Int. J. Production Economics 143 (2013) 574–579

Contents lists available at SciVerse ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Single-vendor single-buyer inventory model with discrete delivery order, random machine unavailability time and lost sales

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ARTICLE INFO

Available online 23 November 2011

Keywords:
JIT
Inventory
Unreliable machine
Buyer risk
Integrated model

ABSTRACT

Integrated single-vendor single-buyer inventory model with multiple deliveries has proved to result in less inventory cost. However, many researchers assumed that the production run is perfect and there is no production delay. In reality, production delay is prevalent due to random machine unavailability and shortages. This study considers lost sales, and two kinds of machine unavailability distributions—uniformly and exponentially distributed. A classical optimization technique is used to derive an optimal solution and a numerical example is provided to illustrate the theory. The results show that delivery frequency has significant effect on the optimal total cost, and a higher lost sales cost will result in a higher delivery frequency.

1. Introduction

Due to unreliable production system, vendors may not deliver products to the buyers when needed, resulting in lost sales. However, excessive supplies to fulfill customer's requirement results in higher inventory cost. The inventory cost is one of the dominant costs for many industries. It represents approximately 25% of the total assets (Philips and White, 1981), while the business investment on inventory is from 15–20% of the annual gross national product in the United States (Tersine, 1994). Industries should plan their strategy to provide products and services to the customers at a minimum cost. The order quantity and the time to order are critical decisions for both the manufacturing and the service industries. Some industries implement Just in Time (JIT) system to reduce their inventory cost. In order to support an efficient JIT system, it is important to ensure the reliability of the vendor's production system.

Since JIT concept can reduce inventory cost, extensive researches on vendor–buyer inventory problems with small batch deliveries have been done recently. A finite rate of production for product with lot shipment policy was initially introduced by Banerjee (1986). Goyal and Nebebe (2000) extended the model by developing a single-vendor single-buyer inventory model with small and equal sized shipment. Hoque and Goyal (2000) developed a single-vendor single-buyer integrated production-inventory system by considering the capacity of transport equipment. A JIT model in a single-vendor single-buyer inventory system with imperfect product quality was developed by Huang (2004). Nieuwenhuyse and Vandaele (2006) proved that lot splitting policies have benefited both the supplier and the buyer. A coordinating vendor–buyer inventory model with permissible delay in payments as trade credit scenario was developed by Jaber and Osman (2006). Ertogral et al. (2007) developed an integrated vendor–buyer model under equal-size shipment and incorporated transportation cost explicitly into the model. Zhou and Wang (2007) built a single-vendor single-buyer inventory model with shortages, wherein the buyer's unit holding cost is not required to be greater than the vendor's unit holding cost and deteriorating items. Pasandideh and Niaki (2008) developed a production inventory model with multiple deliveries, multiple products and warehouse space limitation. A single-vendor single-buyer inventory model with linearly decreasing demand was developed by Omar (2009). Lin (2009) developed an integrated single-vendor single-buyer inventory model with backorder price discount and variable lead time.

All the studies above assumed that the production process is perfect and there is no delay in the production process. However in reality, there are possibilities that the production process is delayed due to machine unavailability and shortages of materials and facilities. Abboud et al. (2000) developed EPQ models by considering random machine unavailability with backorders and lost sales. The models were extended by Jaber and Abboud (2001) who assumed learning and forgetting in production. Later Chung et al. (2011) extended the work of Abboud et al. (2000) by considering deteriorating items. Some researchers have

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doi:10.1016/j.ijpe.2011.11.019

considered preventive maintenance time in a production inventory model (Meller and Kim, 1996; Chen, 2006; El-Ferik, 2008). The effects of machine breakdown and corrective maintenance were first studied by Groenevelt et al. (1992). Machine breakdown and corrective maintenance for a production inventory model have been extended recently by Abboud (2001), Aghessaf et al. (2007) and Chiu et al. (2008).

