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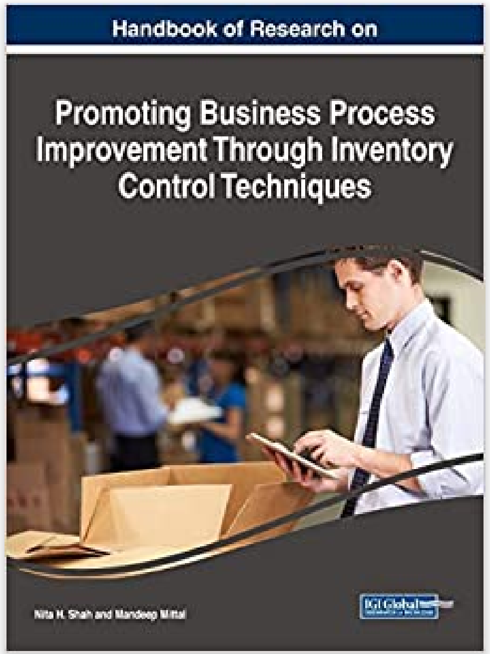
Handbook of Research on Promoting Business Process Improvement Through Inventory Control Techniques (Advances in Logistics, Operations, and Management Science) 1st Edition

by Nita H. Shah (Author, Editor), Mandeep Mittal (Editor)



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The **Handbook of Research on Promoting Business Process Improvement Through Inventory Control Techniques** is a critical scholarly resource that examines optimization techniques, data mining concepts, and genetic algorithms to manage

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Nita H. Shah is a professor in the Department of Mathematics, Gujarat University, Ahmedabad, India. She received her Ph.D. in inventory control management, operations research. Currently, she is engaged in research in inventory control and management, supply chain management, forecasting and information technology and information systems, neural networks, sensors and image processing. She has more than 275 papers published in international and national journals. She is author of four books. She is serving as a member of the editorial board of Investigation Operational, *Journal of Social Science and Management, International Journal of Industrial Engineering* and *Computations and Mathematics Today*.
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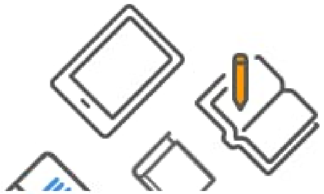

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Preface

Promoting business process improvement through inventory control techniques is a critical component to the success and overall financial well-being of an organization. Through the application of innovative practices and technology, businesses are now able to effectively monitor their operations and manage their inventory by evaluating sales patterns and customer preferences.

We believe that this book will be a pioneering text book focusing on the research in the inventory control and management. It features diverse perspectives on the implementation of various optimization techniques, fuzzy system, genetic algorithms, and datamining concepts, as well as research on big data applications for inventory management. This publication is a comprehensive reference source for practitioners, educators, and researchers in the fields of logistics, supply chain management, operations management, and retail management. It is organized in such a way that it is starting from deterministic inventory models and move towards advance inventory models. We are very pleased to see that this book has generated a lot of important insights and new research results on the inventory control and management related problems.

Chapter 1 is formulation of mathematical formulations of optimal replenishment policies for items with trapezoidal demand rate. In Chapter 2, realistic scenario is analyzed. In today's competitive and global business scenario there is always a race to boost demand of your product over others. This can be achieved by different means and allowing permissible delay in payments is one of them. Researchers have proposed number of inventory models with trade credit that actually help to understand effect of trade credit on total profit and overall demand. This model can be applied to a huge range of products like readymade garments, fashion accessories, electronics, furniture and home furnishing products. Chapter 3 deals with boosting the sale. For this purpose they generally allow credit period. Here, an inventory model is formulated in which supplier gives credit period to retailer and to increase the sale, retailer passes it to end customers. This phenomenon is known as two level trade credits. By allowing credit period we may encounter with the issue of default risk which has been taken care of while calculating profit function for the system. Also each and every inventory product gets deteriorated over the time as per its nature and such deteriorating products have its maximum life time as well. In Chapter 4, the problem of determining the optimal selling price and lot size for an inventory system with non-instantaneous deteriorating item is considered in this chapter. In order to provide general framework, the pricing and lot sizing problem is modeled assuming a general price and time dependent demand function by allowing shortages. The model allows for backlogging of demand which is characterized by decreasing function of waiting time. As the problem involves revenue and costs, a natural objective function for the model is profit per period. Chapter 5 is about an integrated model of supply chain where units in inventory are subjected to time dependent deterioration and demand is inversely proportional to selling price of the item, it is assumed selling price dependent. Delay on payments is offered only on purchase of a certain amount of quantity.

In Chapter 6, the practical observation of retail industries every ordered lot carry some fraction of imperfect quality items which can vary depending upon production and handling conditions. The situation is even more subtle when the items are prone to deterioration. However, an inspection process can spare us from such a criticality by bifurcating the defectives from the good quality lot. Thus, a screening process is mandatory. The Chapter 7 deals with optimal dynamic pricing and ordering for items with synchronized deterioration of quality and physical quantity. Qualitative deterioration is an instantaneous process while physical deterioration-a non-instantaneous process. The dynamic nature of the problem, selling price is assumed to be a time-dependent function of the initial price and discount rate. Initially with no physical deterioration, the product is sold at initial price value in the time period, successively in order to enhance customer's demand, price is exponentially discounted. In Chapter 8, inventory control system of non-instantaneous deteriorating item with maximum fixed life-time and two tiered pricing policy is adopted. The selling prices of product for the non-deteriorating period and the deteriorating period are different. Demand is a function of time and two tiered selling prices which is more suitable for food industry. In the former, we consider the concept of preservation technology investment to preserve the product and reduce deterioration rate in the inventory system when deterioration start. Chapter 9 deals with supply chain inventory model is formulated with for the demand which increases linearly for some times when a product is launched and when with new substitute available demand decreases exponentially. The inventory of every player is subject to deteriorate after a fixed life time. Chapter 10 focuses on uncooperative supply chain inventory models when a supplier offers a credit period to the retailer for a fixed period of time. The models are studied with trade credit in Nash game and Supplier-Stackelberg game respectively. First, the authors have presented optimal results for centralized and decentralized decisions with selling price dependent demand and without trade credit. Second, the authors have obtained optimal results under the two games using classical optimization.

