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Journal paper

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The Effect of Unreliable Machine for Two Echelons Deteriorating Inventory Model

I Nyoman Sutapa¹, I Gede Agus Widyadana^{1*}

Abstract: Numerous studies have focused on two-echelon supply chains, but few consider models involving deteriorating items and unreliable machinery. This paper develops an inventory model for a two-echelon supply chain that incorporates machine unreliability, where the machine's downtime is uniformly distributed. Due to the complexity of the system, a simple heuristic method is employed to solve the model, as a closed-form solution is not feasible. A numerical example illustrates the model's functionality, and a sensitivity analysis is conducted to explore the impact of varying lost sales costs. The results indicate that increasing lost sales costs leads to higher costs for both the manufacturer and the buyer. However, the buyer's total cost rises more significantly, particularly in scenarios where the manufacturer's machine is more unreliable. This analysis highlights the importance of considering machine reliability in supply chain models, especially when dealing with deteriorating items.

Keywords: EPQ, deteriorating items, two-echelon supply chain, unreliable machine.

Introduction

The Economic Production Quantity (EPQ) model for deteriorating items has been a focal point of research since its initial development by Misra [1]. Misra's pioneering model, although groundbreaking, was built on several simplifying assumptions, such as constant demand and a perfect production process. Over time, researchers have sought to enhance the realism of the EPQ model by incorporating elements that reflect the complexities of actual production environments, such as unreliable production processes. Zequeira et al. [2], for instance, introduced a model that optimizes maintenance policies and buffer inventory, accounting for the reality that production processes are often unreliable and require maintenance. During maintenance, a buffer inventory is essential to meet demand during periods of production interruption.

Building on this foundation, Abboud et al. [3] developed an EPQ model for deteriorating items that considers stochastic machine unavailability. In their model, production is ideally initiated when inventory levels reach zero. However, the potential for machine unavailability introduces the risk of shortages, particularly when inventory depletes and the machine is not operational. Chung et al. [4] extended Abboud's work by explicitly incorporating deteriorating items into the model, adding another layer of complexity and realism. Further, Wee and Widyadana [5] advanced this line of research by considering stochastic timing for preventive maintenance and machine breakdowns, acknowledging that machines can fail unexpectedly during production.

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Wee and Widyadana [6] later extended their research to develop a two-echelon supply chain model that included the consideration of unreliable machines. However, their model did not account for deteriorating items, a critical factor in many supply chains. In this paper, we build upon the work of Wee and Widyadana [6] by integrating the consideration of deteriorating items and the occurrence of lost sales into the two-echelon supply chain model. Lost sales are particularly significant when a buyer has an urgent need that cannot be deferred until the next replenishment cycle. The primary objective of this study is to determine the optimal production uptime that minimizes the total cost, taking into account both machine unreliability and the impact of deteriorating items.

This paper unfolds with Section 1, where we delve into the research motivation and review relevant literature. In Section 2, we explore the development of the mathematical model. Section 3 brings the theory to life with a practical numerical example and finally the paper concludes with key insights and exciting directions for future research.

Methods

The assumptions: Production rate is greater than demand rate. No machine breakdown occurs in production run period. Production and demand rate are constant. Deteriorating rate is constant. There is no repair or replacement for a deteriorated item.

Parameters:

 \mathbf{S}

Hb = buyer holding cost

= lost sales cost

i arameters.			variables:		
p	=	production rate	Ι	=	inventory level
d	=	demand rate	T	=	replenishment period
θ_{ν}	=	manufacture deteriorating rate	T_1	=	production period
θ_{b}	=	buyer deteriorating rate	T_d T_s	=	non production period shortage period
Cs	=	setup cost	m	=	delivery quantity unit
Ct	=	transportation cost	Κ	=	frequency of delivery
Hv	=	manufacture holding cost			

The manufacture inventory model is shown in Figure 1. The manufacturer produces goods for a duration of wT_N/K time and delivers *m* units with each shipment. The manufacture's production unit in one replenishment period is:

$$Q = wP\frac{T}{K} \tag{1}$$

For small θ_v , the manufacture's total inventory can be modeled as:

$$I_T = \frac{m^2 K (K - w + 1)}{2d}$$
(2)

