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1 1 On the Accuracy and Convergence of the Hybrid FE- 2 meshfree Q4-CNS Element in Surface Fitting Problems 3 Foek Tjong Wong1, Richo Michael Soetanto2 & Januar Budiman3 5 4 Master Program

of Civil Engineering, Petra Christian University, Surabaya, Indonesia Email: 1 **wftjong@petra.ac.id, 2** [9]

hore_yippie@yahoo.co.id, 3 januar@petra.ac.id 7 6 8 Abstract. In the last decade, several hybrid methods combining the finite 10 9 element and meshfree methods have been proposed for solving elasticity problems. Among these methods,

a novel quadrilateral four-node element with 11 **continuous nodal stress (Q4-CNS) is of** [1]

our interest.

In this method, the shape 12 **functions are constructed using** [29]

the combination

of the 'non-conforming' shape 13 functions **for the** Kirchhoff's plate **rectangular element** and **the** [1]

shape functions 14 obtained using

an orthonormalized and constrained least-squares method. [1]

The 15 key advantage of the Q4-CNS element is that it provides the continuity of the 16 gradients at the element nodes so that the global gradient fields are smooth and 17 highly accurate.

This paper presents a numerical **study on the** accuracy **and** 25

18 convergence of the Q4-CNS interpolation and its gradients in surface fitting 19 problems. Several functions of two variables were employed to examine the 20 accuracy and convergence. Furthermore, the consistency property of the Q4- 21 CNS interpolation was also examined. The results show that the Q4-CNS 22 interpolation possess a bi-linier order of consistency even in a distorted mesh. 23 The Q4-CNS gives highly accurate surface fittings and possess excellent 24 convergence characteristics. The accuracy and convergence rates are better than 25 those of the standard Q4 element. 26 Keywords: continuous nodal stress; finite element; meshfree; Q4-CNS; quadrilateral 27 four-node element; surface fitting. 28

1 Introduction 29 **The finite element method** (FEM) **is** now a **widely-used,** well-establish 30 **numerical method** 14

for solving mathematical models of

practical problems 31 **in engineering and science.** 35

In practice, FEM users often prefer to use 32 simple, low order triangular or

quadrilateral elements in 2D problems and 33 **tetrahedral elements in 3D problems** 15

since these elements can be Received ________, Revised _________, Accepted for publication __________ 34 automatically generated with ease for meshing complicated geometries. 35 Nevertheless, the standard low order elements produce discontinuous 36 gradient fields on the element boundaries and their accuracy is sensitive 37 to the quality of the mesh. 38 To overcome the FEM shortcomings, since the early 1990's up to present 39 a vast amount of

meshfree (or meshless) methods [1], [2], **which do not** 40 require **a mesh** in discretizing **the problem domain,** 5

have been proposed. 41 A recent review on meshfree methods presented by Liu [3]. While these 42 newer methods are able to eliminate the FEM shortcomings, they also 43 have their own, such as: (i) the computational cost is much more 44 expensive than the FEM, and (ii) the computer implementation is quite 45 different from that of the standard FEM. 46 To synergize the strengths of the finite element and meshfree methods 47 while avoiding their weaknesses, in the last decade several hybrid 48 methods combining the two classes of

methods based on the concept of 49 partition-of -unity have been 1

developed [4]-[8]. Among several hybrid 50 methods available in literature, the authors are interested in the

four-node 51 quadrilateral element with continuous nodal stress (Q4-CNS) proposed 1

52 by Tang el al. [6] for the reason that this work is the pioneering hybrid 53 method possessing the property of continuous nodal stress. The

Q4-CNS 54 can be regarded as an improved version of the FE- 19

LSPIM Q4 [4], [5]. In 55 this novel method, the nonconforming shape functions for the 56 Kirchhoff's plate rectangular element are combined with the shape 57 functions obtained using

an orthonormalized and constrained least- 58 squares method. 1

The advantages of the Q4-CNS are [6], [9], [10]: (1) the 59

shape functions are C1 continuous at nodes so that it 1

naturally provides a 60 globally smooth gradient fields. (2) The Q4-CNS can give higher 61

accuracy and faster convergence rate than the standard quadrilateral 62 element (Q4). 1

(3) The

Q4-CNS is more tolerant to mesh distortion. 1

63 The Q4-CNS has been developed and applied for the

free and forced 64 vibration analyses of 2D 33 solids [9] and for 2D crack propagation analysis 65 [10]. Recently the Q4-CNS has been further developed to its 3D 66 counterpart, that is, the

hybrid FE-meshfree eight-node hexahedral 67 **element with continuous nodal stress (Hexa8-CNS)** 10

