A RATIONAL BUCKLING MODEL FOR THROUGH GIRDERS

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ABSTRACT

Buckling of a through girder generally is predicted by the so called U-frame approach which treats the compressed top flanges as compression members restrained elastically by the web stiffness. This study used a line-type finite element analysis to examine buckling behaviour of through girders, both a single span girder and a continuous girder. The elastic buckling loads were plotted for a range of span to height ratio.

Keywords : U-frame, through girder, distorsional buckling.

INTRODUCTION

Through girders provide an alternative design for bridges where a substructure below the deck may be prohibitive because of clearance requirements. Normally they comprise a fabricated I-section girder, with the cross-beams supporting any deck arrangement being attached to the bottom flange, as shown in Fig.1. The girders may be simply supported or continuous over internal supports. This study investigates both cases.

A major design consideration for through girders is instability of the top flange, which is subjected to compression. The compressed flanges are forced to buckle laterally. The buckling mode must necessarily be distortional [1], since the top flange is restrained only by the stiffness of the web, while the bottom flange is restrained against translation and some twist by the cross-beams. A typical distortional buckle is shown in Fig.2 in which the web distorts in its cross-section during buckling.

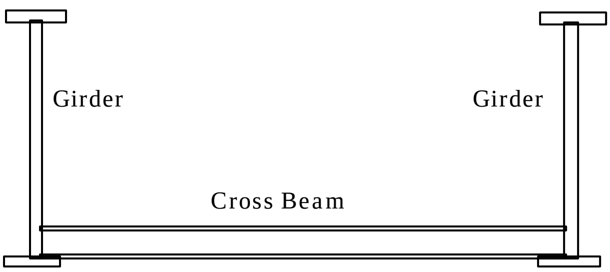

Figure 1. Cross Section of Through-Girder

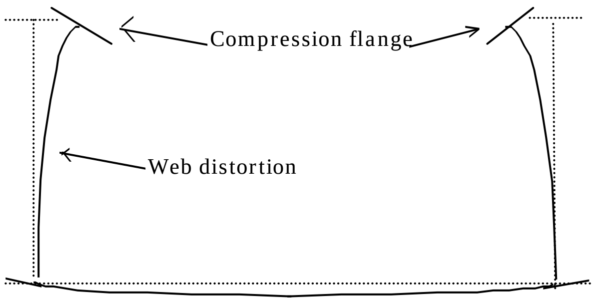

Figure 2. Distortional Buckling Mode

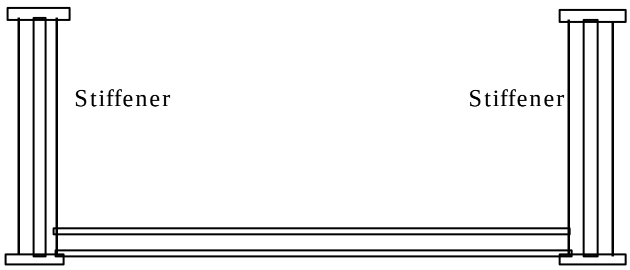

Figure 3. Discrete U-Frame

Since the webs of a through girder are often slender, vertical transverse stiffeners may be required to prevent buckling of the web in shear [2]. Moreover, web yielding and buckling in bearing will generally be alleviated by providing stockier load bearing stiffeners at the supports. The transverse cross-section through a stiffener is often referred to as a "discrete U-frame", shown in Fig. 3 as opposed to the section between these stiffeners, shown in Fig. 1 which is referred to as a "continuous U-frame".

The U-frame model was considered in a practical bridge study by Jeffers [3]. Using this method, the top flanges are modelled as compression members, restrained elastically against translation by the

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web flexibility. The flexibility of this restraint can be obtained by applying a pair of equal and opposite unit forces at the top flange level, as in Fig.4 and calculating the deflection Δ per unit length accounting for web flexing and bending rotation of the cross-beams. This is a conservative model, because in the real flange, the compression force decreases rapidly with distance from midspan. Moreover, there is tensile force in the flange over the internal support for continuous girder, and this resists the buckling [4].

