, however, were not normalized because of the zero exact shear force. 14 F.T. Wong, A. Sulistio, H. Syamsoeyadi The results showed that the K-beam-DSG produced values of the deflection, rotation, and bending moment with at least seven-digit accuracy (nearly exact values)

for both thick and thin beams with regular node distribution. The 1

maximum error for the shear force, however, was of the order of 10?5 (for the case of the thin beam analyzed using the P1-3-G option). Very accurate

results for the deflection, rotation, and bending moment of the thick and thin 10

beams were also

obtained for the case of irregular node distribution, 83

with an accuracy of at least five digits (for the thin beam analyzed using the P1-3-G option). Table 3 tabularizes the results for the thin beam modeled with an irregular node distribution (the most critical case). It was seen that in this case the accuracy of the shear force (the maximum error is of the order of 10?2) was lower than that of the thin beam modeled with regular node distribution. It is worth mentioning here that the standard

finite element and **Kriging-** based **Timoshenko beam** elements with **selective reduced integration** 2

are unable to produce correct values for the shear force computed using Eq. (14) [Prathap (1993); Wong and Syamsoeyadi (2011)]. Table 3. Analysis results for the extremely thin cantilever beam modeled with an irregular node distribution using various K-beam-DSG options. K-beam-DSG options wL / wL exact P1-1-QS 1.0000000 P1-2-QS 0.9999997 P1-3-QS 1.0000000 θL / θL exact 1.0000000 0.9999997 1.0000000 M0 / M0 exact 0.9999999 0.9999997 1.0000000 V 0 –5.36E-09 4.93E-05 –2.02E-05 P2-2-QS 1.0000000 1.0000001 1.0000000 –9.21E-07 P2-3-QS 0.9999999 0.9999999 0.9999998 3.41E-06 P3-3-QS 1.0000001 1.0000001 1.0000002 –1.12E-07 P1-1-G 1.0000000 1.0000000 0.9999999 –8.61E-09 P1-2-G 1.0000000 0.9999998 1.0000004 2.64E-03 P1-3-G 1.0000056 1.0000006 1.0000367 –6.28E-02 P2-2-G 0.9999999 0.9999999 0.9999999 2.71E-03 P2-3-G 0.9999991 0.9999994 0.9999990 –9.55E-03 P3-3-G 1.0000002 1.0000001 1.0000005 9.21E-03 Overall, the results indicate that the K-beam-DSG with different analysis options virtually reproduce the exact solutions. The K-beam-DSG elements with the Gaussian correlation function generally produce less accurate results than those with the QS. In this simple problem, there is no indication of shear locking. 5.1.2. Investigation of shear locking

To investigate the effectiveness of the DSG technique **in** eliminating **shear locking,** 1

we considered a clamped-clamped beam with the finite element model as shown in Fig. 5. The

height of the beam was varied from moderately **thick, L/h = 5, to extremely thin, L/h** 2

= 104. The geometrical and

material properties of the beam were **the same as in** the previous test **(Section** 45

5.1.1) with a

distributed load of **q = 1 kN/m.** The beam **was analyzed** 2

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15 Fig. 5. Clamped-clamped beam modeled with mesh of eight elements. using K-beam-DSG elements with different K-FEM options. The

resulting deflections of the beam mid-span were observed and normalized to the exact 1

solution, that is, wexact ? 38q4LE4I ? qL2 8GAs (35) The results are presented in Table 4. Only the results obtained from the use of K-beam- DSG with the QS correlation function are reported here to save space. The corresponding results with the Gaussian correlation function have similar locking behavior. Table 4. Normalized deflections of the beam mid-span using various K-beam-DSG options. K-beam-DSG options