Chapter 11 considers a supply chain for a single supplier to two retailers. Chapter 12 is about the formulation of an inventory model of supplier and retailer. In practice, suppliers and retailers do not opt to have collaboration and they try to optimize their own decision so we develop a Stackelberg Game model. Two models are developed wherein the first model supplier acts as the leader and in the second model, the retailer acts a leader. Since the models are complex, a hybrid Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) is developed to solve the model. The results show that a Stackelberg Game model for progressive permissible delay of payment is sensitive in varies values of the first and second delay interest rate if supplier acts as a leader. The retailer gets less inventory cost when he acts as a leader compared to when vendor acts a leader at high interest rate of the first and second delay period. Chapter 13 considers supply chain model for imperfect quality items in which retail price of the buyer influence the demand of the product. The seller offers fix credit period to the buyer to stimulate his sales. Inspection process at the buyer's end is assumed. After inspection process, items are separated in two parts, perfect items and imperfect items. The perfect quality items are sold at selling price and the imperfect items are sold at a discounted price immediately after the inspection process. The credit period offered by the seller and selling price both are considered as a decision variable. Relationship between seller and buyer is derived by the non-cooperative seller Stackelberg game approach. Chapter 14 assumes fuzzy demand rate, deterioration rate and inspection parameter of non-defective parameter based on as triangular fuzzy numbers to fit the real word. The total fuzzy cost function has been defuzzified using signed distance and centroid method. Chapter 15 is about a coordinated single-manufacturer and multiple heterogeneous buyers inventory model under fuzzy demand is developed. Here two cases are considered: (1) ex-site delivery case considers manufacturer dominance where manufacturer produce items and delivers to the group of heterogeneous buyers at common replenishment time through common shipment, (2) ex-factory delivery case considers buyer's dominance. Fuzzy set theory is used to handle the uncertainty in the demand variable. The model is analyzed using triangular membership function. In Chapter 16, the basic assumption of an EOQ model is that 100% of items in an ordered lot are perfect is relaxed. This assumption is not always pertinent for production processes because of process deterioration or other factors. Data mining is a technique to identify valid novel, potentially useful, and understandable correlations and patterns in existing data. Data mining techniques, such as clustering, association rule mining, classification, and sequential pattern mining, have attracted a great deal of attention in the information industry and in society as a whole in recent years. Some research studies have also extended the usage of this concept in inventory management. Yet, not many research studies have considered the application of data mining approach on determining both optimal order quantity and loss profit of frequent items. This helps inventory manager to determine optimum order quantity of frequent items together with the most profitable item for optimal inventory control. In Chapter 17, two different cases for determining ordering policy and inventory classification based on loss rule are presented. An example is illustrated to validate the results. In Chapter 19, the integrated marketing-production planning problem and propose an optimal control approach to derive the optimal solution. The state variables are the inventory level and the stock of goodwill and the control variables are the production rate and the advertising rate. Both cases where the firm adopts a continuous-review and a periodic-review policy are considered. The optimal states and controls are obtained explicitly. In every production system, malfunctioning or breakdown during run time can incur heavy loss to the organization, to overcome such a situation it is crucial to use maintenance actions which can be either corrective or preventive depending upon the condition of the system. Also, warranty policy is extensively used world-wide to increase customer confidence in the product and to uplift sales. On account of this, Chapter 20 presents a problem of a manufacturer dealing with an imperfect production system considering maintenance actions and warranty policy by trading off the rework cost, holding cost, warranty cost and corrective/preventive maintenance cost so as to minimize the manufacturer's total cost. Customer's aim is to obtain good quality products with less effort. The preference of online selling is very high compare to offline selling. A dual channel supply chain model is introduced to control the quality of products with more profit using buyback contract by reducing lost sale costs. Manufacturer sells product through retail and e-tail channel, i.e., by dual channel. Demand of products depends on e-tail price, retail price, demand sensitivity, advertisement of retail channel, service level of e-tail channel, and delivery cost e-tail channel. This delivery cost has inverse impact on demand of e-tail chain. Chapter 21 finds the maximum profit for each case and compares results when advertisement and service level are not present in supply chain. The model is solved for centralized and decentralized ways for RC (retail channel) using Stackelberg game policy, EC (e-tail channel), and DC (dual channel). Companies that have been following the innovation mantra have also understood the importance of coming up with successive generations for their offerings. Continuous enhancement of existing technologies or emergence of new technologies has made this improvement possible. In today's neck to neck competitive environment firms introduce their products in more than one market so as to widen their spectrum of visibility and availability. This has allowed customers to avail the facility from more than one place (segment) simultaneously. Chapter 22 deals with mathematical framework to capture the impact of multi-generations on multiple markets and vice-versa. Further, consideration of cross-generation shifting makes our analysis more comprehensive. The presence and management of inventories in the industrial field can be seen from different perspectives, starting from the information that can be shared along the supply chain to the environmental effect and costs of carbon emissions and even incorporating, in addition to environmental aspects, the quality and passing by the bullwhip effect and the analysis of multiple levels in the supply chain. Or by the effect of the four seasons and the deterioration of the products, by the impact of the restrictions of the carbon footprint in the inventory management, by the time of lead time and a better customer service. But in general, when there appear these problems caused by regulations of the state, is under uncertain scenarios, for which the decision making is usually under risk and even worse, under uncertainty. A model that can help solve problems of decision making under uncertainty is The Amplitude Model (TAM [EMA]). Therefore, with respect to this work, it will also use the MoLoBaC, but not through independent positions, but that will be use one of their areas, Inventories. And on the other hand the alternatives will be analyzed from the point of view of the problems of decision making under uncertainty. Specifically be used TAM. From the above the general objective of the Chapter 23: Make use of the Inventories area, of the Logistics Model Based on Positions and with The Amplitude Model, to analyze the possible actions to take, to confront the consequences of the restrictions imposed by the state, which affect the inventory control. From this general objective three specific objectives arise: It shows in which the MoLoBaC Inventory area consists, with emphasis on one of the positions that integrate it, the Spare and Equipment manager (SEM). Explain The Amplitude Model through its parameters and characteristics. Apply TAM to guide the decisions to be taken, when an organization is affected by its control of inventories, as a product of measures taken by the state. Chapter 24 exhibits review article. Inventory and supply chain management is a real concern for business community in today's globally competitive scenario. Various inventory models are proposed, significant parameters are analysed and finally optimized by researchers in order to give managers an insight for the different parameters. Mathematical and logical analysis of different inventory and supply chain models helps mangers in overall cost reduction and further higher revenue generation. Members often encounter conflicting interest and unforeseen scenario. So, all this make supply chain very complex and dynamic process. Complex and uncertain nature of inventory and supply chain, many times either it is not feasible to solve the issue with traditional methods or it is not cost effective. Thus many researchers are using artificial intelligence approach for investigation. Genetic algorithm is one among them that works efficiently with complex nature of the inventory and supply chain management. This chapter provides an up to date review about the role of GA in overall inventory and supply chain management. Fuzzy system was altered from a 'buzz word' to an important technological area, with various publications in international conferences and transactions. Several Japanese products applying fuzzy logic concepts, such as household appliances and electronic equipment, power engineering, robotics, and optimization have been manufactured. This system is capable to process and learn mathematical data as well as linguistic data. Fuzzy system uses linguistic explanations for the variables and linguistic procedures for the I/P-O/P behavior. In this chapter, present the application of fuzzy system with data mining, neural networks, fuzzy automata, and genetic algorithms. It also presents the foundation of fuzzy data Mining, with the fuzzification inference procedure and defuzzification procedure, fuzzy systems and neural networks with feed forward neural network, FNN with it features generalization of Fuzzy Automata, and sixth fuzzy systems and genetic algorithms. The Chapter 25 explores a popular fuzzy system model to show complex systems and an application of fuzzy system. A pragmatic innovation diffusion model is proposed in the present Chapter 26 that interpolates stochasticity in the logistic formulation of the widely-acknowledged Bass model with dynamic market size. These irregular changes are caused due to uncertainty attached to the socioeconomic and political environment in which an innovation is positioned that affects the action of potential adopters leading to their non-uniform behavior. The aim of the current study is to find the analytical solution for the two dynamic market expansion structures, namely, linear and exponential under the influence of irregular fluctuations whose closed-form solutions were not possible in the existing literature. In addition to the changeable market size, the proposed innovation diffusion also incorporates the concept of repeat purchase. The anticipated stochastic differential equation based new product diffusion model is then expounded methodically using the Itô process and Itô's integral equation. Further, the model has been used to study the growth pattern of different consumer durable products. Data mining discovers the knowledge and patterns which are potentially useful and previously unknown. Utility mining is one of the recent emerging fields in data mining. It incorporates usefulness of the knowledge or pattern retrieved. Such usefulness of the knowledge retrieved is termed as utility. In retail industry, utility mining can be used to retrieve highly profitable product in a market. Identifying such profitable products helps in better decision in inventory management. Chapter 27 discusses how utility mining concepts can be implemented in inventory control.