According to the author's extensive literature studies, there are no researches that analyze a single-vendor single-buyer (SV–SB) inventory model with JIT system and stochastic machine unavailability time. In an integrated SV–SB model, the vendor and the buyer decide jointly as a team while for a non-integrated model, the vendor and the buyer make their own decision without consulting the other. Our study on an integrated (SV–SB) model with stochastic inventory is confirmed by some researchers who have shown that an integrated SV–SB model performs better than a non-integrated model (Ben-Daya and Hariga, 2004; Lo et al., 2007).

In this study, we assume a JIT system where the buyer who pays the transportation cost, decides the order quantity size of items and requests items delivery in multiple shipments. The vendor produces the items using an economic production quantity (EPQ) model. Ideally, the machine starts a production run when the inventory level is equal to zero. In some periods, there is a possibility that the machine may not be available. If this situation occurs, the vendor cannot deliver the predetermined quantity ordered by the buyer, resulting in the buyer's lost sales. We consider two distribution models for the random machine unavailability case. The distribution models represent two different types of distribution: uniformly distributed means constant number of machine unavailability over a period of time while exponentially distributed means machine unavailability may increase with time. Both cases can occur in real life. Similar distribution types were used by Abboud et al. (2000) and Giri and Dohi (2005).

The paper has four sections. Section 1 introduces the research motivation and literature review. Section 2 shows the development of the model. Section 3 illustrates the example and sensitivity analysis. Finally, conclusions are drawn in Section 4.

2. Problem definition and formulation

2.1. Assumptions

a. A single vendor and single buyer are considered.
b. The set-up and transportation times are insignificant and can be ignored.
c. The demand rate is constant and the time horizon is infinite.
d. All costs are known and constant.
e. The buyer pays the transportation cost.
f. The unsatisfied demands of the buyer will be lost sale.

2.2. Notations

T	cycle time
T_N	total production and non production time
T_s	lost sales time
T_d	production down time
Q	the vendor's production quantity, units/cycle
q	shipment quantity, units/delivery
K	number of shipments placed during a period T_N
w	number of shipments placed during the production time
P	production rate, units/year
D	buyer's demand rate, units/year
A	buyer's ordering cost, \$/order
Av	vendor's setup production cost, \$/cycle
S_v	vendor's late delivery cost, \$/year/delivery
S_b	buyer's lost sales cost, \$/unit/year
c_t	buyer's transportation cost, \$/delivery
h_v	vendor's holding cost, \$/unit/year
h	buyer's holding cost, \$/unit/year
TBC	total buyer cost
TVC	total vendor cost
$TBUC$ ($TVUC$)	total buyer (vendor) cost per unit time
TUC	total vendor–buyer unit cost
$TBUC_{NL}$ ($TVUC_{NL}$)	total buyer (vendor) cost per unit time for no lost sales case
TUC_{NL}	total vendor–buyer unit cost for no lost sales case
$TBUC_U$ ($TVUC_U$)	total buyer (vendor) cost per unit time for uniform distribution case
TUC_U	total vendor–buyer unit cost for uniform distribution case
$TBUC_E$ ($TVUC_E$)	total buyer (vendor) cost per unit time for exponential distribution case
TUC_E	total vendor–buyer unit cost for exponential distribution case

The vendor inventory model can be seen in Fig. 1. The vendor produces products for wT_N/K time and delivers q units every shipment, where $q=Q/K$. The vendor's production quantity unit per replenishment cycle is

$$Q=wP\frac{T_N}{K} \tag{1}$$

Referring to Wang and Sarker (2006), we modify the total inventory cost to consider the case for one inventory cycle, one has

$$I_T=\frac{q^2K(K-w+1)}{2D} \tag{2}$$

The vendor's total cost consists of the vendor's setup, the holding and the shortage cost. The vendor should pay a penalty cost to the buyer when the items are delivered late. The penalty cost depends on the delivery delay time and is independent of the product quantity. The vendor's total cost in one production cycle, $T=1$, can be modeled as follows:

$$E(TVC)=Av+\frac{h_v q^2K(K-w+1)}{2D}+S_v\int_{t=T_d}^{\infty}(t-T_d)f(t)dt \tag{3}$$

The total replenishment time consists of the production up time and production down time, and the expected shortage time.