The inventory level at the end of production period is equal with inventory level at the beginning of non production period, and one has:

$$(P - (d + \theta v))\frac{wT}{\kappa} = (d + \theta v)\left(T - \frac{wT}{\kappa}\right)$$
(3)

After some simplifications, we have:

$$w = \frac{(d+\theta v)K}{P} \tag{4}$$

The manufacture's total cost consists of the manufacture's setup cost and holding cost. The manufacture's total cost can be modelled as follows:

$$TCM(T, K) = \frac{\frac{Cs + \frac{m^2 K(K - \frac{(d + \theta_v) K}{p} + 1) H_v}{2d}}{T + \int_{t=T_d}^{\infty} (t - T_d) f(t) dt}$$
(5)

Figure 2 illustrates the buyer's inventory level. When inventory reaches zero, m units are delivered by the manufacturer. This model assumes zero delivery lead time; however, shipment delays from the manufacturer may occur, leading to lost sales for the buyer during the period Ts. The buyer's inventory cost, as modelled using the approach from Rau et al. [6], can be expressed as follows:

$$IB = \left(\frac{d(2+\theta_b^T/K)}{\theta_b^2(2-\theta_b^T/K)} - \frac{d+d\theta_b^T/K}{\theta_b^2}\right) \left(\frac{H_bK}{T}\right)$$
(6)

The buyer's total inventory cost consists of setup cost, transportation cost, holding cost, and lost sales cost.

$$TCB(T,K) = \frac{CtK + \left(\frac{d\left(2 + \frac{\theta_b T}{K}\right)}{\theta_b^2 \left(2 - \frac{\theta_b T}{K}\right)} - \frac{d\left(1 + \frac{\theta_b T}{K}\right)}{\theta_b^2}\right) (H_b K) + S_b d\int_{t=T_a}^{\infty} (t - T_d) f(t) dt}{T + \int_{t=T_a}^{\infty} (t - T_d) f(t) dt}$$
(7)



Figure 1. Manufacture inventory level



Figure 2. The buyer inventory level

The total supply chain cost can be modeled by combining (5) and (7), one has:

$$\frac{CS + \frac{m^2 K (K - \frac{(d + \theta_v) K}{p} + 1) H_v}{2d} + CtK + \left(\frac{d \left(2 + \frac{\theta_o T}{K}\right)}{\theta_v^2 \left(2 - \frac{\theta_o T}{K}\right)} - \frac{d \left(1 + \frac{\theta_o T}{K}\right)}{\theta_v^2}\right) (H_o K) + S_a D \int_{s - T_a}^{\infty} (t - T_a) f(t) dt$$

$$TCT(T, K) = \frac{T + \int_{v - T_a}^{\infty} (t - T_a) f(t) dt}{T + \int_{v - T_a}^{\infty} (t - T_a) f(t) dt}$$
(8)

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The Model of Uniform Distribution Unavailability Time

In this paper, we consider the unavailability time t as a random variable that is uniformly distributed over the interval [0, b]. This assumption reflects a scenario where the machine's downtime can occur at any moment within this interval with equal likelihood. The uniform distribution is a practical choice in modelling such uncertainty, as it allows for a straightforward analysis of the impact of downtime on the supply chain's overall performance. The corresponding probability density function f(t) for this uniform distribution is defined as:

$$f(t) = \begin{cases} 1/b, & 0 \le t \le b \\ 0, & otherwise \end{cases}$$

By substituting the uniform probability density function in equation (8), the total supply chain cost per unit time can be modelled as follows:

$$E(TCT(T,K)) = \frac{Cs + \left((d+\theta_i)T - mk\right)\frac{H_r}{\theta_r} + CtK + \left(\frac{d\left(2 + \frac{\theta_s T}{K}\right)}{\theta_s^2 \left(2 - \frac{\theta_s T}{K}\right)} - \frac{d\left(1 + \frac{\theta_s T}{K}\right)}{\theta_s^2}\right)(H_s K) + S_s D\left(\frac{(b-T_s)^2}{2b}\right)}{T + \left(\frac{(b-T_s)^2}{2b}\right)}$$
(9)

The non production period (T_d) is equal with total time minus production period. So, the non-production period can be modelled as:

$$T_{d} = 1 - \frac{wT}{\kappa}$$
(10)