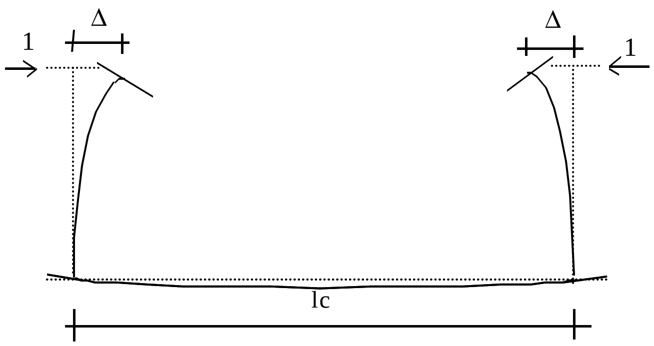


Figure 4. Flange Flexibility **D**

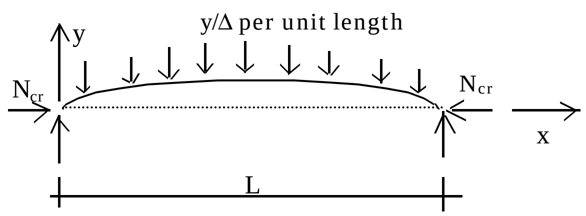


Figure 5. Laterally restrained pin-ended strut

Figure 5 shows a laterally restrained pin-ended strut subjected to a uniform compression force. An approximate solution can be obtained by assuming a sinusoidal buckling mode. Hence, it can be shown that the elastic critical load is [5]

$$N_{cr} = \frac{\pi^2 EI_f}{L^2} + \frac{L^2}{\pi^2 \Delta} \tag{1}$$

further solving Eq. 1 produces

$$\left(N_{cr}\right)_{min} = 2\left(\frac{EI_f}{\Delta}\right)^{1/2} \tag{2}$$

Equation 2 thus forms the basis of a design method for calculating the elastic critical stress $(\sigma_{cr})_{min} = (N_{cr})_{min}/b_f t_f$ in the top flange.

The above U-frame model is conservative since it is assumed that the top flange is uniformly stressed and it ignores the twist restraint provided by the web to the flange and also discrete U-frames. Svensson [6] analysed the strut as elastically restrained and subjected to several different longitudinal variations of compression, reflecting different distributions of bending moment along the girder. Because the beam cross section is uniform, the variations of compression force can be obtained by scaling the bending moment diagram by a constant. After setting up the differential equations for buckling of the strut, Svensson [6] solved these numerically to obtain the critical force $(N_{cr})_{min}$ in non-dimensional tabular form as a function of the elastic restraint $(1/\Delta)$ and distribution of bending moment. The method was subsequently refined by Williams and Jemah [7]. They presented various curves that give the lowest critical buckling load based on bending moment distributions and combinations of end conditions.

The U-frame model, despite being a useful structural idealization, is inaccurate and unable to adequately model discrete U-frames that occur at positions of web stiffeners. In order to overcome these limitations, an efficient and accurate line-type finite element analysis of distortional buckling developed some time ago by Bradford and Trahair [8] is augmented herein to include the elastic translational and torsional restraint provided to the bottom flange of the girder by the stiffness of the cross-beams. This element without the proposed augmentation has proven to be efficient for studying a variety of distortional buckling problems [1].

FINITE ELEMENT THEORY

General

The finite element deployed for this study has been outlined fully by Bradford and Trahair [8], and only a very brief summary is given herein. This model differs from the more usual finite element model of beams [9] in which the beam-column element is developed by idealizing the flanges as individual line elements, and the web as a two-dimensional element that may distort as a cubic curve in the plane of the cross section. The way in which the element is augmented to model through girders is then presented.

Figure 6 shows an elastically restrained cross-section of a beam or line element, which displaces u_T at the top and u_B at the bottom flanges and twists ϕ_T and ϕ_B at these flanges during buckling. These displacements are represented as cubic polynomials in terms of the nodal degrees of freedom.

$$\{q\} = \langle u_{T1}, u_{T2}, u'_{T1}, u'_{T2}, u_{B1}, u_{B2}, u'_{B1}, u'_{B2}, \phi_{T1}, \phi_{T2}, \phi_{B1}, \phi_{B2} \rangle^T \qquad (3)$$

where ϕ_T and ϕ_B are written as linear departures from the average twist $(u_T - u_B)/h$ and primes denote differentiation with respect to the longitudinal coordinate Z.

