Measuring Wavelet Suitability with Symmetric Distance Coefficient

Nemuel Daniel Pah
Department of Electrical Engineering, Universitas Surabaya,
Gedung TB 2.3, Jl. Raya Kalirungkut,
Surabaya, Indonesia, 60293
Tel. (+62 31) 298 1254, Fax. (+62 31) 298 1151
Email: nemuelpah@ubaya.ac.id

ABSTRACT

Wavelet network introduced by Zhang and Benveniste in 1992, is a network that combines wavelet transform and artificial neural network to perform wavelet transform in parallel and adaptive approach with learning capability. In its learning process, wavelet network iteratively adjust wavelet transform parameters to accurately transform significant components in input signal to wavelet space. Wavelet function is the most difficult parameter to be adjusted, since there is no analytic parameter that links various type of wavelet function. This paper reports the experiments conducted to investigate the possibility of using symmetric distance as a parameter to measure wavelet suitability, analytically. The results suggested that there was a strong correlation between symmetric distance coefficients and qualitative inspection of wavelet suitability. The result also suggested that the technique could also be used in high density signal.

Keywords: Wavelet Transform, Wavelet Network, Dymmetricd.

1. INTRODUCTION

Wavelet network introduced by Zhang and Benveniste [1] in 1992 has opened a new direction in the development and application of Artificial Neural Networks (ANN). Wavelet network combines wavelet transform and ANN to perform wavelet transform in parallel and adaptive approach with learning capability. There are two categories of wavelet network [2-4]. Approximation Wavelet Network, which is designed for the purpose of function approximation. It is a modification of radial basis network [5] by replacing the sigmoid activation function with wavelet function. Classification Wavelet Network is designed for classification purposes. It uses wavelet function to extract significant features in the input signal and correlate this feature to the class of the input signal. Its weights and biases are used to value the contribution of each wavelet coefficient to the classification task.

The learning process in wavelet network is the process to iteratively adjust wavelet transform parameters to accurately transform the significant components in the input signal to wavelet space (time shift-scale space). The parameters are the wavelet scale, time shift, wavelet function, threshold levels, weights, biases, and other secondary parameters. Among the above parameters, wavelet function is the most difficult parameter to be adjusted. There are two main reasons for this fact. Firstly, there are abundant varieties of wavelet function to be selected. Secondly, each wavelet function is characterized by a large number of function properties (frequency support, singularities, smoothness, vanishing moments, time-support, and symmetric) that are not analytically related. The absence of parameters that can analytically link one wavelet to the other, makes it impossible for wavelet network to select wavelet function in gradual-based approaches. Most of the currently available wavelet networks use discrete approach such as competitive algorithms.

The process can only be possible if there is a common parameter that can measure wavelet suitability. Mallat [6] measured wavelet suitability by the number of near-zero wavelet coefficient. The most suitable wavelet function is the wavelet function that can concentrate significant energy of the input signal into a small number of wavelet coefficients. The technique is useful, especially in approximation wavelet network. However, it cannot be applied in iterative learning process since the number of near-zero wavelet coefficient is only known after completing the transformation (or the learning) process. The technique is also useless in noisy signal where the energy of the significant signal component are immersed in noisy background [7]. Arbi and Flandrin [8] suggested that the suitability of wavelet function can be measured by the symmetricity of wavelet coefficient around the center of wavelet function. The reason was because the correlation between two similar function is always symmetric. The approach is more applicable, however it needs a parameter to measure the degree of symmetricity.

Motivated by the above shortcomings, this paper reports the experiments conducted to investigate the possibility of using symmetric distance as a parameter to measure wavelet suitability. The experiments include the construction of algorithm to calculate wavelet coefficient's symmetric distance, the analysis of the correlation between symmetric distance coefficient and the qualitative closeness of input signal and wavelet function, and the effectiveness of using symmetric distance in determining suitable wavelet for various type of signals. The results suggested that there was a strong correlation between symmetric distance coefficients and qualitative inspection of wavelet suitability. The result also suggested that the technique could also be used in high density signal, where the other technique fail to measure.