Substitute (4) to (10), one has;

 $T_d = 1 - \frac{(d+\theta_v)T}{P}$

Substitute (11) to (9), one has:

$$Cs + \left((d + \theta_v)T - mk\right)\frac{H_v}{\theta_v} + CtK\left(\frac{d\left(2 + \frac{\theta_b T}{K}\right)}{\theta_b^2\left(2 - \frac{\theta_b T}{K}\right)} - \frac{d\left(1 + \frac{\theta_b T}{K}\right)}{\theta_b^2}\right)(H_bK)$$

$$E(TCT(T, K)) = \frac{T + \left(\frac{\left(b - \left(1 - \frac{(d + \theta_v)T}{P}\right)\right)^2}{2b}\right)}{T + \left(\frac{\left(b - \left(1 - \frac{(d + \theta_v)T}{P}\right)\right)^2}{2b}\right)}$$

$$+ \frac{S_b D\left(\frac{\left(b - \left(1 - \frac{(d + \theta_v)T}{P}\right)\right)^2}{2b}\right)}{T + \left(\frac{\left(b - \left(1 - \frac{(d + \theta_v)T}{P}\right)\right)^2}{2b}\right)}$$

(11)

(12)

The optimal replenishment period is found by deriving (12) in term of T and set the value equal to 0, one has;

$$\frac{d\mathrm{TCT}(\mathrm{T},\mathrm{K})}{d\mathrm{T}} = \begin{pmatrix} \frac{\mathrm{Sd}\left(\mathrm{b}-1+\frac{\mathrm{d}+\theta_{\mathrm{v}}}{\mathrm{P}}\right)}{\mathrm{b}} + \frac{\mathrm{H}_{\mathrm{v}}(\mathrm{d}+\theta_{\mathrm{b}})^{2}\mathrm{TK}\left(\mathrm{K}-\frac{(\mathrm{d}+\theta_{\mathrm{v}})\mathrm{K}}{\mathrm{P}}+1\right)}{\mathrm{d}\mathrm{K}^{2}} \\ + \left(\frac{\mathrm{d}}{\theta_{\mathrm{b}}\mathrm{K}\left(2-\frac{\theta_{\mathrm{b}}\mathrm{T}}{\mathrm{K}}\right)} + \frac{\mathrm{d}\left(2+\frac{\theta_{\mathrm{b}}\mathrm{T}}{\mathrm{K}}\right)^{2} - \frac{\mathrm{d}}{\mathrm{K}\theta_{\mathrm{b}}}\right)\frac{\mathrm{KH}_{\mathrm{b}}}{\mathrm{T}} \\ & \vdots \\ + \left(\frac{(2+\frac{\theta_{\mathrm{b}}\mathrm{T}}{\mathrm{K}})}{\theta_{\mathrm{b}}^{2}\left(2-\frac{\theta_{\mathrm{b}}\mathrm{T}}{\mathrm{K}}\right)} - \frac{\mathrm{d}\left(1+\frac{\theta_{\mathrm{b}}\mathrm{T}}{\mathrm{K}}\right)}{\theta_{\mathrm{b}}^{2}}\right)\frac{\mathrm{KH}_{\mathrm{b}}}{\mathrm{T}^{2}} \\ - \left(\frac{\mathrm{C}_{\mathrm{s}} + \mathrm{KC}_{\mathrm{t}} + \frac{\mathrm{Sd}\left(\mathrm{b}-1+\frac{\mathrm{d}+\theta_{\mathrm{v}}}{\mathrm{P}}\right)}{\theta_{\mathrm{b}}^{2}} - \frac{\mathrm{d}\left(1+\frac{\theta_{\mathrm{b}}\mathrm{T}}{\mathrm{H}\right)}{\theta_{\mathrm{b}}^{2}}\right)}{\mathrm{KH}_{\mathrm{b}}} \\ + \frac{\mathrm{H}_{\mathrm{v}}(\mathrm{d}+\theta_{\mathrm{b}})^{2}\mathrm{T}^{2}(\mathrm{K}-\frac{(\mathrm{d}+\theta_{\mathrm{v}})\mathrm{K}}{\mathrm{h}})}{2\mathrm{Kd}} \\ + \left(\frac{(2+\frac{\theta_{\mathrm{b}}\mathrm{T}}{\mathrm{K})}}{\theta_{\mathrm{b}}^{2}\left(2-\frac{\mathrm{ch}}{\mathrm{K}}\right)} - \frac{\mathrm{d}\left(1+\frac{\theta_{\mathrm{b}}\mathrm{T}}{\mathrm{H}\right)}}{\theta_{\mathrm{b}}^{2}}\right)\frac{\mathrm{KH}_{\mathrm{b}}}{\mathrm{T}} \\ + \left(\frac{(2+\frac{\theta_{\mathrm{b}}\mathrm{T}}{\mathrm{K})}}{\theta_{\mathrm{b}}^{2}\left(2-\frac{\mathrm{ch}}{\mathrm{K}}\right)} - \frac{\mathrm{d}\left(1+\frac{\theta_{\mathrm{b}}\mathrm{T}}{\mathrm{H}\right)}}{\theta_{\mathrm{b}}^{2}}\right)\left(1+\frac{(\mathrm{b}-1+\frac{\mathrm{d}+\theta_{\mathrm{v}}}{\mathrm{P}})\mathrm{T}^{2}}{2b}\right)\right)} \right) \right) = 0$$
 (13)