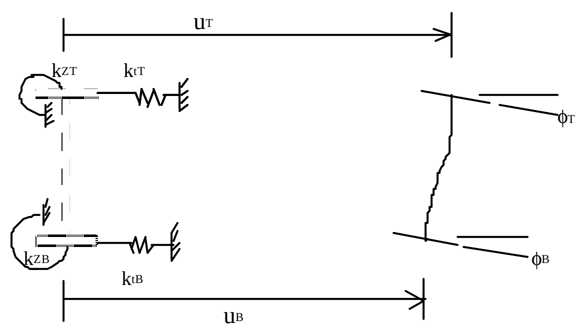
The elastic strain energy U in the unrestrained element produced by the buckling displacements can be written as

$$U = ½\ \{q\}^T[k]\{q\} \qquad (4)$$

while the bending moments and shears move in a conservative fashion [10] with the buckling displacements and do work

$$V = ½\{q\}^T \lambda\,[g]\{q\} \qquad (5)$$

where λ = the buckling load factor. The matrices [k] and λ [g] in Eqs. 4 and 5 are the stiffness and stability matrices respectively, and have been given explicitly by Bradford and Trahair [8]. Additional stiffness matrices for restraints are developed in the following.



Restraints and displacements

Figure 6. Line Element

Strain Energy Stored in Restraints

The elastic restraints shown in Fig. 6 inhibit lateral displacements of the top and bottom flanges with stiffness k_{tT} and k_{tB} per unit length respectively, and inhibit the top and bottom twists with stiffnesses k_{zT} and k_{zB} per unit length respectively. The elastic strain energy stored in these restraints is thus [10]

$$U_r = ½ \int_0^L \{u\}^T \begin{vmatrix} k_{tT} & 0 & 0 & 0 \\ 0 & k_{tB} & 0 & 0 \\ 0 & 0 & k_{zT} & 0 \\ 0 & 0 & 0 & k_{zB} \end{vmatrix} \{u\}\, dz \qquad (6)$$

in which $\{u\} = \langle u_T, u_B, \phi_T, \phi_B \rangle^T$. Upon expressing {u} in terms of the nodal degrees of freedom {q} and performing the integration, the strain energy stored in the restraints may be expressed as

$$U_r = ½\{q\}^T[k_r]\{q\} \qquad (7)$$

in which $[k_r]$ is the restraint stiffness matrix.

Strain Energy Stored in Discrete U-Frame

In formulating the elastic strain energy U in Eg. 4, one of the components is the strain energy stored in the deformable web due to its plate flexure in the plane of the cross section, and is given by

$$U_{wF} = \frac{1}{2}\left(\frac{E\,t_w^3}{12(1-v^2)}\right)\int_0^L\int_{-h/2}^{h/2}\left(\frac{\partial^2 u_w}{\partial y^2}\right)^2 dydz \qquad (8)$$

where u_w is the buckling displacement of the web which may be represented in terms of the vector {u} in Eq. 6 and hence {q} in Eq. 3, by using displacement and slope compatibility at the flange-web junctions. If distortion of the web is to be fully suppressed, as would occur at a position of a stiff stiffener in a discrete U-frame, the term U_{wF} in Eq. 8 should approach infinity [11]. Hence a stiffener strain energy may be written as

$$U_s = (\gamma_s - 1)\ U_{wF} \qquad (9)$$

where γ_s = a restraint factor (≥1), and the subtraction of unity in Eq. 9 is because U_{wF} is already included in Eq. 4. Equation 9 may therefore be written in the form

$$U_s = ½\ \{q\}^T(\gamma_s - 1)\,[k_s]\{q\} \qquad (10)$$

where $[k_s]$ is the stiffener stiffness matrix for a stiffener over a short element length L in the integration limit in Eq. 8. If b_s is the web stiffener width, then γ_s may be expressed as [9]

$$\gamma_s = \left(\frac{b_s}{t_w}\right)^3 \left(1 - v^2\right) + 1 \qquad (11)$$

Clearly if there is no stiffener in this short element, then $b_s = 0$, $\gamma_s = 1$, $U_s = 0$ and the strain energy resulting from web distortion is merely U_{wF} in Eq. 8 is included in Eq.4.