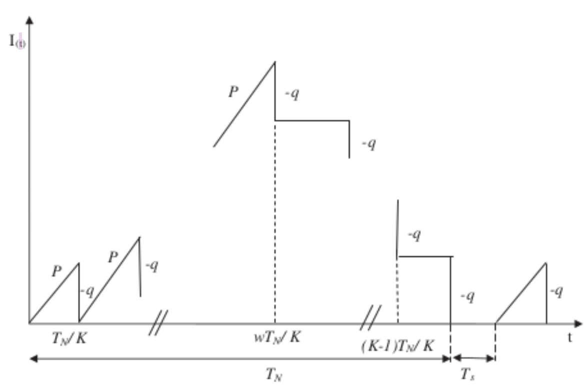

Fig. 1. The vendor inventory model.

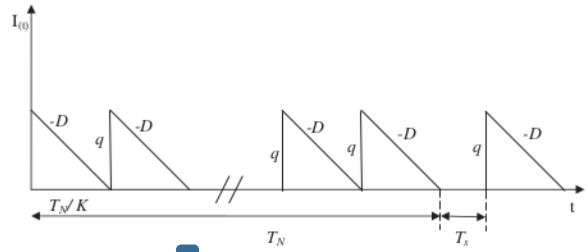

Fig. 2. The buyer inventory level.

The total replenishment time is

$$E(T) = T_N + T_S = T_N + \int_{t=T_d}^{\infty} (t - T_d) f(t) dt \tag{4}$$

Using renewal reward theorem, the total cost per unit time can be modeled as

$$TVUC = \frac{Av + (h_v q^2 K(K - w + 1)/2D) + S_v \int_{t=T_d}^{\infty} (t - T_d) f(t) dt}{T_N + \int_{t=T_d}^{\infty} (t - T_d) f(t) dt} \tag{5}$$

The buyer's inventory level can be represented in Fig. 2. When the inventory level is equal to zero, q units of product will be requested by the buyer. However, there is a possibility that the vendor delays his shipment resulting in the buyer's lost sales during the period T_s.

The buyer's total inventory cost consists of ordering cost, transportation cost, holding cost, lost sales cost and penalty revenue from the vendor. One has

$$E(TBC) = A + c_t K + \frac{hq^2 K}{2D} + (S_b D - S_v) \int_{t=T_d}^{\infty} (t - T_d) f(t) dt \tag{6}$$

The expected buyer cost per unit time can be modeled as

$$TBUC = \frac{A + c_t K + (hq^2 K/2D) + (S_b D - S_v) \int_{t=T_d}^{\infty} (t - T_d) f(t) dt}{T_N + \int_{t=T_d}^{\infty} (t - T_d) f(t) dt} \tag{7}$$

Substitute w from (9) and T_N from (8) into (10), one has

$$T_d = \frac{qK}{D} \left(1 - \frac{D}{P} \right) \tag{11}$$

2.3. *Uniform distribution case*

Assume that the unavailability time t is a random variable uniformly distributed over the interval $[0,b]$. The probability density function, $f(t)$, is given as

$$f(t) = \begin{cases} 1/b, & 0 \le t \le b \\ 0, & otherwise \end{cases}$$

Substitute the uniform probability density function in (5), the vendor's total cost per unit time can be written as

$$TVUC_U = \frac{Av + (h_v q^2 K(K(1-(D/P)) + 1)/2D) + S_v((b - (qK/D)(1-(D/P)))^2/2b)}{(qK/D) + ((b - (qK/D)(1-(D/P)))^2/2b)} \tag{12}$$