Chapter 29 focuses on incorporating dependency factor to optimal spare parts grouping, spare parts group prediction, determining optimal stock levels, and joint replenishment of spare parts. We describe different areas of available literature in different sections such as data mining studies related to item dependency, spare parts classification and grouping, maintenance interval prediction studies, spare parts inventory control, joint replenishment studies and inventory control studies with correlated demand. To define huge datasets, the term of big data is used. The considered "4 V" datasets imply volume, variety, velocity and value for many areas especially in medical images, electronic medical records (EMR) and biometrics data. To process and manage such datasets at storage, analysis and visualization states are challenging processes. Recent improvements in communication and transmission technologies provide efficient solutions. Big data solutions should be multithreaded and data access approaches should be tailored to big amounts of semi-structured/unstructured data. Software programming frameworks with a distributed file system (DFS) that owns more units compared with the disk blocks in an operating system to multithread computing task are utilized to cope with these difficulties. Huge datasets in data storage and analysis of healthcare industry need new solutions because old fashioned and traditional analytic tools become useless.

We would like thank all the authors who have contributed their interesting research articles to this book. We are indebted to the anonymous reviewers who reviewed the manuscripts and provided us with very constructive and timely review comments. Last but not least, we are grateful to our family, colleagues, and students, who have been supporting us during the development of this important research book.

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Chapter 12
Stackcelberg Game Inventory Model With Progressive Permissible Delay of Payment Scheme

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ABSTRACT

Supplier has many schemes to motivate retailer to buy more and of them one is a progressive permissible delay of payment. Instead of analyst from the retailer side alone, in this chapter, we develop the inventory model of supplier and retailer. In reality, some suppliers and retailers cannot have collaboration and they try to optimize their own decision so we develop a Stackelberg Game model. Two models are developed wherein the first model supplier acts as the leader and in the second model, the retailer acts a leader. Since the models are complex, a hybrid Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) is developed to solve the model. A numerical analysis and sensitivity analysis are conducted to get management insights of the model. The results show that a Stackelberg Game model for progressive permissible delay of payment is sensitive in varies values of the first and second delay interest rate if supplier acts as a leader. The retailer gets less inventory cost when he acts as a leader compared to when vendor acts a leader at high interest rate of the first and second delay period.

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INTRODUCTION

In order to motivate customers to buy more from supplier, supplier apply some scheme such as dynamic pricing, quantity discount gift on purchase and permissible delay of payments. Trade credit in the form of permissible delay of payment affects the conduct of business significantly (Jaggi *et al.*, 2013). In permissible delay of payment buyer does not have to pay the supplier immediately after receiving the goods but can delay the payment until the allowable time period. Buyer can get benefit from interest earn form goods that have been sold. Many research have been conducted by considering permissible delay of payment in inventory. Huang (2005) developed a buyer's inventory model by considering delay of payment and cash discount. Inventory model with collaboration between supplier and retailer by considering permissible delay of payment was developed by Jaber and Osman (2006). They concluded that coordination with permissible delay in payment is better than no-coordination system. Similar research of inventory model with permissible delay of payment under collaboration between single-vendor and single-buyer for deteriorating items was developed by Yang and Wee (2006). They found that permissible delay of payment is a win-win strategy when implemented under collaboration system. Liao (2007) developed deteriorating economic production quantity model by considering permissible delay in payments. Tsao and Sheen (2008) did not consider permissible delay of payment for deteriorating inventory model but they also include other schemes which are dynamic pricing and promotion. Instead only collaboration between supplier and retailer, Jaggi *et al.* (2008) developed an inventory model with two levels of credit policy. In their model supplier gives a fixed credit period to the retailer and retailer offer credit period to customers. Inventory model with permissible delay of payment by considering allowable shortage was introduced by Chung and Huang (2009). Huang *et al.* (2010) developed single-vendor single buyer integrated inventory model considering permissible delay of payments and order time reduction that can be attained through procedural changes, worker training and specialized equipment acquisition. Sarkar (2012) developed an EOQ model by considering stock dependent demand, production defective items and delay of payment scheme.

Soni and Shah (2008) develop an inventory model with stock dependent demand and progressive payment scheme. Progressive scheme is a variant of permissible delay of payment. In progressive payments scheme, there is more than one payment period. The supplier does not charge any interest if the buyer pays before the first payment period but interest will be charged after the first payment period and become higher for the next payment period. Similar payment scheme model using two-levels of credit policy was developed by Jaggi *et al.* (2012). This chapter tries to extend the work of Soni and Shah (2008) by considering not only retailers but supplier and retailer decisions simultaneously in just in time inventory model. Since supplier and the retailer try to optimize their own decision, the model is developed as a non-cooperative model. Single vendor-single buyer non-cooperative models with permissible delay in payments are developed using Stackelberg equilibrium (Chern *et al.*, 2013) and under Nash equilibrium (Chern *et al.*,2014). Teng *et al*. (2012) studied vendor-buyer inventory model with credit financing for both integrated and non-cooperative environment. They concluded that vendor should offer short permissible delay payment to reduce its cost. Li *et al.* (2014) introduced an inventory model with a transferable utility game under permissible delay of payment scheme. Supplier sells the same commodities and gives the retailers delay of payments.

Stackelberg Game can be described as follows: a leader of this game, for example a supplier, who knows the decision process of his buyers will react to maximize his own profit. The buyer, as a follower, answers the supplier's decision by setting new decision to improve his profit. On the other side, buyer can act as a leader and supplier acts of the follower depend on the bargaining power of the buyer and the supplier. Even though collaboration system is better than the competitive system, Stackelberg game theory has attracted many researchers' attention for practical reason. Esmaeili *et al.* (2008) developed Stackelberg game and cooperative game between several buyers and several sellers where marketing expenditure and unit price charged by the buyer influence the demand of the product being sold. Yu *et al.* (2009) discussed a Stackelberg Game theory in a Vendor Manage Inventory (VMI) system. They concluded that vendor can get benefit from his leadership in Stackelberg game model. In this chapter, Stackelberg game model between the supplier and the buyer is developed. Supplier produces an item continuously and then sends it to the buyer using just in time concept.

From our extensive literature research there are few papers discussing about inventory models with progressive delay of payment scheme and no research that consider two players in progressive delay of payment scheme using Stackelberg game approach. In this chapter, supplier will give two period permissible delay of payment to the buyer and the buyer decides replenishment period and delivery frequency at each replenishment time. The item will send to the buyer in discrete frequency and buyer can sell it directly to customer with constant demand rate. Buyer can pay the supplier at the first delay period, between first and second delay period and after the second delay period. In this chapter, we will show the effect of progressive delay in payment when supplier acts a leader and buyer acts as a leader. Since the model is complex, a hybrid of Genetic Algorithm (GA) and Simulate Annealing (SA) is used to solve the problem. This chapter is divided into five sections. The first section discusses the research gap and literature study. Inventory model development is shown in section two and the method to solve the model is described in section three. Section four shows a numerical example and sensitivity analysis to give management insights of the model and section five concluded the results.

MATHEMATICAL MODEL

The entire of this chapter using assumptions, parameters and decision variables as follows:

Assumptions

1. Demand rate is constant and deterministic.
2. Shortage is not allowed
3. Production rate is higher than demand rate
4. Delivery lead time is zero
5. Supplier does not charge any interest if buyer pay before delay period (M_1). Supplier charges buyer with interest I_{c1} if buyer pay between first delay period (M_1) and second delay period (M_2) and interest (I_{c2}) if the buyer pay after second delay period (M_2), where ($I_{c2} > I_{c1}$).
6. Planning horizon is infinite.