Buckling Solution

Each element in the girder is assigned elastic restraint stiffnesses k_{tT}, k_{tB}, k_{zT} and k_{zB} and a stiffness γ_s. The work done during buckling is thus, from Eqs. 4,7,10 and 5

$$\tfrac{1}{2}\{q\}^T([k]+[k_r]+(\gamma_s - 1)[k_s]-\lambda[g])\{q\}$$

From the usual considerations of equilibrium and compatibility at the nodes, the matrices [k], $[k_r]$ and $(1-\gamma_s)[k_s]$ for each element may be assembled into the global stiffness matrix [K], while the stability matrices $\lambda[g]$ for each element may be assembled into the global stability matrix $\lambda[G]$. The total work done during buckling of the whole girder Π is thus

$$\Pi = \tfrac{1}{2}\{Q\}^T([K]-\lambda[G])\{Q\} \tag{12}$$

where {Q} = the global buckling degrees of freedom assembled from the {q} vectors for each element. Minimising Π with respect to the vector {Q} in accordance with the Ritz method yields the familiar buckling equation

$$\frac{\partial \Pi}{\partial \{Q\}} = ([K] - \lambda[G])\{Q\} = \{0\} \tag{13}$$

which is solved in this study for the buckling load factor or eigenvalue λ and buckling mode shape or eigenvector {Q} by deploying standard eigensolvers, such as that outlined by Hancock [12].

Validation of Solution

The finite element method has been validated against closed form solutions for lateral torsional buckling when the elements are each considered to have a large value of γ_s, so that a condition of suppressed distortion is realised while maintaining a web thickness t_w [11]. In addition, the ability of the model to handle conditions of large buckling-induced web distortions has been demonstrated by a comparison with test data [1].

BEHAVIOUR OF A THROUGH GIRDER

General

A through girder with dimensions h = 1500 mm, b_f = 250 mm, t_f = 25 mm and with t_w = 10 mm or 15 mm has been studied. The lengths were chosen to give the ratio L/h of 10, 15, 20, 25, and 30. In the continuous girder these length were employed for one span length. Each girder was assumed to be subjected to a uniformly distributed load to represent an adverse bending moment distribution. The behaviour of single span girders under uniform moment were also examined. The bottom deck was represented by setting u_B, u'_B equal to zero because of the restraint provided by the deck. The finite element distortional buckling analysis has also been used to study the effect of different end conditions on the buckling of through girders. These end conditions are deployed by setting $u_T = 1$, $u'_T = 1$, $u_B = 0$, $u'_B = 0$, $\phi_T = 1$, $\phi_B = 0$ at the ends which allows top flanges to move laterally at the ends, these degrees of freedom were also applied to the other internal nodes. If the top flange are not allowed to move laterally (fixed) the degrees of freedom were set to $u_T = 0$, $u'_T = 1$, $u_B = 0$, $u'_B = 0$, $\phi_T = 0$, $\phi_B = 0$ at the ends and the internal nodes were as previously.

Buckling of The Unstiffened Girders

Figures 7 and 8 show the buckling loads for single span girder under uniform moment for web thickness (t_w) equal to 10mm and 15mm, respectively. The dimensionless buckling load is the ratio of the elastic distortional buckling load factor λ_{cr} to the load factor for the lateral-torsional buckling λ_{ob}. The curves obtained from Svensson [6] show conservatism compared to the finite element results. This is because Svensson does not take the web into account in his analysis.