[11]. However, 68 examination of the Q4-CNS interpolation in fitting surfaces defined by 69 functions of two variables has not been carried out. Thus, it is the 70

purpose of this paper to present a numerical study **on the** on **the** 18

accuracy 71 and convergence of the Q4-CNS shape functions and their derivatives in 72 surface fitting problems. Furthermore, the consistency (or completeness) 73 property of the Q4-CNS shape functions is numerically examined in this 74 study. 75 2 The Q4-CNS Interpolation 76 As in the standard finite element procedure, a 2D problem domain, ? , is 77 firstly divided into four-node quadrilateral elements to construct the Q4- 78 CNS shape functions. Consider a typical element ?e with the local node 79 labels 1, 2, 3 and 4. The unknown function u on the interior and boundary 80 of the element is approximated by 81 uh(x, y) ? ? 4i?1wi(?,?)ui(x, y) (1) 82 where wi(ξ,η)

and ui(x, y) **are the weight** functions **and nodal** 83 approximations, **respectively, associated with node i,** 1

i=1,…,4. Note that 84 in the classical isoparametric four-node quadrilateral element (Q4), the 85 weight functions are given as the shape functions and the nodal 86 approximations are reduced to nodal values ui. The weight functions in 87 the Q4-CNS are defined as the non-conforming shape functions for the 88 Kirchhoff's plate rectangular element [6], [12], that is, 89 wi (?, ?) ? 18 (1 ? ?0)(1 ? ?0)(2 ? ?0 ? ?0 ? ?2 ? ?2) , (2a) 90

?0 ? ?i ? , ?0 ? ?i ? , i=1,2,3,4. 21

(2b) 91 where ξ and η are the natural coordinates of the classical Q4 with the 92 values in the range of –1 to 1. The weight functions satisfy the partition 93 of unity property, that is, ?14 wi (?, ?) ? 1 . The nodal approximations 94 ui(x,y) are constructed using

the orthonormalized and constrained least- 95 **squares method (CO-LS)** 5

as presented by Tang et al. [6] and Yang et al. 96 [9], [10]. Here the CO-LS is briefly reviewed. 97 To construct the CO-LS approximation, nodal support domains of node i, 98 ?i , i=1,…,4 of a typical quadrilateral element ?e are firstly defined 99 using the neighboring nodes of node i. For example, the nodal support 100 domain of node 3 of element e is shown in Fig. 1(a). The element support 101 domain ? ˆe is then defined as the union of the four nodal support 102 domains, that is, ?ˆe ? ?14 ?i , as shown in Fig. 1(b). 103 Consider a nodal support domain

of node i, ?i with **the total number of** 104 supporting **nodes** n. Let **the** 22

labels for the nodes be j, j=1,…, n. Using the 105

least-squares method, the nodal approximation ui(x, y) **is** 1

given as 106 ui

(x, y) ? pT **(x, y)A?** 1Ba (3) 107 **where p(x, y) is a vector of** polynomial **basis** 3

functions, viz. 108 pT (

x, y) ? ?1 x y x2 xy y2 ? (1? m) **(4)** 6

109 110 111 112 113 114 115 116 117 118 119 120 121 (a) ?3 (b) ?ˆe Figure 1 Definitions of: (a) the nodal support domain of node 3

of element e and (b) **the** element support **domain of element** 6

e. Here

m is the number of monomial bases **in p.** 4

Following the original work [6], in this study the ‘serendipity’ basis function pT

(x,y)??1 x y x2 xy y2 x2y xy2? 16

is used if n?8 and the bi- linearbasisfunction pT(x,y)??1 x y xy? isusedif n?8.MatricesA andBarethemomentmatrixandthebasismatrix,respectively,givenas B