Lost sales in the machine unavailability time does not occur if the production down time, T_d, is greater or equal to the upper bound of the machine unavailability time, b. To convey condition without lost sales, (12) can be remodeled as

$$TVUC_{NL} = \frac{DAv + (h_v q^2 K(K(1-(D/P)) + 1)/2)}{qK} \tag{13}$$

The buyer's total cost per unit time for the uniform machine unavailability time is

$$TBUC_U = \frac{A + c_t K + (hq^2 K/2D) + (S_b D - S_v)((b - (qK/D)(1-(D/P)))^2/2b)}{\frac{qK}{D} + ((b - (qK/D)(1-(D/P)))^2/2b)} \tag{14}$$

Similar to the expected vendor total cost per unit time, the expected buyer total cost when the non production period (T_d) is greater or equal to the upper bound of machine unavailability time, b, is

$$TBUC_{NL} = \frac{AD + c_t KD + (hq^2 K/2)}{qK} \tag{15}$$

The vendor and the buyer total cost can be modeled as

$$TUC_U = TVUC_U + TBUC_U$$

$$TUC_U = \frac{Av + (h_v q^2 K(K(1-(D/P)) + 1)/2D) + A + c_t K + (hq^2 K/2D) + S_b D((b - (qK/D)(1-(D/P)))^2/2b)}{(qK/D) + ((b - (qK/D)(1-(D/P)))^2/2b)} \tag{16}$$

The replenishment time is

$$T_N = \frac{qK}{D} \tag{8}$$

The optimal order quantity can be derived when the equation below is fulfilled:

$$\frac{dTUC_U}{dq} = \frac{(h_v qK(K(1-(D/P)) + 1)/D) + (hqK/D) - S_b K(1-(D/P))((b - (qK/D)(1-(D/P)))/2b)}{(qK/D) + ((b - (qK/D)(1-(D/P)))^2/2b)} - \frac{R_U(A_V + (h_v q^2 K(K(1-(D/P)) + 1)/2D) + A + c_t K + (hq^2 K/2D) + S_b D((b - (qK/D)(1-(D/P)))^2/2b))}{((qK/D) + ((b - (qK/D)(1-(D/P)))^2/2b))^2} = 0 \tag{17}$$

From (1) and (8), the value of w can be modeled as

$$w = \frac{KD}{P} \tag{9}$$

Since the production up time is wT_N/K, then the production down time is

$$T_d = \left(1 - \frac{w}{K} \right) T_N \tag{10}$$

where

$$R_U = \frac{K}{D} - K\left(1 - \frac{D}{P} \right)((b - (qK/D)(1-(D/P)))/bD)$$

The vendor and buyer total cost when the production down time is bigger than the upper bound of the machine unavailability time is

$$TUC_{NL} = TVUC_{NL} + TBUC_{NL}$$

$$TUC_{NL} = \frac{A_v + (h_v q^2 K(K(1-(D/P))+1)/2D) + A + c_t K + (hm^2 K/2D)}{(qK/D)} \tag{18}$$

Taking the derivative of (18) with respect to q and set the value equal to zero, one has

$$q^*_{NL} = \sqrt{\frac{2(A_v + c_t + A)}{K(h_v(K(1-(D/P))+1)+h)}} \tag{19}$$

The single-vendor single-buyer inventory model with uniformly distributed machine unavailability time can be solved using the following procedure:

Step 1 Set $K=1$ and TUC$(0,q)$.
Step 2 Calculate q^*_{NL} using (19).
Step 3 Calculate (11) using q^*_{NL} from step 2. If T_d is less than b, go to step 4, otherwise q^* is found.
Step 4 Calculate (17) to derive q^*. If $TUC(K^*-1,q) \geq TUC(K^*,q) \leq TUC(K^*+1,q)$, the optimal solution is found, otherwise $K=K+1$ and go to step 2.

The cost function (16) is a nonlinear equation, and no closed form solution can be derived. However, the optimal solution can be guaranteed when $0 \leq q \leq Db/(K(1-D/P))$. The detailed calculations are given in Appendix A.