L/h = 5 L/h = 10 L/h = 100 L/h = 1000 L/h = 10000 1

P1-1-QS 0.958 0.944 0.938 0.938 0.938 P1-2-QS 0.999 1.000 0.959 0.206 0.003 P1-3-QS 0.997 0.996 0.983 0.517 0.012 P2-2-QS 1.001 1.001 0.993 0.540 0.011 P2-3-QS 1.003 1.003 0.994 0.505 0.010 P3-3-QS 1.001 1.001 1.001 1.001 1.001 It can be seen

from Table 4 that for moderately thick to thin beams (i .e., 63

L/h = 5 to 100), all K-beam-DSG options could produce accurate results. However, when the beam becomes thinner, it is apparent that only the K-beam-DSG elements with P1-1-QS and P3-3-QS options show no locking. Further testing of the P1-1-QS and P3-3-QS options with length-to-thickness ratios L/h greater than 104 reveals that there is indeed no locking up to L/h = 107. For L/h over 107, the results become inaccurate because of the ill- conditioned stiffness matrix. It is worth noting that K-beam-DSG P1-1-QS is actually identical to the locking-free linear Timoshenko beam element with DSG presented in

[Bischoff et al. (2003); Bletzinger et al. (2000)]. 4

To further investigate why the shear locking phenomenon remains in the K-beam-DSG elements with linear (except with one layer) and quadratic basis functions, we considered the shear forces calculated using the ‘successful’ options, namely P1-1-QS and P3-3-QS, and ‘unsuccessful’ options, namely P1-3-QS and P2-3-QS, for the extremely thin beam (L/h = 104). The shear force diagrams are plotted in Figs. 6a and 6b, respectively. Figure 6a shows 16 F.T. Wong, A. Sulistio, H. Syamsoeyadi that the shear force distribution obtained from the K-beam-DSG element with P1-1-QS is, as expected, piecewise constant over each element. This indicates

that the element is able to give an optimal approximate solution in the 77 constant strain function space. The shear force distribution corresponding to the P3-3-QS option oscillates about the exact distribution. This oscillation, however, is very mild compared to the violent shear force oscillations obtained from the K-beam-DSG elements with P1-3-QS and P2-3-QS (Fig. 6b). The total area under the shear force curves of P1-1-QS and P3-3-QS seems to be

equal to the total area under the exact **curve.** 72

Furthermore, it was found that with P1-1-QS and P3-3-QS options, the shear force distributions for the beams of L/h greater than 104, up to L/h = 107, remain the same as those for the beam of L/h = 104. This is in contrast to the oscillation of shear force with the other K-beam element options, which are much heavier as the beams become thinner. This is the reason why the K-beam-DSG elements with P1-1-QS and P3-3-QS options are free of locking while they suffer from shear locking with the other options.

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17 (a) (b) Fig. 6. Shear force diagram for the clamped-clamped beam with L/h = 104 obtained using K-beam-DSG elements with options: (a) P1-1-QS and P3-3-QS; (b) P1-3-QS and P2-3-QS. 18 F.T. Wong, A. Sulistio, H. Syamsoeyadi To assess the locking behavior and accuracy of the K-beam-DSG elements in comparison to the original K-beam elements (without any locking treatment) and to the K- beam elements with the selective reduced integration (SRI) technique [Wong and Syamsoeyadi (2011)], the results obtained from a series of analyses using these K- beam elements for the beams with L/h = 5 and L/h = 10000

are listed in Table 5. **It can be seen** from Table 5 **that** 12

without any locking treatment, the K-beam elements suffer from shear locking even for the element with a cubic polynomial basis. The

SRI technique is effective at eliminating **the locking** 6

for all K-beam element options. However, this technique produces less accurate results

for the thick beam (L/h ? 5). **The** DSG technique produces **the** 2

most accurate results for the thick beam, but it is only effective at eliminating the shear locking for the K-beam elements with the P1-1-QS and P3-3-QS options. Table 5. Normalized deflections

obtained using the original **and** SRI K **-beam elements** and **the** K **-beam-** DSG **elements.** 1