2. THEORETICAL BACKGROUND

2.1. Wavelet Transform

Wavelet transform [6] of a function $f \in L^2(R)$ is defined as the correlation between the function f and a wavelet function ψ dilated and translated by scale s and time-shift σ.

$$Wf(s,\sigma) = < f, \psi^*_{s,\sigma} > = \int_{-\infty}^{+\infty} f(t) \frac{1}{\sqrt{s}} \psi^*(\frac{t-\sigma}{s}) dt \qquad (1)$$

The magnitude of wavelet coefficient $|Wf(s,\sigma)|$ is depending on how the properties of the wavelet function at (s,σ) matches the properties of the analyzed function f around $t = \sigma$. By properly selecting a wavelet function, significant features of the analyzed signal can be extracted.

2.2. Wavelet Networks

Wavelet network [4] is a class of ANN [5] that includes wavelet transform in its algorithm. The main reason for combining wavelet transform and ANN is to perform wavelet transform in parallel and adaptive approach.

The need for parallel and adaptive computation in wavelet transform is because wavelet transform involves massive calculations with many parameters to be selected and adjusted. Parallel computation reduces the computation time of wavelet transform by performing the calculation simultaneously, while the adaptive computation allows the automatic selection of the transformation parameters. One of the difficulties in using wavelet transform is the selection of its parameters [7]. It involves the selection of wavelet function ψ, range of scales s and time-shift σ.

2.3 Wavelet Suitability by Symmetric Coefficient

The accuracy of wavelet function ψ selection determines how accurate wavelet coefficients represent important features in input function f, since the magnitude of wavelet coefficients represents the closeness between the features of wavelet function ψ and the features of the input function f.

Abry and Flandrin [8] suggested that the better time-coincidence between wavelet function ψ and input function f if the wavelet coefficient in the neighborhood of σ is symmetric. The argument for this suggestion is because the correlation of two similar functions will always be symmetric. Wavelet suitability can be measured by how symmetric its wavelet coefficient is. Suppose $\psi_{s,\sigma}(t)$ is similar to $f(t)$:

$$\frac{1}{\sqrt{s}} \psi(\frac{t}{s}) = f(t) \qquad (2)$$

then

$$Wf(s,\sigma) = \int_{-\infty}^{+\infty} f(t) f(t-\sigma) d \qquad (3)$$

It can be proved that the coefficient is symmetric in the neighborhood of σ:

$$Wf(s,-\sigma) = Wf(s,\sigma) \qquad (4)$$

$$\int_{-\infty}^{+\infty} f(t) f(t+\sigma) d = \int_{-\infty}^{+\infty} f(t) f(t-\sigma) d \qquad (5)$$

for $x = (t - \sigma)$

$$\int_{-\infty}^{+\infty} f(t) f(t+\sigma) dt = \int_{-\infty}^{+\infty} f(x+\sigma) f(x) dx \qquad (6)$$

Since x is the shifted version of t, both functions are compactly supported and the integration is for the whole duration of t, then the above equation applied.

2.4 Symmetric Distance

A function, $f(t)$, is considered symmetric around $t = \sigma$ if

$$f(\sigma + \tau) = \pm f(\sigma - \tau) \qquad (7)$$

The above equation strictly distinguish between symmetric and non symmetric function. It cannot further grade the unsymmetrical function based on how far they are from a symmetric condition. It cannot measure the condition some times known as 'nearly symmetric' or 'pseudo symmetric'.

To overcome the above shortcoming, the author introduced another parameter to measure the level close to symmetric. The parameter is inspired by the concept of symmetric distance in image analysis. Symmetric distance, D, is defined as a minimum afford required to turn a function into its symmetric shape [9]. Theoretically, it is the sum-squared of the difference between negative and positive side of f around σ for the whole support of $\tau = [\tau_1, \tau_2]$ in a ratio to the total energy of the function as follows:

$$D(\sigma)=\frac{\int_{\tau_1}^{\tau_2}\left(f(\sigma-\tau)-f(\sigma+\tau)\right)^2 d\tau}{\int_{\tau_1}^{\tau_2} f^2(\sigma-\tau)d\tau} \quad (8)$$

The range of $D(\sigma)$ is $[0, 2]$ with zero represents even-symmetric, *2* represents odd-symmetric, and the center point, *1*, represents asymmetric. The above range of $D(\sigma)$ does not meet common sense, therefore, difficult to interpret or remembered.