Figures 9 and 10 show the buckling loads under uniformly distributed load for a simply supported girder, while figures 11 and 12 show the continuous girders. The lateral-torsional buckling load λ_{ob} was found by setting the degree of freedom $u_T = 0$, $u'_T = 1$, $u_B = 0$, $u'_B = 1$, $\phi_T = 0$, $\phi_B = 0$ at the supports and setting all the restraint parameters equal to 1 at the other nodes. Both twist stiffness of top and bottom flanges (k_{zT}, k_{zB}) were set equal to zero. The web rigidity was set equal to 4E + 3 to represent very rigid web, thus the cross section of the member remain undistorted. The value of λ_{ob} could also be found by [2] for simply supported girder:

$$M_{ob} = 1.13\frac{\pi\sqrt{(EI_y GJ)}}{L}\sqrt{\left(1+\frac{\pi^2 EI_w}{GJ\,L^2}\right)} \tag{14}$$

for continuous girder:

$$M_{ob} = 2.25\frac{\pi\sqrt{(EI_y GJ)}}{L}\sqrt{\left(1+\frac{\pi^2 EI_w}{GJ\,L^2}\right)} \tag{15}$$

where EI_y and GJ = minor axis flexural and torsional rigidities respectively, and EI_w = warping rigidity.

The approximate U-frame approach in Eq. 2 was also applied to the problem. Under a unit force applied laterally at the top flange, the web displaces Δ as a cantilever by

$$\Delta = \frac{h^3}{3EI_{web}} \quad (16)$$

where $I_{web} = t_w^3/12$. The critical moment is therefore $(N_{cr})_{min}$ x h, where $(N_{cr})_{min}$ is determined from Eq. 2. The critical moments obtained by the simplified U-frame model are also plotted in Fig. 7-12 with the legend 'Min'.

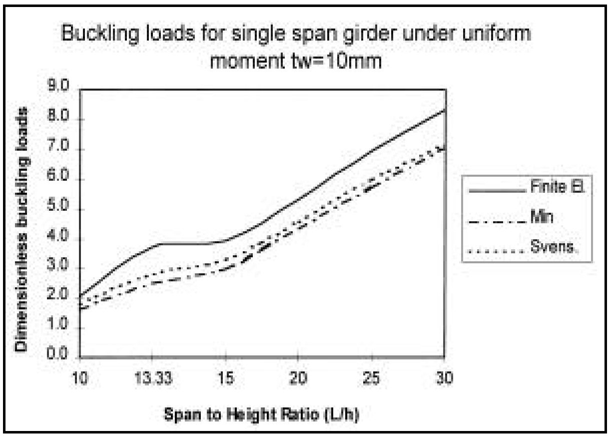


Figure 7. Buckling loads for single span girder under uniform moment for web thickness = 10mm

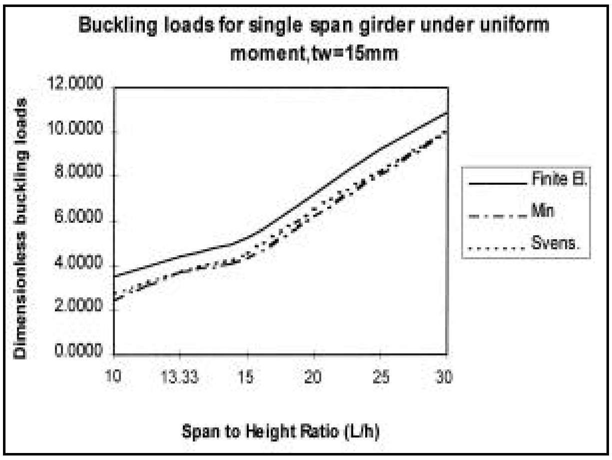


Figure 8. Buckling loads for single span girder under uniform moment for web thickness = 15mm

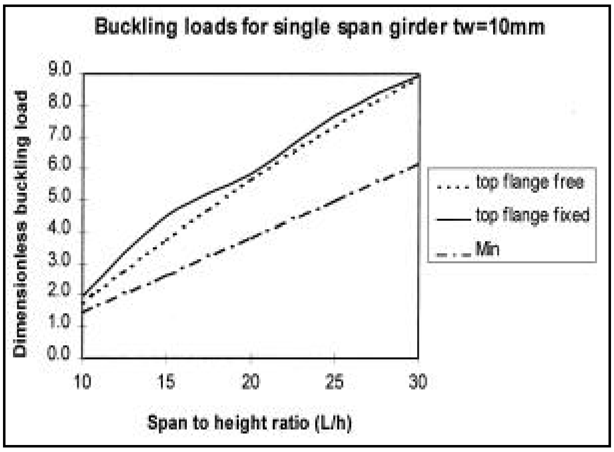


Figure 9. Buckling loads for single span girder for web thickness = 10mm under uniformly distributed load.