Parameters

I: Product quantity
Q: Order quantity
q: Delivery quantity

K: Delivery frequency
w: Delivery frequency during production up time
P: Supplier's production rate (unit/unit time)
D: Buyer's demand rate (unit/unit time)
A: Buyer's ordering cost
A_v: Supplier's setup cost
C_t: Transportation cost
h: Buyer's inventory cost
h_o: Buyer's opportunity cost
h_v: Supplier's inventory cost
h_{vo}: Supplier's opportunity cost
IP: Supplier average inventory
I_{c1}: Supplier's interest rate for buyer when the buyer pay between M_1 and M_2 Period
I_{c2}: Supplier's interest rate for buyer when the buyer pay after M_2 period
I_e: Bank's interest rate
c: product unit cost
p_r: product price
TI_{ev}: Total supplier's opportunity cost
TI_{eb}: Total buyer's interest profit
TI_{c1}: Total interest paid by the buyer if buyer pays between M_1 and M_2 period
TI_{c2}: Total interest paid by the buyer if buyer pays after M_2 period
$TBUC$: Buyer's total cost
$TVUC$: Supplier's total cost

Decision Variables

T: Replenishment period
K: Delivery quantity during replenishment period
M_1: The first period of delay of payment
M_2: The second period of delay of payment

This chapter discusses an inventory model with progressive payment between single supplier and single buyer. Supplier has a specific production rate and delivers the buyer's order in a specific quantity (q) to the buyer in a discrete delivery frequency (K) as seen in Figure 1. Buyer receives items from the supplier in a specific T/K period and then sells the items with a constant demand rate. Figure 2 show the inventory rate at the buyer's side. The buyer does not have to pay directly when the items are delivered. The supplier gives delay of payment to the buyer to give opportunity for buyer to buy in higher volume. Supplier has two deadline payment period which are M_1 and M_2. When the buyer pay before M_1 period, there is no interest charged by supplier to the buyer. If the buyer pays after M_1 and before M_2, supplier charges a certain interest rate to the buyer. If the buyer pays after M_2 period then the supplier charges the buyer with higher interest rate. The buyer can sell the items directly and can get cash payment from his customer. The buyer can save his income to banks and get extra interest.

Figure 1. Supplier's inventory model

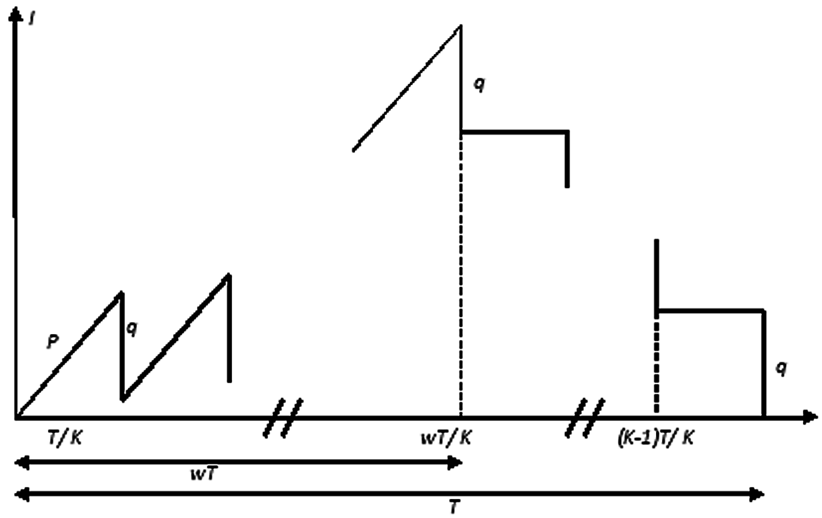
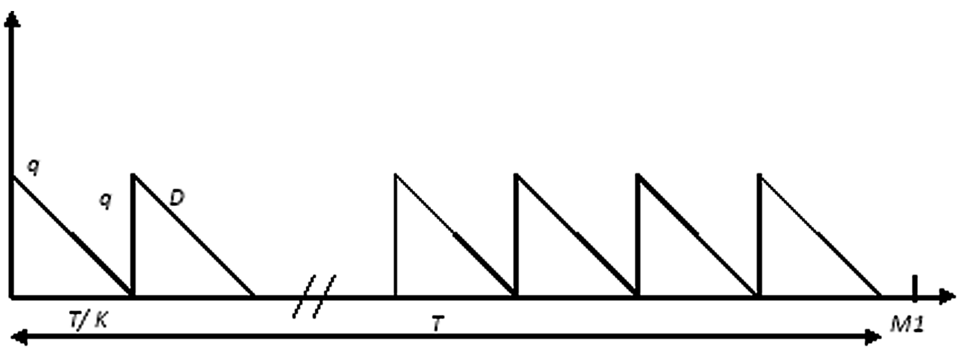


Figure 2. Buyer inventory level



There are three cases in this model. In the first model, the total order quantity (Q) in replenishment time (T) has shorter time than the first delay period (M_1). The second case where replenishment time in between the first delay period and second delay period and the last case where replenishment period is longer than the second delay period.

Using Figure 1, the supplier has production rate P and has production period as long as (wT/K) period. Supplier delivers m unit/delivery every T/K period for K deliveries. Average supplier inventory can be modeled as:

$$IP = \frac{\frac{1}{2} \times q \times \left(\frac{(K-1)T}{K} - \frac{wT}{K} \right)}{\frac{T}{K}} = \frac{q(K-w-1)}{2} \tag{1}$$

and the buyer average inventory as seen in Figure 2, can be modeled as

$$IB = \frac{q}{2} \tag{2}$$

Case 1

($T \le M_1$)

In the first case replenishment time (T) is shorter than the first delay time (M_1). Buyer can has payment at last in the first delay time (M_1). When buyer pay at M_1, buyer can have opportunity cost as shown in Figure 3.

When vendor does not apply for delay of payment, supplier can receive payment directly after the products are sold to buyer. Since supplier applied delay of payment, supplier has opportunity cost as below.

$$TI_{ev} = \frac{h_{v0}PwT}{K}\left(M_1 - \frac{wT}{2K}\right) \tag{3}$$

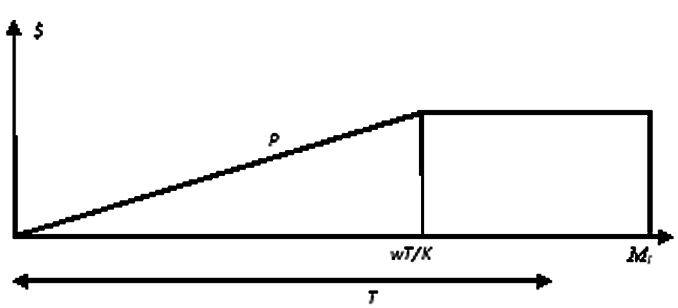
In the other side, buyer receives payment directly from his buyer and delay payment to supplier until the first delay period (M_1). With bank interest rate (I_e), buyer can get total interest gain as:

$$TI_{eb} = I_e DT\left(M_1 - \frac{T}{2}\right) \tag{4}$$

The total supplier cost consists of setup cost, inventory cost and opportunity lost cost, and one has:

$$TVUC_1 = \frac{A_v D}{mK} + \frac{h_v q\left(K - w + 1\right)}{2} + \frac{h_{v0}PwT}{K}\left(M_1 - \frac{wT}{2K}\right) \tag{5}$$

Figure 3. Buyer opportunity cost for case 1



The buyer cost consists of setup cost, transportation cost, inventory cost and buyer's opportunity gain as modeled below.

$$TBUC_1 = \frac{AD}{qK} + \frac{C_t D}{m} + \frac{hq}{2} - I_e DT\left(M_1 - \frac{T}{2}\right) \tag{6}$$

Case 2

$(M_1 \leq T \leq M_2)$

In the case 2, there are two possibilities of buyer opportunity cost as shown in Figure 4. The first case occurs when the vendor production uptime longer than the first delay of payment period and the second case when the vendor production uptime is shorter than the first delay of payment period.

Case 2.1.