K-beam L/h = 5 options Original SRI DSG P1-1-QS 0.887 0.958 0.958 1.96E-06 0.938 0.938 P1-2-QS 0.979 1.018 0.999 5.34E-05 1.031 0.003 P1-3-QS 0.983 1.031 0.997 4.33E-04 1.049 0.012 P2-2-QS 0.994 1.001 1.001 P2-3-QS 0.991 1.036 1.003 P3-3-QS 0.999 1.011 1.001 L/h = 10000 Original SRI DSG 5.28E-05 0.999 0.011 0.001 1.049 0.010 0.002 1.006 1.001 In the subsequent tests, we only considered the K-beam-DSG element with the P3-3-QS option since this element is free from shear locking and truly Kriging-based finite elements (the element with the P1-1-QS option is essentially identical to the conventional DSG linear beam element [Bletzinger et al. (2000)]). 5.1.3. Assessment of accuracy and convergence

To assess the accuracy and convergence characteristics of the K **-beam** element with **the** 1

P3-3-QS option, we considered

a cantilever beam subjected to a linearly varying **distributed load, as** illustrated **in** 1

Fig. 7, under different beam thickness conditions, that is, normal (L/h ? 8), extremely thick (L/h ? 1), and extremely thin (L/h ? 10000). The problem

parameters were taken as L ? 4 **m, b ? 2 m, E** ? 1000 **kN/** m2, **v ? 0.3, and** q0 ? **1 kN/m.** The beam **was** 1

analyzed using different degrees of mesh refinement, that is, 4, 8, 16, and 32 elements. The deflection

at the free end, the bending moment, and the **shear force at the** clamped **end were observed and** normalized **to the corresponding exact solutions,** 1

viz. [Friedman and Kosmatka (1993)]: wL ? 3q00LE4I (1 ? ?) 5 (36a) 12

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19 Fig. 7. Cantilever beam subjected to linearly varying load. ? ? 5 (12 ? 11?) ??? hL ???2 1 (36b) M0 ? 16 q0L2 , Q0 ? 12 q0L (37a, b) The results are presented in Tables 6, 7, and 8 for the normal-thickness, extremely thick, and extremely thin beams, respectively, along with the results obtained using the K-beam P3-3-QS element with the SRI technique and using the original K-beam (without any shear locking treatment). Tables 6 and 7 show that the K-beam-DSG P3-3-QS element can produce remarkably accurate deflection and bending moment and reasonably accurate shear stress, even using relatively course mesh (four elements), for the extremely and moderately thick beams. For the extremely thin beam (Table 8), while

the deflection remains very accurate, the accuracy of the bending moment drops a little. The resulting shear stresses, however, are not very accurate for the extremely thin beam. Tables 6–8 show that all the results converge very well to the corresponding exact solutions.

While the SRI technique can eliminate the shear **locking,** 1

it deteriorates the accuracy in the cases of thick beams (compared to the original K-beam element solutions). Overall,

the results of the K **-beam-** DSG **element** 6

are better in comparison to those obtained from the original and SRI K-beam elements. Thus,

application of the DSG technique to **the** K **-beam elements** has improved **the** 1

elements' performance. Table 6. Results obtained for the normal-thickness beam (L/h ? 8) using the K-beam P3-3-QS elements with the DSG technique, the SRI technique, and the original element. (a) Normalized free-end deflection Number of K-beam P3-3-QS elements DSG SRI Original 4 0.99989 1.00534 0.99989 8 0.99999 1.00069 0.99999 16 1.00000 1.00017 1.00000 32 1.00000 1.00006 1.00000 20 F.T. Wong, A. Sulistio, H. Syamsoeyadi Table 6. Results obtained for the normal-thickness beam (L/h ? 8)… (continued) (b) Normalized clamped-end bending moment Number of K-beam P3-3-QS elements DSG SRI Original 4 0.99972 1.07761 0.92760 8 1.00190 1.04268 0.99075 16 1.00074 1.02153 0.99917 32 1.00022 1.01071 0.99993 (c) Normalized clamped-end shear force Number of K-beam P3-3-QS elements DSG SRI Original 4 1.03397 4.25756 1.62985 8 1.00210 1.90156 1.11862 16 1.00022 1.44091 1.01848 32 1.00003 1.34918 1.00251 Table 7. Results obtained