??p(x1,y1) p(x2,y2) p(xn,yn)? (m **?n)** 17

A??nj?1p(xj,yj)pT(xj,yj) (m?m) (5) (6) Vector a??a1 a2 an?Tis the vector of nodal parameters. Note that in general vector a is not a vector of nodal values because the approximation ui(x,y)

does not necessarily **pass through the nodal values.** Defining **the** 24

inner product for any two basis functions f

(x,y) and **g(x,y)** as 122 ? **f (x, y),g(x, y)?** ? ?nj?1 **f** (xj, yj **)g(** 11

xj, yj) (7) 123 and

using the Gram-Schmidt orthonormalization algorithm [6], **the** basis 124 **vector p can be** 4

transformed into an orthonormal basis function vector r 125 so that the moment matrix A becomes the identity matrix. Subsequently, 126 the nodal approximation is constrained using the Lagrange multiplier 127 method so that the nodal parameter ui(x,y) at node i is equal to the nodal 128 value ui. Going through the abovementioned process, the nodal 129 approximation, Eqn. (3), turns into 130

ui(x, y) ? Φ **(x, y)** a ? ?nj **?1?** ij **(x, y)** 31

aj (8) 131 where 132 Φ

(x, y) ? ???i1(x, y) ?i2(x, y) ?in **(x, y)??** ? rT **(x, y)** 13

Bi (9) 133 Bi ? ??B1i Bi2 Bin?? (10) 134 Bij ? r(xj, yj) ? fjir(xi, yi) , j=1, …, n (11) 135 f ji ? ????? (r(Tr(Tx(ix,iy,iy)ir)(rx(jx, jy, jy)j?))1)(r(Tr(Tx(ix,iy,iy)ri)(rx(ix,iy,iy))i)) iiff jj ? i ? i (12) 136 Note that n,

the number of nodes in the nodal **support domain of node i,** 3

137 in general varies with i. 138 Consider now the element support domain of element e, ?ˆe , with the 139 total number of nodes N. Let the node labels in ?ˆe be I=1, …, N. Using 140 this element level labelling system and substituting Eqn. (8) into Eqn. 141 (1), the approximate function can be expressed as 142 uh(x, y) ? ?4i?1wi(?,?)?NI?1?iI(x, y)aI ? ?NI?1?I(x, y)aI (13) 143

in which ?I (x, y) **is the** Q4-CNS **shape function** associated with **node I** 4 in 144 the element support domain. In this equation, if node I is not

in the nodal 145 **support domain of node i,** 3

then ?iI (x, y) is defined to be zero. It is obvious 146 that the shape function is the product of the nonconforming rectangular 147 element shape functions wi(ξ,η) and the CO-LS shape functions ?iI (x, y) , 148 that is, 149 150 ?I (x, y) ? ?i4?1wi(?,?)?iI (x, y) (14) 151 3 Numerical Tests 152 In this section, the accuracy and convergence of the Q4-CNS 153 interpolation in fitting surfaces of z ? f (x, y) and their derivatives are 154 examined. To measure the approximation errors, the following relative L2 155 norm of error is used 156 rz ? ??h(z?zh)2dA ??hz2dA (15) 157 in which z is the function under consideration, zh is the approximate 158 function, and ?h is the approximate domain with the element 159 characteristic size, h. This expression is also applicable to measure the 160 relative error of the function partial derivatives (replacing z and zh with 161 their derivatives). The integral in Eqn. (15) is

evaluated numerically 162 **using Gaussian quadrature rule. The number of** 20

quadrature sampling 163 points is taken to be 5?5.

For the purpose of comparison, the **accuracy** 164 **and convergence** of **the** 27

standard Q4 interpolation and its partial 165 derivatives are also presented. 166 3.1 Shape function consistency property 167 In order to be applicable as the basis functions in the Rayleigh-Ritz based 168 numerical method, a set of shape functions is required to be able to 169 represent exactly all polynomial terms of order up to m in the Cartesian 170 coordinates [13], where m is the variational index (that is, the highest 171 order of the spatial derivatives that appears in the problem functional). A 172 set of shape functions that satisfies this condition is called m-consistent 173 [13]. This consistency property is a necessary condition for convergence 174 (that is, as the

mesh is refined, the solution approaches to **the** exact 175 **solution** of **the** 7

corresponding mathematical model). 176 To examine the consistency property of the Q4-CNS shape functions, 177 consider a 10 ?10 square domain shown in Fig. 2. The domain is 178 subdivided using 4 ? 4 regular quadrilateral elements, Fig. 2(a), and 179 irregular quadrilateral elements, Fig. 2(b). The functions under 180 consideration are the polynomial bases up to the quadratic bases, that is, 181 z ?

1 , z ? x , z ? y , **z** ? xy , **z** ? x2 **and** 34 z ? y2 .