2.4. *Exponential distribution case*

In the second case, the machine unavailability time is a random variable that is exponentially distributed. Exponential probability density function with mean $(1/\lambda)$ is given as

$f(t) = \lambda e^{-\lambda t}$ for $\lambda > 0$

The expected machine unavailability time is

$$E(T_s) = \frac{e^{-\lambda T_d}}{\lambda} \tag{20}$$

The vendor total cost per unit time is

$$TVUC_E = \frac{Av + (h_v q^2 K(K(1-(D/P))+1)/2D) + (S_v e^{-\lambda((qK/D)(1-(D/P)))}/\lambda)}{(qK/D) + (e^{-\lambda((qK/D)(1-(D/P)))}/\lambda)} \tag{21}$$

The buyer total cost per unit time can be modeled as

$$TBUC_E = \frac{A + c_t K + (hq^2 K/2D) + (S_b D - S_v)(e^{-\lambda((qK/D)(1-(D/P)))}/\lambda)}{(qK/D) + (e^{-\lambda((qK/D)(1-(D/P)))}/\lambda)} \tag{22}$$

The total vendor and buyer cost is

$$TUC_E = \frac{Av + (h_v q^2 K(K(1-(D/P))+1)/2D) + A + c_t K + (hq^2 K/2D) + S_b D(e^{-\lambda((qK/D)(1-(D/P)))}/\lambda)}{(qK/D) + (e^{-\lambda((qK/D)(1-(D/P)))}/\lambda)} \tag{23}$$

The optimal order quantity can be found by taking the derivative of (23) with respect to q, one has

$$\frac{dTUC_E}{dq} = \frac{qK(h_v(K(1-(D/P))+1)+K)}{qK + (e^{-\lambda((qK/D)(1-(D/P)))}/\lambda D)} - \frac{S_b(1-(D/P))Ke^{-(\lambda qK(1-(D/P))/D)}}{(qK/D) + (e^{-\lambda((qK/D)(1-(D/P)))}/\lambda)}$$
$$-R_E\left(\frac{A_V + A + c_t K + (q^2 K(h_v(K(1-(D/P))+1)+h)/2D) + (S_b D e^{-\lambda((qK/D)(1-(D/P)))}/\lambda)}{((qK/D) + (e^{-\lambda((qK/D)(1-(D/P)))}/\lambda))^2}\right) = 0 \tag{24}$$

where

$$R_E = \frac{K}{D} - K\left(1 - \frac{D}{P}\right)\left(\frac{e^{-\lambda((qK/D)(1-(D/P)))}}{D}\right)$$

Table 1
The optimal solution for different production rate.

K	**q**	**Buyer cost**	**Vendor cost**	**Total cost**
1	1730.247	11586.98	9380.29	20967.27
2	1068.940	6581.28	8820.60	15401.88
3	771.229	4780.77	8430.03	13210.80
4	603.226	3885.32	8186.86	12072.18
5	495.534	3367.82	8026.41	11394.23
6	420.672	3043.52	7914.84	10958.36
7	365.631	2831.08	7834.21	10665.29
8	323.467	2689.16	7774.28	10463.44
9	290.138	2594.51	7728.86	10323.37
10	263.132	2533.08	7693.98	10227.06
11	240.806	2495.86	7667.00	10162.86
12	222.042	2476.83	7646.07	10122.90
13	206.050	**2471.78**	7629.90	10101.68
14	192.259	2477.70	7617.53	**10095.23**
15	180.243	2492.40	7608.24	10100.64
16	169.682	2514.21	7601.49	10115.70
17	160.325	2541.86	**7596.83**	10138.69
18	151.978	2574.34	7593.96	10168.30

The closed form solution of the total cost per unit time for the exponential distribution cannot be derived. However, the optimal solution can be guaranteed when some conditions as shown in Appendix A are fulfilled.