The equation was modified to shift its range to [-1,1] where *-1* represents odd-symmetric, *0* represents asymmetric, and *1* represents even-symmetric as follows:

$$D(\sigma)=1-\frac{\int_{\tau_1}^{\tau_2}\left(f(\sigma-\tau)-f(\sigma+\tau)\right)^2 d\tau}{\int_{\tau_1}^{\tau_2} f^2(\sigma-\tau)d\tau} \quad (9)$$

3. EXPERIMENTS

The experiments were designed to investigate the effectiveness of measuring wavelet suitability using the introduced symmetric distance coefficient. The first experiment used a function *f(t)* consists of two transients were similar to *'db2'* and *'db8'* wavelet at scale *16* and located at $t = 225$ and $t = 569$, respectively, as shown in Figure 1. The function was then transformed using *'db2'* wavelet at scale *16* (Figure 2). The symmetric distance of the wavelet coefficient was calculated and presented in Figure 3.

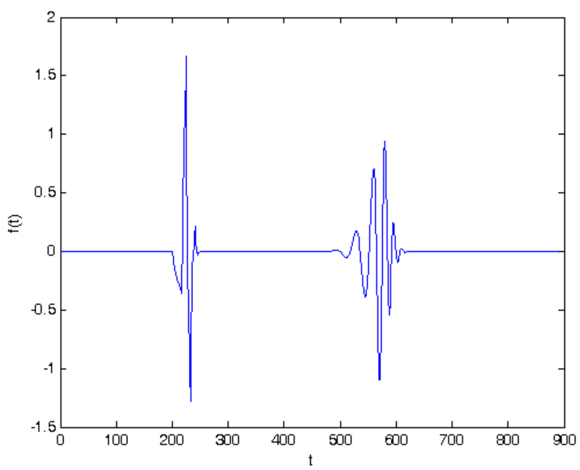

Figure 1. The analyzed function with two transient similar to *'db3'* at $t=225$ and *'db8'* at $t=569$.

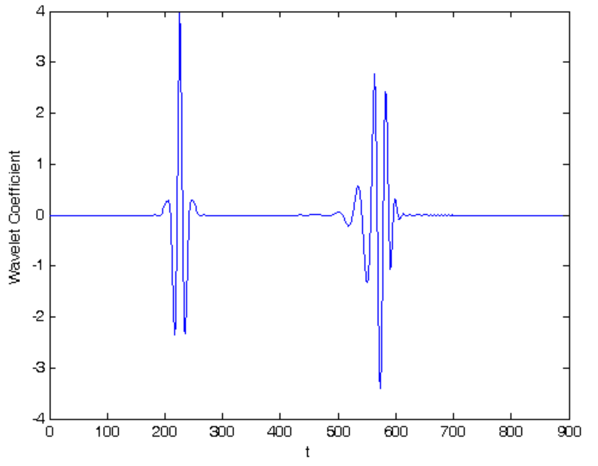

Figure 2. The wavelet coefficient of the function shown in Figure 1 calculated with *'db3'* at scale *16*.

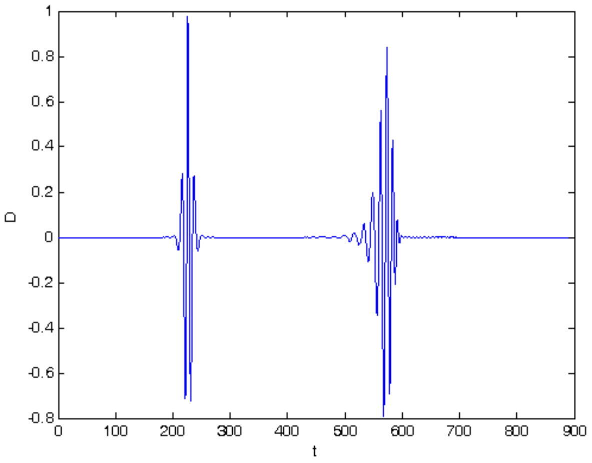

Figure 3. The symmetric distance coefficient, *D*, of the wavelet coefficient shown in Figure 2.