The results of the relative 182 **errors for the** **30**

Q4-CNS interpolation and its nonzero partial derivatives 183 are listed in Tables 1 and 2, respectively, together with those of the 184 standard Q4 interpolation. 185 186 187 188 (a) Regular mesh (b) Irregular mesh Figure 2 Square function domain of size 10-by-10 subdivided into: (a) regular and (b) irregular quadrilateral elements. On the Accuracy and Convergence of the Hybrid FE- … 11 189 Table 1 Relative L2 norm of errors for the approximation of different 190 polynomial basis functions using the regular and irregular meshes. Function Regular Mesh Irregular Mesh Q4-CNS Q4 Q4-CNS Q4 191 192 193 194 195 196 197 198 199 200 201

z=1 z **=x** z **=y** **z=xy** z **=x2** z **=y2** **23**

9.98E-16 1.41E-15 1.20E-15 1.39E-15 1.22% 1.22% 1.32E-17 0 0 1.49E-16 2.55% 2.55% 1.88E-15 2.82E-15 1.45E-15 4.59E-15 2.65% 2.33% 1.35E-17 0 0 2.37% 5.83% 5.37% Table 2 Relative L2 norm of errors for the approximation of nonzero polynomial basis function derivatives using the regular and irregular meshes. (a) Basis function derivatives with respect to x Function Derivative to Regular Mesh Irregular Mesh x Q4-CNS Q4 Q4-CNS Q4 z,x=1 9.11E-15 2.25E-16 2.15E-14 2.82E-16 z,x=y 9.36E-15 2.55E-16 3.06E-14 11.32% z,x=2x 6.70% 12.50% 10.94% 16.58% (b) Basis function derivatives with respect to y Function Derivative to Regular Mesh Irregular Mesh y Q4-CNS Q4 Q4-CNS Q4 z,y=1 8.71E-15 1.98E-16 9.61E-15 2.11E-16 z,y=x 1.02E-14 2.93E-16 3.58E-14 12.53% z,y=2y 6.70% 12.50% 10.30% 15.90% The tables show

that the Q4-CNS interpolation **is** capable **to reproduce exact** **1**

solutions up to the xy basis both for the domain with regular and irregular meshes. In other words, the Q4-CNS interpolation is consistent up to the xy basis. On the other hand, the Q4 interpolation is consistent 202 up to the same basis for the regular mesh, but it is only purely linear 203 consistent for the irregular mesh. This finding may partly explain the 204 reason the Q4

-CNS has higher tolerance to mesh distortion **32**

[6]. For the x2 205 and y2 bases, both the Q4-CNS and Q4 interpolations are not able to 206 produce the exact solutions, as expected. For these bases, the Q4-CNS 207 interpolation is consistently more accurate than the standard Q4. 208 The tables clearly reveals that the Q4-CNS interpolation is not consistent 209 up to all of the quadratic bases. As a consequence, the Q4-CNS is not 210 applicable to variational problems possessing variational index m=2, 211 including the Love-Kirchhoff plate bending and shell models. This is in 212 contradiction to the statement made in the original paper [6], which 213 mentioned that the Q4-CNS “is potentially useful for the problems of 214 bending plate and shell models”. If the Reissner-Mindlin theory is 215 adopted, however, the Q4-CNS is of course applicable. 216 3.2 Accuracy

and Convergence 217 3.2.1 Quadratic function 218 The accuracy and convergence of the Q4-CNS interpolation in fitting 219 functions in 2D domain are firstly examined using quadratic function 220 (adapted from an example in Wong and Kanok-nukulchai [14]) given as 221 z ?1? x2 ? y2 (16) 222 with two different domains, viz. 223 ?S ?

?(x, y) 0 ? x ? 1, 0 ? y ? 1? (17) 224 ?C ? **?(x, y)** x2 ? y2 ? 1, **x** ? 0, **y** **12**

? 0? (18) 225 The first domain, Eqn. (17), is the unit square while the second one, Eqn. 226 (18), is a quarter of the unit circle, both of which are located in the first 227 quadrant of the Cartesian coordinate system. The unit square is 228 subdivided using regular meshes of 2 ? 2 , 4 ? 4 , 8 ? 8 , and 16 ?16 square 229 elements. The quarter of the unit circle is subdivided into 3, 12, 27, and 230 48 quadrilateral elements as shown in Fig. 3 (taken from an example in 231 Katili [15]). 232 The relative error norms of the Q4-CNS and Q4 interpolations in 233 approximating the quadratic function, Eqn. (16), and its partial 234 derivatives, are presented