3. Numerical example

In this section, a numerical example is shown to illustrate the model. The numerical example is partly adopted from Kim and Ha (2003). Let the production rate $P=19{,}200$ units/year, demand rate $D=4800$ units/year, vendor setup cost $A_v=\$600$/cycle, ordering cost of buyer $A=\$25$/order, vendor holding cost $h_v=\$6$/unit/year, buyer holding cost $h=\$7$/unit/year, transportation cost $F=\$50$/delivery, vendor lateness delivery $=\$50$/year/delivery and buyer lost-sales cost $=\$10$/unit/year. The result shows that the optimal supply chain cost per unit time is \$10,095.23, where the vendor total cost per unit time is \$7617.53 and the buyer total cost per unit time is \$2477.70. The optimal solution is derived when the units per delivery, $q=192.259$ and the number of delivery, $K=14$. In our example (see Table 1), if the buyer act as the leader, then he prefers to set $K=13$. If the vendor acts as the leader, he will prefer to set $K=17$; if the vendor and the buyer use the service of a third party decision making, then $K=14$ will result in a least total supply chain cost. Table 1 shows that the optimal shipment frequency depends on who will act as leader in the decision making process.

For perfect machine, there is no machine unavailability time; the solutions derived are shown in Table 2. The optimal solution is derived when the unit per delivery, $q=192.354$ and the number of delivery (K) is equal to 6. It is clear that the total supply chain cost (\$7694.15), the vendor cost (\$5669.24) and the buyer cost (\$2024.91) in the perfect machine condition are lower than the costs in the machine unavailability time model.

Table 3 shows the optimal solutions for different lost sales costs. When the lost sales cost increases, as expected the number

Table 2
Optimal solution for no machine unavailability time.

K	q	Buyer cost	Vendor cost	Total cost
1	608.511	2721.40	7927.54	10648.94
2	397.721	2146.32	6603.54	8749.86
3	305.917	1985.99	6120.80	8106.79
4	252.727	1952.89	5881.65	7834.54
5	217.541	1974.96	5747.73	7722.69
6	**192.354**	**2024.91**	**5669.24**	**7694.15**
7	173.344	2090.13	5623.68	7713.81

Table 3
The optimal solution for different lost sales costs.

S_b	K	q	Buyer cost	Vendor cost	Total cost
4	11	190.1635	3091.25	6227.45	9318.70
7	13	192.3884	2675.99	7192.10	9868.09
10	14	192.2585	2477.70	7617.53	10095.23

of shipments tends to increase and the optimal order quantity tends to be stable. This situation indicates that the expected replenishment time is increasing and the lost sales probability is decreasing. Since the lost sales probability is decreasing, the lost sales cost also decreases but the inventory cost increases. As a consequence, the total supply chain cost and the vendor cost both increase.

The numerical example in the exponential distribution model uses similar data as the uniform distribution model, except for the machine unavailability rate, where we set $\lambda=4$. The optimal solution is derived for the machine with exponentially distributed unavailability time when the optimal order quantity (q) is 192.127 units and the number of shipment (K) is 16. The total supply chain cost is $12,423.94, the buyer cost is $4448.28 and the vendor cost is $7975.66.

4. Conclusions

In this study, a single-vendor single-buyer inventory model with stochastic machine unavailability time has been developed. The machine unavailability time is assumed to be uniformly and exponentially distributed. The numerical example illustrates how the multiple deliveries result in a lower cost than the single delivery model. The stochastic machine time model results in a higher cost and more delivery frequencies when compared to a perfect machine model. The optimal delivery frequency increases when the lost sales cost increases. This study provides managerial insight into enterprises that employ JIT systems and production delay (lost sales) due to machine unavailability. The proposed model helps enterprises to optimize their profit by coordinating the number of deliveries for various machine unavailability time and lost sales cost. The models can be extended to consider Poisson distribution machine breakdown and stochastic delivery time.