It was expected (as confirmed by Figure 3) that the wavelet function used in the above transform (*'db2'* at scale *16*) was more suitable for the first transient compare to the second. The symmetric distance coefficient of the first transient was *0.974*, while only *0.840* for the second transient. The wavelet coefficient of the first transient was symmetric around $\sigma = 225$, while asymmetric for the second transient. The result suggested that wavelet function *'db2'* at scale *16* was the best wavelet for the first transient.

In the second experiment, a function was generated with only one transient similar to *'db1'* wavelet at scale 16. The symmetric distance at $t = 200$ of its wavelet coefficient was calculated when the function was transformed using various different wavelet function (*'db1'*, *'db2'*, …, *'db10'*). The experiment was repeated with different transient that similar to *'db2'* and *'db3'* at scale *16*. The recorded symmetric distance coefficients are presented in Figure 4.

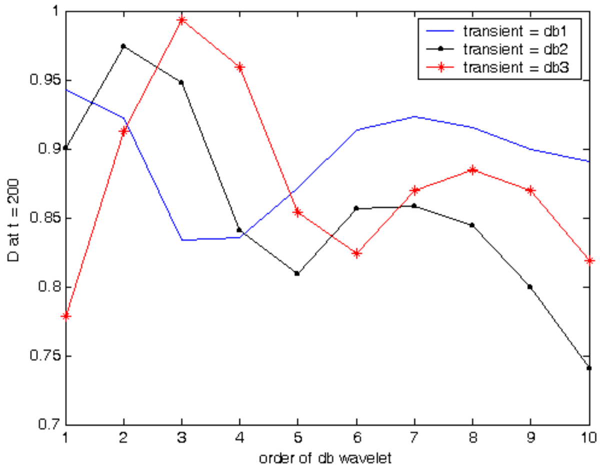

Figure 4. The symmetric distance coefficient, *D*, of *f(t)*'s wavelet coefficient transformed using ten different wavelet function.

The result showed that symmetric distance, *D*, coefficient could accurately measure wavelet suitability. When the transient was similar to *'db1'* wavelet, the symmetric distance coefficient suggested that the best wavelet to be used was *'db1'*. The highest *D* appeared at *'db1'*. The tendency was also shown by the experiment when the transient was similar to *'db2'* and *'db3'*.

The third experiment was conducted to investigate the ability of symmetric distance coefficient, *D*, to measure wavelet suitability of high density function. A function, *f(t)*, was generated by inserting *10* transients similar to *'db3'* scale *16* at random location. Gaussian white noise with *SNR=2dB* was added to the function as shown in Figure 5. Since the function consisted of noise and transients similar to *'db3'* at scale *16*. It was expected that *'db3'* was the most suitable wavelet function to be used as mother wavelet.

The function was then transformed using various different wavelet function (*'db1'*, *'db2'*, …, *'db10'*), and the maximum symmetric distance coefficient was calculated and recorded for each case.

The result, shown in Figure 6, suggested that wavelet transform with *'db3'* created the most symmetric pattern in wavelet coefficients, with *D = 0.987*. In other words, *'db3'* is the most suitable wavelet function for the analyzed function *f(t)*.

The experiment demonstrated the effectiveness of using symmetric distance as a measure of wavelet suitability in high density signal.