for the extremely **thick beam (L/h** = 1) using **the** K **-beam** 2

P3-3-QS elements with the DSG technique, the SRI technique, and the original element. (a) Normalized free-end deflection Number of elements DSG SRI Original 4 0.99995 1.01852 0.99995 8 1.00000 1.00864 1.00000 16 1.00000 1.00359 1.00000 32 1.00000 1.00162 1.00000 (b) Normalized clamped-end bending moment Number of elements DSG SRI Original 4 1.00055 1.07761 0.98400 8 1.00192 1.04268 0.99783 16 1.00074 1.02153 0.99974 32 1.00022 1.01071 0.99997 (c) Normalized clamped-end shear force Number of elements DSG SRI Original 4 1.00097 1.43572 1.01403 8 1.00019 1.36542 1.00234 16 1.00004 1.33960 1.00034 32 1.00001 1.32864 1.00005

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21 Table 8. Results obtained for the extremely thin beam (L/h = 10000) using the K-beam P3-3-QS

elements with the DSG technique, the SRI technique, and the original element. (a) Normalized free-end deflection Number of elements DSG SRI Original 4 0.99989 1.00486 0.99989 8 0.99999 1.00041 0.99999 16 1.00000 1.00004 1.00000 32 1.00000 1.00001 1.00000 (b) Normalized clamped-end bending moment Number of elements DSG SRI Original 4 0.95561 1.07761 0.69987 8 0.95087 1.04268 0.70047 16 0.98499 1.02153 0.72160 32 0.99917 1.01072 0.91343 (c) Normalized clamped-end shear force Number of elements DSG SRI Original 4 2.79 4.5E+06 4.7 8 2.59 8.5E+05 16.5 16 0.61 1.6E+05 28.6 32 0.98 3.3E+04 17.6 To further demonstrate the accuracy of the K-beam-DSG P3-3-QS element in comparison to the original K-beam P3-3-QS element, the profiles of deflection, rotation, and bending moment for the extremely thin beam case, obtained from the analyses using eight elements, are plotted in Figs. 8a to 8c. The shear force profiles obtained from the K-beam- DSG and K-beam elements have not been plotted because they oscillated heavily about the exact shear force. The figures confirm that the K-beam-DSG P3-3-QS element is able to produce accurate

results for the deflection, rotation, and bending moment 10

even

in the case of the extremely thin beam. The shear force, however, is 19

not accurate. The

performance of the element is superior compared to the 10

original K-beam element. 22 F.T. Wong, A. Sulistio, H. Syamsoeyadi

(a) Deflection profile (b) Rotation profile (c) Bending moment profile Fig. 15

8. Profiles of

(a) deflection, (b) rotation, and (c) bending moment along the 15

extremely thin beam (L/h = 10000), obtained using the K-beam P3-3-QS element with the DSG technique in comparison to the exact profile and that obtained using the original K-beam P3-3-QS.

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5.2. Free vibration analysis 5.2.1. 73

Thick and thin clamped-clamped beams

To assess the performance of the K-beam-DSG **element** with **the** 19

P3-3-QS in free vibration analyses, we first considered

free vibration analysis **of** a **clamped-** clamped **beam with two different** 78

thickness conditions, that is, moderately

thick (L/h = 5) and very **thin (L/h** 6

= 1000). The parameters of the beam were

L = 10 m, b = 1 m, E 6

= 2 × 109 N/m2,

v = 0.3, and ρ = 10 **kg/m3. The** beam **was** 52

modeled using meshes of

4, 8, 16, and 32 elements. The resulting natural frequencies **were expressed in the form of** dimensionless **frequency** 2

parameters [Lee and Schultz (2004)], given as ?i ? ?i L2 ?EAI (38) where ωi

is the natural frequency of the i-th **vibration mode. The** natural frequencies **obtained from** 17

the

pseudo-spectral method [Lee and Schultz (2004)] were taken as **the** 80

reference solution for the moderately thick beam, while those obtained from the exact solution based on the Euler-Bernoulli beam theory 56