If the optimal production down time is bigger than the upper bound of the uniform distribution unavailable time, then lost sales will not occur. In this situation, equation (12) is not fulfilled. Equation (12) should be revised as follow: $(a_1(a_2), a_3(a_4), a_4(a_4), a_4($

$$Cs + \left((d + \theta_{v})T - mk\right)\frac{H_{v}}{\theta_{v}} + CtK + \left(\frac{d\left(2 + \frac{\theta_{b}T}{K}\right)}{\theta_{b}^{2}\left(2 - \frac{\theta_{b}T}{K}\right)} - \frac{d\left(1 + \frac{\theta_{b}T}{K}\right)}{\theta_{b}^{2}}\right)\left(H_{b}K\right)$$

$$E(TCT_{NL}(T, K)) = \frac{T}{T}$$
(14)

The optimal replenishment time for total cost without lost sales can be found by deriving (14) in term of T, and one has:

$$\frac{\mathrm{dTCT}_{\mathrm{NL}}(\mathrm{T},\mathrm{K})}{\mathrm{dT}} = \begin{pmatrix} \frac{\mathrm{Sd}\left(\mathrm{b}-1+\frac{\mathrm{d}+\theta_{\mathrm{v}}}{\mathrm{P}}\right)}{\mathrm{b}} + \frac{\mathrm{H}_{\mathrm{v}}(\mathrm{d}+\mathrm{t}_{\mathrm{b}})^{2}\mathrm{TK}\left(\mathrm{K}-\frac{(\mathrm{d}+\theta_{\mathrm{v}})\mathrm{K}}{\mathrm{P}}+1\right)}{\mathrm{dK}^{2}} \\ + \left(\frac{\mathrm{d}}{\theta_{\mathrm{b}}\mathrm{K}\left(2-\frac{\theta_{\mathrm{b}}\mathrm{T}}{\mathrm{K}}\right)} + \frac{\mathrm{d}\left(2+\frac{\theta_{\mathrm{b}}\mathrm{T}}{\mathrm{K}}\right)}{\theta_{\mathrm{b}}\mathrm{K}\left(2-\frac{\theta_{\mathrm{b}}\mathrm{T}}{\mathrm{K}}\right)^{2}} - \frac{\mathrm{d}}{\mathrm{K}\theta_{\mathrm{b}}}\right)\frac{\mathrm{KH}_{\mathrm{b}}}{\mathrm{T}} \\ + \left(\frac{\left(2+\frac{\theta_{\mathrm{b}}\mathrm{T}}{\mathrm{H}_{\mathrm{b}}}\right)}{\theta_{\mathrm{b}}^{2}\left(2-\frac{\theta_{\mathrm{b}}\mathrm{T}}{\mathrm{H}_{\mathrm{b}}}\right)} - \frac{\mathrm{d}\left(1+\frac{\theta_{\mathrm{b}}\mathrm{T}}{\mathrm{H}_{\mathrm{b}}}\right)}{\theta_{\mathrm{b}}^{2}}\right)\frac{\mathrm{KH}_{\mathrm{b}}}{\mathrm{T}^{2}} \end{pmatrix} \right) \\ - \left(\frac{\mathrm{C}_{\mathrm{s}} + \mathrm{KC}_{\mathrm{t}}}{\left(+\frac{\mathrm{Sd}\left(\mathrm{b}-1+\frac{\mathrm{d}+\theta_{\mathrm{v}}}{\mathrm{P}}\right)}{\mathrm{b}} + \frac{\mathrm{H}_{\mathrm{v}}(\mathrm{d}+\theta_{\mathrm{b}})^{2}\mathrm{T}^{2}\left(\mathrm{K}-\frac{(\mathrm{d}+\theta_{\mathrm{v}})\mathrm{K}}{\mathrm{P}}+1\right)}{2\mathrm{Kd}}}{\mathrm{H}_{\mathrm{b}}} \\ + \left(\frac{(2+\frac{\theta_{\mathrm{b}}\mathrm{T}}{\mathrm{H}_{\mathrm{b}}}}{\theta_{\mathrm{b}}^{2}\left(2-\frac{\theta_{\mathrm{b}}\mathrm{T}}{\mathrm{H}_{\mathrm{b}}}\right)} - \frac{\mathrm{d}\left(1+\frac{\theta_{\mathrm{b}}\mathrm{T}}{\mathrm{H}_{\mathrm{b}}}\right)}{\mathrm{H}_{\mathrm{B}}}}{\mathrm{H}_{\mathrm{b}}\left(\frac{\mathrm{d}}{\mathrm{H}_{\mathrm{b}}}\mathrm{T}\right)} \right) \left(\frac{1}{\mathrm{T}}\right)^{2} \tag{15}$$