in Table 3 **for** the **square** domain **and in Table** 4 235 **for** **28**

the quarter circle domain. The tables show that the Q4-CNS 236 interpolation converges very well to the quadratic function z both for the 237 regular mesh in the unit square domain and for the relatively irregular 238 mesh in the quarter of the unit circle domain. The tables also confirm that 239 the Q4-CNS interpolation is consistently more accurate than the Q4 240 interpolation. The finer the mesh the more accurate the Q4-CNS 241 interpolation compared to the Q4. 242 3 elements 12 elements 27 elements 48 elements 243 Figure 3 A quarter of the unit circle subdivided into different number of 244 quadrilateral elements (Katili [15], p.1899). 245 246 Table 3 Relative L2 norm of errors for the approximation of the quadratic 247 function, rz, and its partial derivatives, rz,x and rz,y over the unit square 248 domain. M r z rz,x rz,y Q4-CNS Q4 Q4-CNS Q4 Q4-CNS Q4 2 10.18% 16.26% 22.77% 4 1.83% 4.07% 10.62% 8 0.33% 1.02% 4.13% 16 0.06% 0.25% 1.52% 25.00% 26.29% 28.87% 12.50% 12.26% 14.43% 6.25% 4.77% 7.22% 3.13% 1.76% 3.61% M: the number of elements on each edge 249 Table 4 Relative L2 norm of errors for the approximation of the quadratic 250 function, rz, and its partial derivatives, rz,x and rz,y over a quarter of the 251 unit circle domain. Number of rz rz,x rz,y elements Q4-CNS Q4 Q4-CNS Q4 Q4-CNS Q4 252 253 254 255 256 257 258 259 260 261 262 263 264 3 12 27 48 11.06% 2.51% 0.91% 0.44% 16.59% 4.52% 2.04% 1.15% 28.14% 14.56% 8.42% 5.64% 33.92% 16.16% 10.68% 7.99% 22.48% 12.57% 7.37% 4.97% 27.10% 13.96% 9.36% 7.03% (a) Relative error norms of interpolations (b) Relative error norms of interpolation x-partial derivative Figure 4 Convergence of the Q4-CNS and Q4 interpolations in approximating: (a) the quadratic function, (b) the partial

derivatives of the function with respect to x, over **the** **8**

unit square. The number in the legend indicate the average convergence rate. The relative error norms are plotted against the number of elements on each edge, M, in log-log scale as shown in Fig. 4. The convergence graphs for the partial derivatives with respect to y are similar to Fig. 4(b) and have the same convergence rates. The graphs show that the average convergence rate of the Q4-CNS interpolation is about 25% faster than that of the Q4. It is worth mentioning here that the convergence rates of 265 the Q4 interpolation, 2, and its partial derivatives, 1, are exactly the same 266 as predicted by the interpolation theory [16]. 267 3.2.2 Cosine function 268 The second function chosen to examine the accuracy and convergence of 269 the Q4-CNS interpolation is 270 z ? cos(x)cos(y) ? ? 2 2 (19) 271 defined over the square unit domain, Eqn. (17). The meshes used are the 272 same as those in the previous example. 273 The convergence graphs of the relative error norms of the Q4-CNS and 274 Q4 interpolations and their partial derivatives with respect to x are shown 275 in Fig. 5. The graphs confirm the superiority of the Q4-CNS interpolation 276 over the Q4 interpolation both

in terms of the accuracy and convergence 277 **rate.** 1

278 4 Conclusions 279 The consistency property, accuracy and convergence of the Q4-CNS 280 interpolation in surface fitting problems have been numerically studied. 281 The results show that the Q4-CNS interpolation is consistent up to the 282 bilinear basis both

for the regular and irregular meshes. It is 7

more 283 accurate than the Q4 in fitting the functions and their derivatives. In a 284 sufficiently fine mesh, the error norm of the Q4-CNS interpolation is 285 around 3 to 4 times smaller than that of the Q4, and the error norm of its 286 derivatives is around 1.5 to 2 times smaller than that of the Q4. The Q4- 287 CNS interpolation converge very well to the fitted function. Its 288 convergence rate is approximately 25% faster than that of the Q4. The 289 demerits of the present method is that the computational cost to construct 290 the shape function is much higher than the Q4 shape function. 291 292 293 294 295 296 297 298 (a) Relative error norms of interpolations (b) Relative error norms of interpolation of the x-partial derivative Figure 5 Convergence of the Q4-CNS and Q4 interpolations in approximating: (a) the bi-cosine function, (b) the partial

derivatives of the function with respect to x, over **the** 8

unit square. The number in the legend indicate the average convergence rate. 299 Acknowledgement 300 We gratefully acknowledge that this research is partially supported by the 301 research grant of the

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