[Lee and Schultz (2004)] were taken as the reference solution for the very thin beam. The resulting frequency parameters for the first 15 vibration modes obtained using the DSG and original K-beam elements,

normalized with respect to the corresponding reference solutions, are presented in Tables 9 and 8

10 for the moderately thick and very thin beams, respectively. It is seen from Table 9 that for the moderately thick beam, the present K-beam element can give very accurate natural frequencies. Furthermore, the results obtained using the K-beam-DSG element are consistently more accurate than those obtained using the original K-beam. Using only four elements, the DSG K-beam element

can predict the first two natural frequencies within 5% accuracy while the original K-beam element can only predict one natural frequency. 7

Using 32 elements, the present

element can predict the first 15 natural frequencies 7

with an error of less than 0.5%. Table 10, however, indicates that the accuracy deteriorates considerably when the beam becomes very thin. Using a mesh of four elements, only the first natural frequency is predicted accurately. Using the same mesh, the original K-beam element gives erroneous results due to shear locking. For this very thin beam, the K-beam-DSG P3-3-QS element needs a mesh of 32 elements to obtain reasonable results for high natural frequencies. As in the case of the thick beam, in this case the performance of the DSG element is also better than that of the original K-beam element. 24 F.T. Wong, A. Sulistio, H. Syamsoeyadi Table 9.

Normalized frequency parameters for the moderately thick beam (L/h = 5). Mode λi/λi* Shape λi* 4 elements 8 elements 16 elements 32 2

elements DSG Original DSG Original DSG Original DSG Original 1 4.2420 1.0039 1.0370 1.0014 1.0180 1.0016 1.0180 1.0016 1.0180 2 6.4188 1.0144 1.1510 1.0021 1.0290 1.0022 1.0280 1.0023 1.0280 3 8.2853 1.1519 1.1940 1.0046 1.0400 1.0027 1.0340 1.0028 1.0340 4 9.9037 1.3579 1.4320 1.0104 1.0550 1.0029 1.0370 1.0031 1.0370 5 11.3847 1.2451 1.3240 1.0223 1.0690 1.0002 1.0370 1.0001 1.0360 6 12.6402 1.2002 1.2650 1.0475 1.0890 1.0042 1.0430 1.0034 1.0420 7 13.4567 – – 1.0104 1.0610 1.0045 1.0560 1.0044 1.0560 8 13.8101 – – 1.0461 1.0910 1.0054 1.0460 1.0035 1.0430 9 14.4806 – – 1.0385 1.0830 1.0041 1.0470 1.0037 1.0470 10 14.9383 – – 1.0512 1.0900 1.0074 1.0480 1.0036 1.0440 11

15.6996 – – 1.0753 1.1150 1.0041 1.0410 1.0031 1.0400 12 16.0040 – – 1.1284 1.1690 1.0107 1.0520 1.0038 1.0460 13 16.9621 – – 1.1340 1.1710 1.0042 1.0350 1.0025 1.0330 14 16.9999 – – 1.1963 1.2390 1.0152 1.0570 1.0041 1.0480 15 17.9357 – – – – 1.0203 1.0470 1.0044 1.0440

Note: * Convergent solution from the pseudo-spectral **method** 6

[Lee and Schultz (2004)] Table 10.