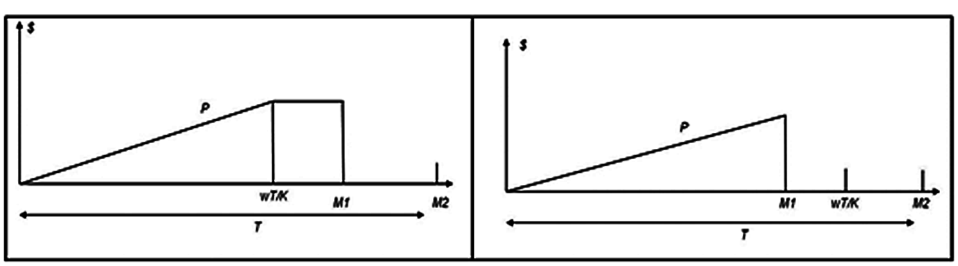
$p_r DM_1 + p_r I_e DM_1^2/2 \geq cDT$

In this case, the buyer can pay with full payment at M_1. For this case, there are two cases of the vendor opportunity cost. In the first cases, the vendor's production up time period is less than the first permissible time and in the second case, the vendor's production period is longer than the first permissible payment. For the first case, the vendor opportunity cost is same as equation (3). For the second case, the vendor opportunity cost is:

$$TI_{cv} = h_{v0}\left(\int_{t=0}^{M_1} Pt\,dt\right) = \frac{h_{v0}P\left(M_1\right)^2}{2} \tag{7}$$

Since the buyer pays all cost at M_1, so the vendor does not get interest earned. The buyer interest earned is equal to:

Figure 4. Buyer opportunity cost for case 2



$$TI_{eb} = I_e \left(\int_{t=0}^{M_1} Dtdt \right) = I_e \left(\frac{DM_1^{\ 2}}{2} \right) \tag{8}$$

Buyer does not have interest cost, since buyer pays all payment at M_1.

Case 2.1.a.

wT/K < M_1

The buyer's total inventory cost consists of setup cost, transportation cost, interest cost, opportunity cost minus interest earned. The total inventory cost per unit can be modeled as:

$$TBUC_{21a} = \frac{AD}{qK} + \frac{C_t D}{Qq} + \frac{hq}{2} - I_e \left(\frac{DM_1^{\ 2}}{2} \right) \tag{9}$$

The vendor total cost consists of setup cost, physical inventory holding cost, interest cost minus interest earned. The vendor total cost is

$$TVUC_{21a} = \frac{A_v D}{qK} + \frac{h_v q(K - w + 1)}{2} + \frac{h_{v0} PwT}{K} \left(M_{1-} \frac{wT}{2K} \right) \tag{10}$$

Case 2.1.b.

wT/K > M_1

The buyer's total inventory cost consists of setup cost, transportation cost, interest cost, opportunity cost minus interest earned. The total inventory cost is same as $TBUC_{21a}$. The vendor total cost consists of setup cost, physical inventory holding cost, interest cost minus interest earned. The vendor total cost can be formulated as:

$$TVUC_{21b} = \frac{A_v D}{qK} + \frac{h_v q(K - w + 1)}{2} + \frac{h_{v0} P\left(M_1\right)^2}{2} \tag{11}$$

Case 2.2.

$p_r DM_1 + p_r I_e DM_1^2/2 < cDT$

In this case, the vendor opportunity cost is equal with the vendor opportunity cost in case 2.1. The vendor interest earned can be modeled as:

$$TI_{ev} = \frac{I_{c1}}{2p_r D}\left(cDT - \left(p_r DM_1 + \frac{p_r IeDM_1^2}{2}\right)\right)^2 \quad (12)$$

The buyer interest cost is equal to the vendor interest earned and the buyer interest earned is equal to case 2.1.

Case 2.2.a.

wT/K < M_1

The buyer's total inventory cost consists of setup cost, transportation cost, interest cost, opportunity cost minus interest earned. The total inventory cost per unit can be modeled as:

$$TBUC_{22a} = \frac{AD}{qK} + \frac{C_t D}{Qq} + \frac{hq}{2} + \frac{I_{c1}}{2p_r D}\left(cDT - \left(p_r DM_1 + \frac{p_r IeDM_1^2}{2}\right)\right)^2 - I_e\left(\frac{DM_1^{\ 2}}{2}\right) \quad (13)$$

The vendor total cost consists of setup cost, inventory holding cost, interest cost minus interest earned. The vendor total cost can be formulated as:

$$TVUC_{22a} = \frac{A_v D}{qK} + \frac{h_v q(K-w+1)}{2} + \frac{h_{v0} PwT}{K}\left(M_{1-}\frac{wT}{2K}\right) - \frac{I_{c1}}{2p_r D}\left(cDT - \left(p_r DM_1 + \frac{p_r IeDM_1^2}{2}\right)\right)^2 \quad (14)$$

Case 2.2.b.

wT/K > M_1

The buyer's total inventory cost consists of setup cost, transportation cost, inventory cost, interest cost, opportunity cost minus interest earned. The total inventory cost is same as $TBUC_{22a}$. The vendor total cost consists of setup cost, inventory holding cost, interest cost minus interest earned. The vendor total cost can be formulated as:

$$TVUC_{22b} = \frac{A_v D}{qK} + \frac{h_v q(K-w+1)}{2} + \frac{h_{v0} P\left(M_1\right)^2}{2} - \frac{I_{ci}}{2p_r D}\left(cDT - \left(p_r DM_1 + \frac{p_r IeDM_1^2}{2}\right)\right)^2 \quad (15)$$

Case 3

($M_2 \leq T$)

There are some possibilities when the replenishment time (*T*) is longer than the second delay of payment period (M_2). The buyer opportunity cost for case 3 has same trend as case 2 as shown in Figure 4.

Case 3.1.

$$p_r DM_1 + p_r I_e DM_1^2 / 2 \geq cDT$$

In this case the vendor opportunity cost is equal with the vendor opportunity cost in case 2, the vendor interest earned is zero since all payment is paid at M_1, The buyer interest cost is zero since all payment is paid at M_1 and the buyer interest earned is equal to case 2.1. There are two possibilities where production period is shorter than the first delay of payment and the production period is longer than the first delay of payment.

Case 3.1.a.

$$wT / K < M_1$$

The buyer's total inventory cost consists of setup cost, transportation cost, inventory cost, minus interest earned. The total inventory is same as $TBUC_{21a}$. The vendor total cost consists of setup cost, inventory holding cost, interest cost minus interest earned. The vendor total cost is same as $TBUC_{21a}$.

Case 3.1.b.

$$wT/K > M_1$$

The buyer's total inventory cost consists of setup cost, transportation cost, inventory cost minus interest earned. The total buyer inventory cost per unit is same as $TBUC_{21a}$. The vendor total cost consists of setup cost, inventory holding cost, and interest cost. The vendor total cost per unit time is same as $TVUC_{21a}$.

Case 3.2.

$$p_r DM_1 + p_r I_e DM_1^2 / 2 < cDT but \begin{bmatrix} p_r D\left(M_2 - M_1\right) \\ + p_r IeD\left(M_2 - M_1\right)^2 / 2 \end{bmatrix} \geq \left[cDT p_r DM_1 + p_r I_e DM_1^2 / 2\right]$$

The vendor opportunity cost is equal with the vendor opportunity cost in case 2. The vendor interest earned, the buyer interest cost and the buyer interest earned is equal to case 2.2.

Case 3.2.a.

$wT/K < M_1$

The buyer's total inventory cost consists of setup cost, transportation cost, inventory cost, opportunity cost minus interest earned. The total inventory cost per unit is same as $TBUC_{22a}$.

The vendor total cost consists of setup cost, inventory holding cost, interest cost minus interest earned. The vendor total cost per unit time is same as $TVUC_{22a}$.

Case 3.2.b.