Appendix A

The second derivative of the uniform distribution unavailable time in q is

$$\begin{aligned}\frac{d^2TUC}{dq^2} &= \frac{h_vK(K(1-(D/P))+1)+hK-S_bK^2(1-(D/P))}{D((qK/D)+((b-(qK/D)(1-(D/P)))^2/2b))}\\ &\quad-\frac{R_U(2h_vqK(K(1-(D/P))+1)+(2hqK/D)-S_bK(1-(D/P))((b-(qK/D)(1-(D/P))/2b)))}{((qK/D)+((b-(qK/D)(1-(D/P)))^2/2b))^2}\\ &\quad+\frac{{R_U}^2(2A_v+(h_vq^2K/D)(K(1-(D/P))+1)+2A+2c_tK+(hq^2K/D)-S_bD(1-(D/P))((b-(qK/D)(1-(D/P)))/b)^2)}{((qK/D)+((b-(qK/D)(1-(D/P)))^2/2b))^3}\\ &\quad-\frac{K^2(1-(D/P))^2(A_v+(h_vq^2K/2D)(K(1-(D/P))+1)+A+c_tK+(hq^2K/2D)-S_bD(1-(D/P))((b-(qK/D)(1-(D/P)))/2b)^2)}{D^2b((qK/D)+((b-(qK/D)(1-(D/P)))^2/2b))^2}\end{aligned} \tag{A1}$$

where

$R_U=(K/D)-K(1-(D/P))((b-(qK/D)(1-(D/P)))/bD)$

For $T_d=0$, (A1) is equal to zero; for $T_d=b$, (A1) can be rewritten as

$$\frac{d^2TUC}{dq^2}=\frac{2D(A_v+A+c_tK)}{q^3K}>0 \tag{A2}$$

Since (A2) is true and (A1) is non-increasing in $T_d>0$ to b, then the total cost per unit time is convex when $0<T_d\le b$. Using (10), we can prove that the total cost per unit time is convex when $0<q\le Db/(K(1-D/P))$.With some simplification, the four conditions that satisfy the convexity of the total cost per unit time for the exponential distribution case are shown in (A3)–(A5)

$$q<\frac{D\ln(((1/\lambda)-(K/P)-(2K/D))(D/K))}{\lambda K(1-(D/P))} \tag{A3}$$

$$2+\left(1-\frac{D}{P}\right)e^{-(\lambda qK(1-(D/P))/D)}\left(-4+\left(1-\frac{D}{P}\right)e^{-(\lambda qK(1-(D/P))/D)}-\left(\frac{qK}{D}\right)\left(1-\frac{D}{P}\right)\lambda\right)>0 \tag{A4}$$

$$\left(\frac{e^{-(\lambda qK(1-(D/P))/D)}}{\lambda}\right)\left(\frac{hK}{D}+S_bK^2\lambda\left(1-\frac{D}{P}\right)^2e^{-(\lambda qK(1-(D/P))/D)}\right)+\left(\frac{qK}{D}\right)\left(S_bK^2\lambda\left(1-\frac{D}{P}\right)^2e^{-(\lambda qK(1-(D/P))/D)}\right)$$

$$> \left(\frac{hqK}{D} - 2S_bK\left(1-\frac{D}{P}\right)e^{-(\lambda qK(1-(D/P))/D)}\right)K\left(\frac{1-(1-(D/P))e^{-(\lambda qK(1-(D/P))/D)}}{D}\right) \tag{A5}$$

(A5) is true if

$$\left(\frac{hqK}{D} - 2S_bK\left(1-\frac{D}{P}\right)e^{-(\lambda qK(1-(D/P))/D)}\right)K\left(\frac{1-(1-(D/P))e^{-(\lambda qK(1-(D/P))/D)}}{D}\right) < 0 \tag{A6}$$

and one has

$$S_b > \frac{hq}{2D(1-(D/P))e^{-(\lambda qK(1-(D/P))/D)}} \tag{A7}$$

From (A3), (A4) and (A7), we can conclude that the total cost per unit time for the exponential distribution case is convex under certain conditions.

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