The single-vendor single-buyer inventory model with uniformly distributed maintenance time can be solved using the following procedure: (1) Set K=1; (2) Calculate T^* by solving equation (13); (3) Use T^* from step 2 to calculate equation (12). If $T_d < b$, proceed to step 4; otherwise, T^* is the optimal solution; and (4) Solve equation (15) to find T^* . If $TUC(K^*-1, T) > TUC(K^*,T) < TUC(K^*+1,T)$, the optimal solution is found. Otherwise, increase K by 1 and return to step 2.

Results and Discussion

A numerical example illustrates the application of the proposed model with the following parameters: setup cost (Cs) of \$40 per setup, transportation cost (Ct) of \$0.5 per delivery, production rate (p) of 150 units per time period, and demand rate (d) of 80 units per time period. The holding cost for the manufacturer (Hv) is \$5 per unit per time period, while the buyer's holding cost (Hb) is \$6 per unit per time period. The buyer incurs a lost sales cost (S) of \$12 per unit, with the manufacturer's deterioration rate (θ v) at 0.05 per time period and the buyer's deterioration rate (θ b) at 0.06 per time period. The manufacturer's unavailability is modelled as being uniformly distributed between 0 and 1. Given the model's complexity, a closed-form solution cannot be derived. Instead, a simple heuristic method, implemented using Maple software, is employed to determine the optimal solution. The results indicate that the optimal total cost is achieved with a small but positive replenishment period, balancing the costs associated with holding, setup, and lost sales.





Figure 3 Total cost in varies of T

Table 1. The computational results for Uniform distribution unavailability time

K	T^*	TCT* (\$	\$) K	T^*	TCT^*
					(\$)
1	0.346	473.77	11	0.457	219.98
2	0.390	330.42	12	0.463	218.70
3	0.409	283.00	13	0.467	217.75
4	0.421	259.71	14	0.470	217.07
5	0.429	246.11	15	0.474	216.60
6	0.436	237.35	16	0.477	216.29
7	0.441	231.36	17	0.480	216.11
8	0.446	227.10	18	0.483	216.06
9	0.451	224.00	19	0.487	216.09
10	0.455	221.70	20	0.490	216.21

Tuble 2. Sensitivity analysis for emitterin distribution anavaliability times
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Decision	Lost sales cost (S)				
variables	9.6	10.8	12	13.2	14.4
K	20	19	18	17	16
T^*	0.552	0.515	0.483	0.456	0.432
Manf.Cost	67.86	73.03	78.06	82.97	87.79
Buyer Cost	124.98	131.76	138.00	143.81	149.22
<u>Total Cost</u>	192.84	<u>204.79</u>	216.06	226.78	237.01

The computational results for the numerical example are presented in Table 1, which illustrates the optimal total cost per unit time across different values of delivery frequency (K). The analysis reveals that the minimum total cost is achieved when the number of deliveries (K) equals 18, corresponding to an optimal total cost per unit time of \$216.06. At this point, the optimal production time (T*) is 0.483. These results demonstrate that careful adjustment of the delivery frequency and production time can significantly impact the overall cost efficiency of the supply chain, particularly when dealing with deteriorating items and machine unreliability.