$wT / K > M_1$

The buyer's total inventory cost consists of setup cost, transportation cost, inventory cost, opportunity cost minus interest earned. The total inventory cost per unit is same as $TBUC_{22a}$. The vendor total cost consists of setup cost, physical inventory holding cost, interest cost minus interest earned. The vendor total cost per unit time is same as $TVUC_{22b}$.

Case 3.3.

$$p_r DM_1 + p_r I_e DM_1^2 / 2 < cDT \text{and} \left[p_r D\left(M_2 - M_1\right) + p_r IeD\left(M_2 - M_1\right)^2 / 2 \right] < \begin{bmatrix} cDT - p_r DM_1 \\ -p_r I_e DM_1^2 / 2 \end{bmatrix}$$

The vendor opportunity cost is equal with the vendor opportunity cost in case 2. The vendor interest earned is:

$$TIE_v = I_{c1}\left(\frac{a+b}{2}\right)\left(M_2 - M_1\right) + I_{c2}\left(\frac{b^2}{2p_r D}\right) \tag{16}$$

where

$$a = \left(cDT - \left(p_r DM_1 + \frac{p_r IeDM_1^2}{2} \right) \right)$$

and

$$b = \left(cDT - \left(p_r DM_1 + \frac{p_r IeDM_1^2}{2} \right) \right) - \left(p_r D\left(M_2 - M_1\right) + p_r IeD\frac{(M_2 - M_1)^2}{2} \right)$$

The buyer interest cost is equal with the vendor interest earned and the buyer interest earned is equal with case 2.2.

Case 3.3.a.

$$wT / K < M_1$$

The buyer's total inventory cost consists of setup cost, transportation cost, inventory, opportunity cost minus interest earned. The total inventory cost per unit can be modeled as:

$$TBUC_{33a} = \frac{AD}{qK} + \frac{C_t D}{Qq} + \frac{hq}{2} + I_{c1}\left(\frac{a+b}{2}\right)\left(M_2 - M_1\right) + I_{c2}\left(\frac{b^2}{2p_r D}\right) - I_{eb}\left(\frac{DM_1^{\ 2}}{2}\right) \quad (17)$$

The vendor total cost consists of setup cost, inventory holding cost, interest cost minus interest earned. The vendor total cost can be formulated as:

$$TVUC_{33a} = \frac{A_v D}{qK} + \frac{h_v q(K - w + 1)}{2} + \frac{h_{v0} PwT}{K}\left(M_{1-} \frac{wT}{2K}\right) - I_{c1}\left(\frac{a+b}{2}\right)\left(M_2 - M_1\right) + I_{c2}\left(\frac{b^2}{2p_r D}\right) \quad (18)$$

where

$$a = \left(cDT - \left(p_r DM_1 + \frac{p_r IeDM_1^2}{2}\right)\right)$$

and

$$b = \left(cDT - \left(p_r DM_1 + \frac{p_r IeDM_1^2}{2}\right)\right) - \left(p_r D\left(M_2 - M_1\right) + p_r IeD\frac{(M_2 - M_1)^2}{2}\right)$$

Case 3.3.b.

$$wT / K > M_1$$

The buyer's total inventory cost consists of setup cost, transportation cost, inventory cost, opportunity cost minus interest earned. The total inventory cost per unit is same as $TBUC_{33a}$. The vendor total cost consists of setup cost, physical inventory holding cost, interest cost minus interest earned. The vendor total cost can be formulated as:

$$TVUC_{33b} = \frac{A_v D}{qK} + \frac{h_v q(K-w+1)}{2} + \frac{h_{v0} P\left(M_1\right)^2}{2} - I_{c1}\left(\frac{a+b}{2}\right)\left(M_2 - M_1\right) + I_{c2}\left(\frac{b^2}{2p_r D}\right) \quad (19)$$

where

$$a = \left(cDT - \left(p_r DM_1 + \frac{p_r IeDM_1^2}{2}\right)\right)$$

and

$$b = \left(cDT - \left(p_r DM_1 + \frac{p_r IeDM_1^2}{2}\right)\right) - \left(p_r D\left(M_2 - M_1\right) + p_r IeD\frac{(M_2 - M_1)^2}{2}\right)$$

THE STACKELBERG GAME SOLUTION

In the Stackelberg game model, the buyer decision variables are replenishment period (*T*) and delivery frequency (*K*). The vendor offers the first and second delay of the payment period (M_1 and M_2). Since the model is complex, hybrid of Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) is applied. The GA method is used to solve leader optimization problem and PSO is applied to solve the follower optimization problem. In the solution we use a simple GA and PSO method. The GA method algorithm as below:

GA Algorithm

Chromosome

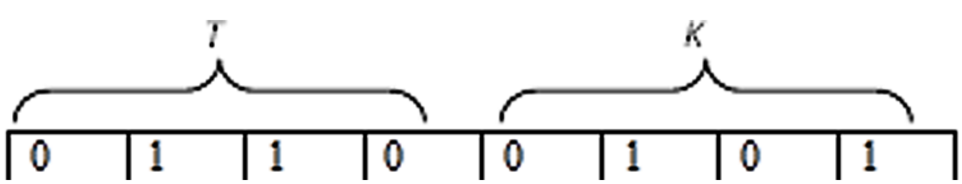
The chromosome (allele) is a binary The chromosome structure for 2 products is shown in Figure 5.

As example, the *T* in Figure 4 can be represented as:

$T = 0x2^0+1x2^1+1x2^2+0x2^3 / 15 = 5/15 = 0.333$

and

Figure 5. Chromosome structure



$K = 0x2^0+1x2^1+0x2^2+1x2^3 = 9$.

Initial Population

The initial chromosome population is generated randomly and the population size is equal to 20.

Evaluation of Fitness

A fitness function is calculated using the total vendor unit cost if vendor act as the leader and the total buyer unit cost if the buyer act as the leader.

The Parent Selection

This model uses the roulette wheel method and a solution with a less fitness value has greater probability to be selected.

Genetic Operators

Genetic operators are used to derive a better solution for each generation. Genetic operators consist of elitism, crossover and mutation. In this study, the population size is set at a constant through successive generation. In each generation, elitism is set and crossover and mutation are used to generate new children. Two point crossover function with crossover is used with probability is equal to 0.8. The mutation scheme is uniform with mutation probability is equal to 0.03.

Stopping Criterion

The stopping criterion for Genetic Algorithm depends on the number of generations. If the number of generations is greater than the value previously determined, then stop. We used 100 number of generations.

PSO Framework

Particle swarm optimization is a population-based computation technique where each particle moves according to its own best position and the best position of the other particle. It is like a flock of birds collectively foraging for food, where the food location is represented by the fitness function.

1. Initialize particle by setting number of particles (*pr*), number of iterations (α) and some initial parameters. Set $\overrightarrow{v_0} = 0$, personal best (pbest) $\overrightarrow{xl_{ps}} = \overrightarrow{x_{ps}}$ and iteration *i*=1.
2. For *i*=1,..., *p* decode $\overrightarrow{x_{ps}}$ to a set of decision variable.
3. For *i*= 1..., *p*.calculate the performance measurement of R_i as Z_i, where Z_i is calculated using total vendor unit cost or total buyer unit cost depend on who acts as the follower. Update pbest by setting $\overrightarrow{xl_{ps}} = \overrightarrow{x_{ps}}$ if $Z_{x_{ps}} < Z_{xl_{ps}}$.