Normalized frequency parameters **for the very thin beam (L/h = 1000).** 2

Shape λi*

4 elements 8 elements 16 elements 32 elements 65

Mode λi/ λi* DSG Original DSG Original DSG Original DSG Original 1 4.7300 1.0044 7.5960 1.0012 1.6020 1.0000 1.0280 2 7.8532 4.2232 12.3080 1.1775 2.1730 1.0017 1.1480 3 10.9956 12.2361 12.6810 1.7695 4.1300 1.0133 1.3170 4 14.1372 173.2836 195.6570 2.2220 6.4780 1.0589 1.4490 5 17.2788 147.5479 160.0830 2.7744 7.9750 1.1689 1.5410 6 20.4204 129.1803 135.4550 4.0242 8.7380 1.3321 1.8260 7 23.5619 – – 8.9171 9.2180 1.4923 2.4440 8 26.7035 – – 76.5291 103.5830 1.6049 3.2090 9 29.8451 – – 80.4533 92.6800 1.6013 3.9900 10 32.9867 – – 75.6397 83.8530 1.5731 4.7140 11 36.1283 – – 71.2762 76.5620 1.7623 5.3280 12 39.2699 – – 66.7072 70.4380 2.2091 5.8200 13 42.4115 – – 61.9115 65.2200 2.9443 6.2270 14 45.5531 – – 57.6457 60.7230 4.2292 6.5440 15 48.6947 – – – – 6.3361 6.6620 1.0000 1.0000 1.0000 1.0040 1.0000 1.0120 1.0002 1.0300 1.0010 1.0560 1.0030 1.0890 1.0074 1.1220 1.0158 1.1520 1.0298 1.1750 1.0508 1.1910 1.0791 1.2050 1.1138 1.2340 1.1521 1.3040 1.1905 1.4360 1.2253 1.6250

Note: * Exact solution of Euler-Bernoulli beam theory 2

[Lee and Schultz (2004)]

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25 5.2.2. Thick simply supported beam To assess the performance of the K-beam-DSG P3-3-QS element in comparison to the exact and consistent

Timoshenko beam element developed by Friedman and Kosmatka 2

[1993] (‘exact’ within static analysis), we adopted the beam problem as described in t h e reference. The problem is a simply supported beam with the parameters b = 0.2, h = 0.2, E = 1, v = 0.3, and L/h = 5. The first six natural frequencies were calculated using three different meshes, that is, 4, 8, and 20 elements, and then

normalized to the first natural frequency based upon the Euler -Bernoulli beam theory, 7

that is, ?I ? ??? ?L ??? 2 EI ? A (39) The

results are presented in Table 11. It can be seen from the table that 13

when only four elements are used, the current element can predict the first two natural frequencies 7

more accurately than the Friedman and Kosmatka element [Friedman and Kosmatka (1993)] . When more

elements are used, the current element can predict the first six natural frequencies 7

consistently more accurately than the Friedman and Kosmatka element. Table 11. First six

normalized natural frequencies of a thick simply supported beam (L/h 18

= 5) obtained using the current K-beam-DSG P3-3-QS element and Friedman and Kosmatka element (F&K) [Friedman and Kosmatka (1993)]. Mode ω/ωT Shape Exact*

4 elements 8 elements 20 elements Current F&K Current F&K Current 7

F&K 1 0.9404 1.0003 1.0024 0.9999 1.0006 2 3.2672 1.0090 1.0281 0.9996 1.0067 3 6.2514 1.3180 1.0952 1.0019 1.0241 4 9.4970 1.8297 1.5101 1.0125 1.0550 5 12.8357 1.4408 1.4682 1.0446 1.0985 6 16.1981 1.3535 1.2714 1.0727 1.1371 1.0001 1.0001 0.9999 1.0010 0.9998 1.0038 0.9997 1.0088 0.9997 1.0161 1.0000 1.0258 * Timoshenko [1922] as cited in Friedman and Kosmatka [1993] 5.3. Buckling analysis