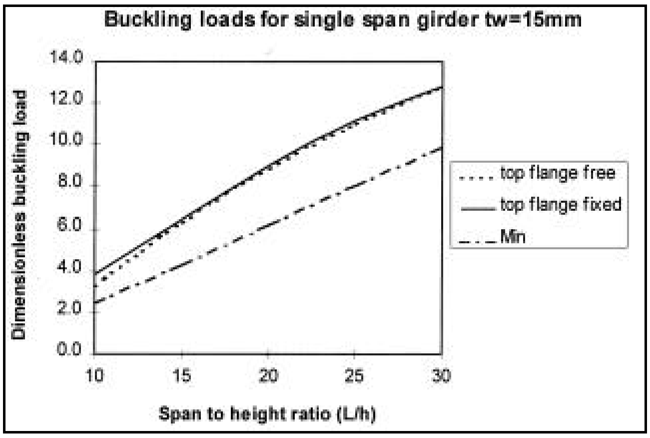


Figure 10. Buckling loads for single span girder for web thickness = 15mm under uniformly distributed load.

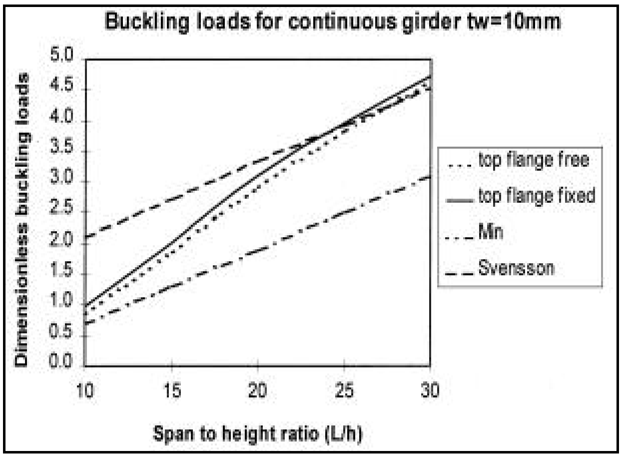


Figure 11. Buckling loads for continuous girder for web thickness = 10mm under uniformly distributed load.

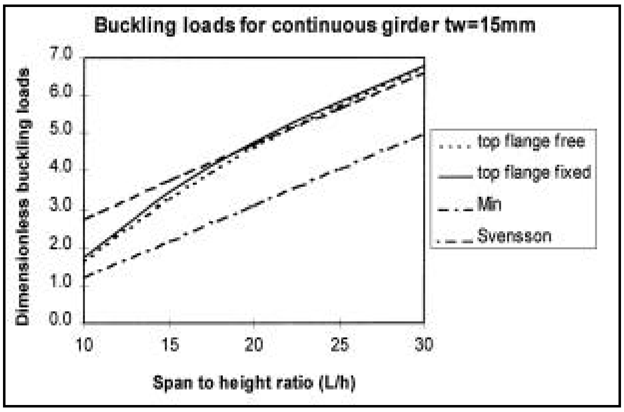


Figure 12. Buckling loads for continuous girder for web thickness = 15mm under uniformly distributed load.

CONCLUSION

A finite element program developed some time ago has been used to study the buckling behaviour of through girders. The girders consist of two parallel plate girders joined at their bottom flange level by a deck which provides twist restraint to the girders.

The elastic buckling loads were plotted for a range of span to height ratio 10, 15, 20, 25, 30 and web thickness t_w=10mm and t_w=15mm. The buckling loads were compared with conservative model of U-frame which analyses the problem as a uniformly compression strut, restrained elastically by the web. The continuous through girders were also compared with Svensson's model which analyses the strut subjected to compression force which has been scaled from the bending moment distribution. Distortional buckling loads under uniform moment obtained using the Svensson model overestimate the loads obtained by finite element analysis. This disparity is because Svensson does not include the web in his analysis.

Analysis of a continuous girder shows a good agreement between the Svensson model and the finite element model as the span to height ratio of the girders increase. However, for small span to height ratio (<20 for t_w=10mm, <15 for t_w=15mm) Svensson overestimates the buckling loads.

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