4. Update gbest by setting $\overrightarrow{xl_{ps}} = \overrightarrow{x_{ps}}$ if $Z_{xl_{ps}} < Z_{xg_s}$ $Z_{xl_{ps}} < Z_{xg_s}$.
5. Update the velocity and the position of each particle

$$\overrightarrow{v_{ps}}\left(i+1\right) = w\left(i\right)\times\overrightarrow{v_{ps}}\left(i\right) + u\left[0,1\right]\times c1\left(i\right)\times\left(\overrightarrow{xg_s} - \overrightarrow{x_{ps}}\left(i\right)\right) + u\left[0,1\right]\times c2\left(i\right)\times\left(\overrightarrow{xl_{ps}} - \overrightarrow{x_{ps}}\left(i\right)\right) \quad (20)$$

Update of the moment inertia using fitness distance ratio (FDR), and it can be can be shown as

$$w\left(i\right) = w\left(F\right) + \left(\frac{i-F}{1-F}\right)\left(w\left(1\right) - w\left(F\right)\right) \quad (21)$$

Calculate the new position using (20)

$$\overrightarrow{x_{ps}}\left(i+1\right) = \overrightarrow{x_{ps}}\left(i\right) + \overrightarrow{v_{ps}}\left(i+1\right) \quad (22)$$

6. If the generation meet the stopping criteria, stop. Otherwise add generation by one and return to step 2.
7. Set gbest of the last solution as the best solution for multi route inventory routing problem for deteriorating items.

A NUMERICAL EXAMPLE

A numerical example is used to show how the model work and a sensitivity analysis is conducted to get management insights of the model. The numerical example use similar data as Goyal *et al.*(2007). The set of data that is used in this numerical example is $[A, A_v, I_e, c, p_r]$ = [200,150,4%,25,35,] and $[P, D, C_t, h_{vo}, h, h_v]$ = [4000,1000,100,110,4,4]. In the first case, vendor acts as a leader and buyer acts as the follower. The decision variables have lower bound and upper bound as shown in Table 1.

The algorithm is run ten times to get a solution that near to the global optimal solution. The best solution with different values of I_{c1} and I_{c2} when vendor acts as a leader is shown in Table 2. Table 2 shows that the buyer cost tends to increase when the second period interest increase. Buyer tries to reduce her cost by reducing replenishment period and delivery frequency. In the other side, vendor tries to reduce as small as possible the first and second delay of payment to get more profit and reduce her total cost. Buyer's and vendor's total cost tend to be stable in varies the interest of the first delay of the payment period. The buyer's and vendor's decisions are not significantly different in varies interest of the first delay of payment.

In the second sensitivity analysis, buyer acts as a leader and vendor acts as a follower. The lower bound and upper bound of each decision are set as shown in Table 3.

Table 1. The decision variables bound

Decision Variables	Upper Bound	Lower Bound
M_1	1,2 year	0 year
M_2	1,2 year	0 year
K	20 times	1 times
T	1,3 year	0 year

Table 2. Sensitivity analysis with varies of Ic_1 and I_{c2} (vendor as a leader)

I_{c1}	I_{c2}	T^*	K^*	M_1^*	M_2^*	*TVUC*	*TBUC*
3,5%	8%	0,5976	3	0,0025	0,1045	$1.355,20	$1.427,00
	9%	0,4762	2	0,0007	0,0007	$1.324,20	$1.497,60
	10%	0,4617	2	0,0002	0,0002	$1.294,30	$1.518,10
	11%	0,4589	2	0,0003	0,0009	$1.268,20	$1.536,50
4%	8%	0,5983	3	0,0010	0,1230	$1.355,40	$1.426,40
	9%	0,4764	2	0,0014	0,0015	$1.325,60	$1.496,80
	10%	0,467	2	0,0001	0,0002	$1.294,20	$1.518,10
	11%	0,4591	2	0,0011	0,0014	$1.269,30	$1.535,70
4,5%	8%	0,5974	3	0,0010	0,1340	$1.350,80	$1.430,10
	9%	0,4761	2	0,0002	0,0002	$1.323,30	$1.498,20
	10%	0,4672	2	0,0002	0,0009	$1.294,90	$1.517,50
	11%	0,4588	2	0,0003	0,0004	$1.267,70	$1.536,90
5%	8%	0,5975	3	0,0030	0,1530	$1.351,50	$1.431,20
	9%	0,4764	2	0,0012	0,0012	$1.325,00	$1.497,10
	10%	0,4671	2	0,0004	0,0004	$1.294,60	$1.517,80
	11%	0,4588	2	0,0005	0,0005	$1.267,80	$1.536,80

The calculation results with varies of interest rate at the first delay of the payment period (I_{c1}) and the second delay of the payment period (I_{c2}) are shown in Table 4. Table 4 shows the buyer's and vendor's decision is not sensitive in different I_{c1} and I_{c2}. Since the buyer is the leader, vendor tries to minimize the inventory cost by set the first delay of payment to be 0 but the second delay of payment period have to be set as long as possible. When the buyer acts as a leader, the progressive payment scheme becomes a single delay of payment scheme.

Figure 6 shows the vendor and buyer total cost for varies of percentage interest rate of the first and second delay of payment. When vendor acts as a leader, total vendor cost (TVUC VL) decreases as the percentage rate of a second delay of payment increase and as a consequence, the buyer total cost increase. The condition is different when the buyer acts as a leader. The vendor total cost and the buyer total cost are not significantly different in value of the percentage rate of the second delay of payment. This condition shows that the total cost is more sensitive when the vendor acts as leader than the buyer acts as a leader. The comparison of the vendor total cost in varies of I_{c1} and I_{c2} can be seen in Figure 7.

Table 3. Lower bound and upper bound variables when buyer acts as a leader

Decision Variables	Upper Bound	Lower Bound
K	20 times	1 times
T	1,3 year	0 year
M_1	1,2 year	0 year
M_2	1,2 year	0 year

Table 4. Sensitivity analysis when buyer acts as a leader

I_{c2}	I_{c1}	T*	K*	M_1*	M_2*	TBUC	TVUC
8%	3,50%	0,6956	3	0	1,2	$ 1.333,74	$ 1.571,60
	4%	0,6638	3	0	1,2	$ 1.353,10	$ 1.506,80
	4,50%	0,6464	3	0	1,2	$ 1.372,30	$ 1.464,70
	5%	0,5995	3	0	1,2	$ 1.394,10	$ 1.388,70
9%	3,50%	0,6724	3	0	1,2	$ 1.333,20	$ 1.538,70
	4%	0,6648	3	0	1,2	$ 1.353,10	$ 1.508,20
	4,50%	0,6441	3	0	1,2	$ 1.372,40	$ 1.461,70
	5%	0,5899	3	0	1,2	$ 1.396,20	$ 1.377,00
10%	3,50%	0,694	3	0	1,2	$ 1.333,60	$ 1.569,30
	4%	0,6523	3	0	1,2	$ 1.353,30	$ 1.491,30
	4,50%	0,643	3	0	1,2	$ 1372,40	$ 1460,30
	5%	0,5997	3	0	1,2	$ 1394,10	$ 1388,90
11%	3,50%	0,7734	4	0	1,2	$ 1.349,40	$ 1.553,80
	4%	0,665	3	0	1,2	$ 1.353,20	$ 1.508,50
	4,50%	0,6472	3	0	1,2	$ 1.372,30	$ 1.465,70
	5%	0,5962	3	0	1,2	$ 1.394,80	$ 1.384,70

Figure 7 shows that the vendor total cost is higher when vendor acts as a leader at small values of I_{c1} and I_{c2}, but mostly the vendor total cost when vendor acts as a leader smaller than the buyer acts as a leader. Similar as previous research, vendor has bigger advantage when he acts as a leader than buyer acts as a leader. The difference of vendor total cost when vendor acts a leader and buyer acts a leader becomes wider as the percentage interest rate of the first delay of payment (I_{c1}) and percentage interest of the second delay of payment (I_{c2}) increase.