To study the accuracy and convergence of the K- 3

beam-DSG P3-3-QS element

in predicting the critical buckling load of axially loaded 60

beams, we performed analyses of simply supported and clamped-clamped beams for a variety of beam

length-to-thickness ratios, that is, L /h = 5, 10, 76

100, and 1000, using different meshes of 4, 8, 16, and 32 elements. A comparison is made with the original K-beam P3-3-QS element. The resulting critical loads (Tables 12 and 13) were normalized to the exact solution [Bažant and Cedolin (1991) as cited in K o s m a t k a (1 9 9 5)] , which

accounts for the effect of shear deformation, that is, 10

26 F.T. Wong, A. Sulistio, H. Syamsoeyadi Pcr ? L22eEffI ??? 1 ? ? ? ? ? 1 ? ? 2 EI ? ? (40) ? L2eff GAs ? ? where Leff is the effective length of the beam (L

for simply supported beams, L/2 for clamped-clamped beams). 15

It is observed from Tables 12 and 13 that all results converged very well to the corresponding exact solutions. While for the thick beam conditions

(L/h = 5 and L/h = 10) the accuracies of the 75

DSG and original elements are comparable, for the thin beams the performance of the DSG is superior to that obtained using the original element. For the clamped-clamped beam with

L/h = 100 and L/h = 1000 modeled using four elements, 18

the results obtained from the original K-beam element present large errors because of the shear locking.

Table 12. Normalized critical buckling loads of a simply supported 53

beam obtained using the K-beam element with the DSG technique and the original K-beam element. Number of

L/h = 5 L/h = 10 L/h = 100 L/h = 1000 1

elements DSG Original DSG Original DSG Original DSG Original 4 0.9986 1.0125 0.9985 1.0417 0.9985 1.2072 0.9985 1.2158 8 0.9997 1.0001 0.9997 1.0002 0.9998 1.0094 1.0002 1.1731 16 1.0000 1.0000

1.0000 1.0000 1.0000 1.0002 1.0000 1.0169 32 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0003 Table 13. Normalized critical buckling loads of a clamped-clamped beam obtained using the K-beam element with the DSG technique and the original K-beam element. Number of

L/h = 5 L/h = 10 L/h = 100 L/h = 1000 1

elements DSG Original DSG Original DSG Original DSG Original 4 1.1254 1.1157 1.1654 1.3451 1.1850 29.9669 1.1852 2891.6 8 0.9972 1.0012 0.9965 1.0029 1.0043 1.0568 1.0612 5.5373 16 0.9997 1.0000 0.9997 1.0001 0.9997 1.0024 1.0028 1.1160 32 1.0000 1.0000 1.0000 1.0000 1.0000 1.0001 1.0000 1.0034 6.

Conclusions The DSG technique has been applied in **the** 1

framework of Kriging-based Timoshenko beam

elements in an attempt to eliminate shear locking. The 2

essence of the **technique is to replace the** original kinematic **shear strain with a substitute shear strain** derived **from the** interpolation **of** discrete **shear** gaps at **the** 1

nodes in the DOI. The developed elements, referred to as K-beam-DSG elements, were then tested in static, free vibration, and bucking analyses. The results showed that the DSG technique was effective at eliminating the shear

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27 locking for the element with a cubic basis function and three element-layer DOI (P3-3- QS) but was not very effective for the other K-FEM options. In all cases, the performance of the K-beam elements was improved by the DSG technique. For a practical range of beam thicknesses, the K-beam-DSG P3-3-QS element can achieve exceptionally accurate defections, rotations, bending moments, natural frequencies, and critical buckling loads and reasonably accurate shear forces with a relatively course mesh. For extremely thin beams, however, the accuracy may deteriorate and the resulting shear force is not accurate

with a small number of elements. The application **of the** 59

DSG technique to the Timoshenko beam model provides understanding and insight regarding its effectiveness at eliminating shear locking.

Future research may be directed to apply **the DSG technique to** 1

Kriging-based shear deformable plate and shell models. Acknowledgments We gratefully acknowledge that

this research has been partly **supported by the** internal **research** 69

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