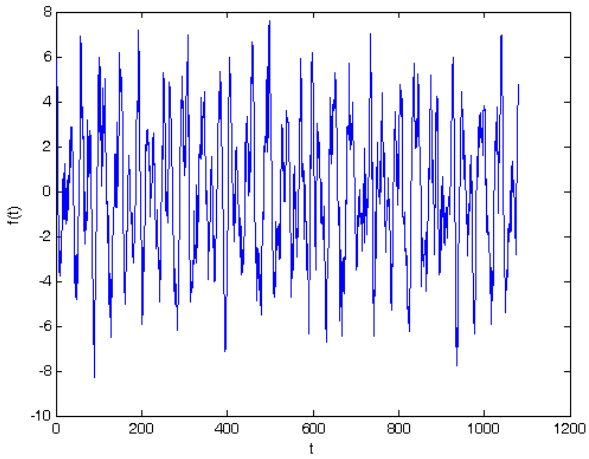

Figure 5. The analyzed high density function.

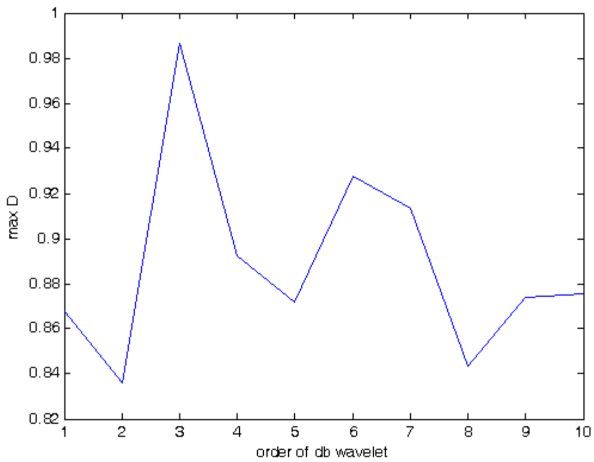

Figure 6. The maximum symmetric distance coefficient, *D*, of wavelet coefficient of the function shown in Figure 4, transformed using ten different wavelet function.

4. CONCLUSION

This paper introduces a new method to measure the suitability of selected wavelet function in wavelet transform. The method is based on symmetric distance coefficient developed by the author. The most suitable wavelet function is the function that creates wavelet coefficients with the highest symmetric distance coefficient.

The experiments, reported in this paper, confirms the efficacy of the method to determine suitable wavelet function. The experiments also demonstrated the effectiveness of the method to measure wavelet suitability in high density function.

The method can be further developed to be included in the adaptive learning process of wavelet network.

REFERENCES

[1] Q. Zhang and A. Benveniste, "Wavelet Networks," *IEEE Transaction on Neural Networks*, vol. 3, pp. 889-898, 1992.

[2] M. Akay, *Time Frequency and Wavelets in Biomedical Signal Processing*. New York: IEEE, Inc,, 1998.

[3] N. D. Pah and D. K. Kumar, "Thresholding Wavelet Networks for Signal Classification," *International Journal of Wavelets, Multiresolution, and Information Technology*, vol. 1, pp. 243-261, 2003.

[4] H. H. Szu, B. Telfer, and S. Kadambe, "Neural Network Adaptive Wavelets for Signal Representation and Classification," *Optical Engineering*, vol. 31, pp. 2907-1916, 1992.

[5] M. T. Hagan, H. B. Demuth, and M. Beale, *Neural Network Design*. Boston: PWS Publishing Company, 1996.

[6] S. Mallat, *A Wavelet Tour of Signal Processing*. London: Academic Press, 1999.

[7] N. D. Pah, "Thresholding Wavelet Network for Surface Electromyography Classification," in *School of Electrical and Computer System Engineering*. Melbourne, Australia: RMIT, 2003, pp. 225.

[8] P. Abry and P. Flandrin, "Multiresolution Transient Detection," presented at Proceedings of the IEEE-SP International Symposium on Time-Frequency and Time-Scale Analysis, 1994.

[9] T. U. Atsushi Imiya, Iris Fermin, "Symmerty Detection By Random Sampling and Voting Process for Motion Analysis," *International Journal of Pattern Recognition and Artificial Intelligent*, vol. 17, pp. 83-125, 2003.