Figure 8 shows a comparison of buyer total cost when vendor acts as leader and buyer acts as a leader. In small first delay of payment interest rate (I_{c1}), the buyer total cost when buyer acts as a leader higher than vendor acts as a leader. The buyer total cost moves faster when vendor acts as leader and starts to become higher when $I_{c1} = 4\%$ and $I_{c2} = 10\%$ and 11%. The trends continue and the buyer total cost when vendor acts as a leader is higher than buyer acts as a leader in high first delay of payment interest rate ($I_{c1} = 5\%$). This situation shows that buyer has benefited even vendor acts as a leader when the vendor set small value of the first and second delay of payment interest rates.

Figure 6. Sensitivity analysis in varies of I_{c1} and I_{c2}

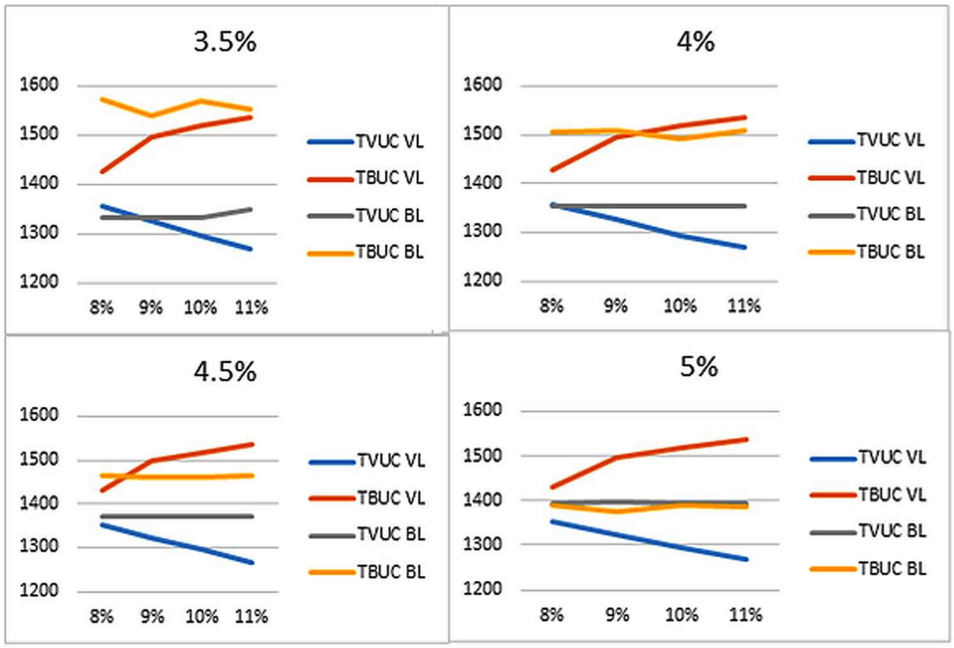


Figure 7. Vendor total cost when vendor and buyer act as a leader

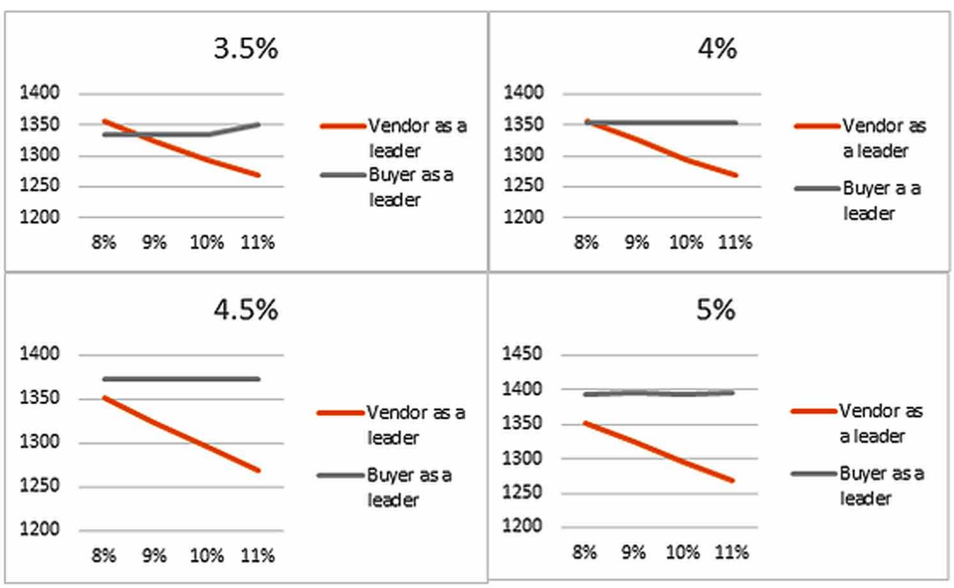
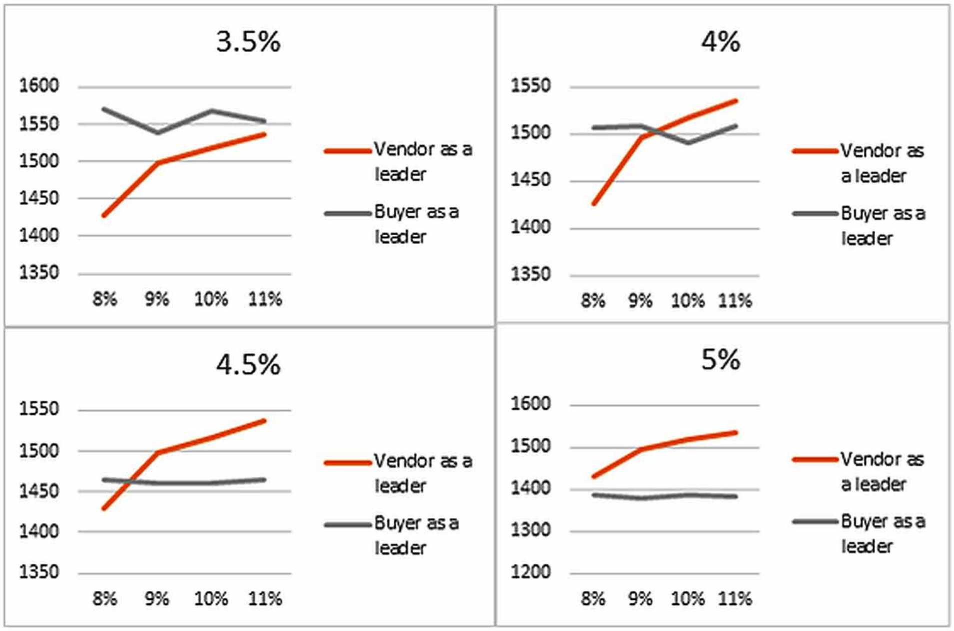


Figure 8. Buyer total cost when vendor act as a leader and buyer act as a leader



The sensitivity analysis shows that the progressive payment scheme works as a single delay of payment when buyer acts as a leader and the decision variables and total cost are more stable. On the other side, when the vendor acts as leader, the decision variables and the total cost are sensitive in varies values of the first and second delay of payment interest rate. The sensitivity analysis shows that the progressive payment scheme can be applied in the Stackelberg game model when vendor acts as the leader and vendor set small values of the first and second delay of payment interest rate.

CONCLUSION

In this chapter, a Stackelberg game model for the progressive payment scheme is developed. The vendor offers to the buyer two delay of the payment period. When the buyer pays before the first delay of payment period, there is no interest charged by the vendor to the buyer. Interest is charged when a buyer pays after the first delay of the payment period and higher interest is charged when a buyer pays after the second delay of payments period. The model is solved using a hybrid of GA and PSO since the model is a nonlinear model. A numerical analysis and sensitivity analysis are conducted to show how the model works and gets some management insights. The sensitivity analysis shows that the progressive payment scheme is similar as a single payment scheme when the buyer acts as a leader. The vendor can get benefit when he acts as a leader, and the buyer can get benefit when he acts as a leader only when the first and second delay of payment interest rate offer by the vendor are high. The Stackelberg game model can work better when vendor acts as a leader and set small values of interest rates.

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