Given the importance of the lost sales cost (S) in determining the total supply chain cost, a sensitivity analysis was conducted to explore how variations in S influence the optimal decisions. The results of this analysis are shown in Table 2, where different values of S are examined to assess their impact on the decision variables and total costs. The sensitivity analysis indicates that the lost sales cost has a pronounced effect on the optimal total cost. Specifically, a 20% increase in the lost sales cost leads to a 9.6% rise in the total supply chain cost. This finding is consistent with existing literature, which suggests that higher lost sales costs generally drive-up total costs within the supply chain.

As the lost sales cost increases, both the frequency of deliveries (K) and the optimal replenishment time (T*) decrease. This response is driven by the need for manufacturers and buyers to minimize their costs under the pressure of higher potential losses due to unmet demand. Consequently, both parties opt for more frequent deliveries and shorter replenishment periods to mitigate the risk of lost sales. However, this strategy results in an increase in the total cost per unit time for both the manufacturer and the buyer.

Interestingly, the analysis shows that the manufacturer's cost per unit time increases by 12.46%, while the buyer's cost per unit time rises by 8.13% with a 20% increase in lost sales cost. This disparity highlights that the percentage increase in the manufacturer's cost is higher than that of the buyer, although the absolute cost borne by the manufacturer remains lower than that of the buyer. This outcome reflects a common observation in supply chain models: the entity that leads the supply chain, typically the manufacturer, often benefits more from the overall supply chain structure.

However, this advantage for the manufacturer comes at a cost to the buyer, who may face higher expenses due to reduced replenishment periods. To maintain a balanced relationship and encourage the buyer to continue purchasing more products, the manufacturer might need to offer compensation or other incentives. This could help reduce the manufacturer's cost per unit time while also ensuring that the buyer's increased costs are managed effectively. This dynamic underscore the importance of collaborative strategies in supply chain management, particularly in scenarios where machine reliability and product deterioration play significant roles.

Conclusion

This paper presents the development of a deteriorating production inventory model for a two-echelon supply chain, taking into account an unreliable production system. The model incorporates uniform distributions for machine unavailability time, reflecting the randomness of production disruptions. Due to the complexity of the system, a simple heuristic method was employed to solve the model, as deriving a closed-form solution was not feasible. The model's effectiveness was demonstrated through a numerical example and further validated with a sensitivity analysis. The analysis revealed that as

the lost sales cost increases, the total supply chain cost per unit time also rises. Specifically, the manufacturer's cost per unit time increases as the buyer shortens the replenishment period to minimize their own costs, highlighting the interdependence between supply chain partners in managing costs under uncertain production conditions.

However, this study has certain limitations. The model assumes uniform distributions for machine unavailability, which may not capture all real-world scenarios where other distributions might better represent downtime variability. Additionally, the heuristic approach, while practical, may not always provide the most optimal solution, especially in more complex or larger-scale supply chains. Another limitation is the focus on a single buyer and manufacturer, which does not fully represent the complexities of multi-echelon supply chains involving multiple stakeholders.

Future research could address these limitations by exploring alternative probability distributions for machine unavailability and developing more sophisticated optimization techniques that could yield more accurate solutions. Additionally, extending the model to multi-echelon supply chains with multiple buyers and manufacturers could provide more comprehensive insights. Furthermore, investigating different manufacturing strategies that could incentivize buyers to purchase more products, even in the presence of an unreliable machine environment, would be a valuable avenue for research. Such strategies could include dynamic pricing, contractual agreements, or collaborative forecasting and replenishment planning, which could enhance the overall efficiency and resilience of the supply chain.

Acknowledgement

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