Comparative Study of Particle Swarm Optimization Algorithms in Solving Size, Topology, and Shape Optimization

D Prayogo[1], K Harsono[1], K E Prasetyo[1], F T Wong[1], D Tjandra[1]

[1]Department of Civil Engineering, Petra Christian University, Jalan Siwalankerto 121-131, Surabaya 60236, Indonesia
E-mail: m21416123@john.petra.ac.id

Abstract. This paper focuses on optimizing truss structures while propose best PSO variants. Truss optimization is one way to make the design efficient. There are three types of optimization, size optimization, shape optimization, and topology optimization. By combining size, shape and topology optimization, we can obtain the most efficient structure. Metaheuristics have the ability to solve this problem. Particle swarm optimization (PSO) is metaheuristic algorithm which is frequently used to solve many optimization problems. PSO mimics the behavior of flocking birds looking for food. But PSO has three parameters that can interfere with its performance, so this algorithm is not adaptive to diverse problems. Many PSO variants have been introduced to solve this problem, including linearly decreasing inertia weight particles swarm optimization (LDWPSO) and bare bones particles swarm optimization (BBPSO). The metaheuristic method is used to find the solution, while DSM s used to analyze the structure. A 10-bar truss structure and a 39-bar truss structure are considered as case studies. The result indicates that BBPSO beat other two algorithms in terms of best result, consistency, and convergence behavior in both cases. LDWPSO took second place for the three categories, leaving PSO as the worst algorithm that tested.

1. Introduction

Truss structures are often seen in buildings. This structure is only subjected to axial force due to releasing the moment of fixity. In civil engineering it is important to have efficient design, especially for truss structures. For civil engineers, construction cost efficiency is considered as priority. There are many ways to minimize construction costs. One way that can be used is structure optimization. There are three types of optimization: size, shape, and topology [1]. Size optimization is used to find the optimal sectional area for each member, topology finds the optimal number of elements in the structure while still paying attention to structural stability, and shape is used to find the optimal node coordinates. Usually researchers only consider one or two optimizations, but by optimizing all of them, we can obtain the most efficient structure [2].

"Trial and error" is commonly used by engineers to gain this efficient design. But this method is not efficient and requires a lot of time due to its many constraints and variables. Fortunately, metaheuristics have the capability to solve this problem [3]. Particle swarm optimization (PSO) [4], proposed by Kennedy and Eberhart, is popular in solving the problem of optimization. It is well known for its simple concept. This algorithm applies the behavior of flocking birds. Each bird tries to find best place in the flock to find food. Like flocks of bird, they use information from the previous direction, the best location that the group ever experienced, and the best location that each bird ever experienced. Although it is easy to understand the concept, this algorithm has some weaknesses. Three parameters that must be set in the beginning is one of them [5]. To resolve this matter, many researchers have proposed some PSO variants like linearly decreasing inertia weight particles swarm optimization (LDW-PSO) [6] and bare bones particle swarm optimization (BBPSO) [7].

2. Literature review

2.1. Particle swarm optimization (PSO)

While bird searching for food, they tend to use information from initial velocity (vi(t)), best location that this particle discovers $X_{pbest}(t)$, best location from population $X_{gbest}(t)$, and its current location $X_i(t)$. This concept is used by PSO to search for the optimum solution. This algorithm is well known for this simple concept. But the one weakness of this algorithm is the need to pre-set the parameters to adapt to each separate problem [7]. First, the algorithm generates a random location for each particle [6]. Then the particle enters the main looping, where each particle updates its location every iteration using Equation (1). Particles use velocity to update the location, which is calculated with Equation (2).

$$X_i(t+1) = X_i(t)+v_i(t+1) \tag{1}$$

$$v_i(t+1) = wv_i(t)+r_1C_1\left(X_{pbest}(t)\text{-}X_i(t)\right)+r_2C_2\left(X_{gbest}(t)\text{-}X_i(t)\right) \tag{2}$$

where $v_i(t+1)$ is the next velocity; w is inertia weight; $v_i(t)$ is the initial velocity; r_1 and r_2 are random numbers between 0 and 1; C_1 and C_2 are constants that have been set (usually 2); $X_{pbest}(t)$ is personal best; *Xi(t)* is the initial location; $X_{gbest}(t)$ is global best; and $X_i(t+1)$ is the particles new location.

2.2. Linearly Decreasing Inertia Weight Particles Swarm Optimization (LDWPSO)

LDWPSO perfects one parameter in PSO: Inertia weight, which is used to adjust local and global searches. For a more global search a large value of inertia weight is needed, while for more local search a small value of inertia weight is needed. By reducing the inertia weight each iteration, PSO searches more in a global scope at the beginning of iteration, and in a local scope at the end of iteration [6]. The inertia weight updates with Equation (3):

$$w= w\text{-}(ws\text{-}we)(t)/(t_{max}) \tag{3}$$

where w is current inertia weight; ws is initial inertia weight; *we* is final inertia weight; *t* is current iteration; and *tmax* is total iteration.

2.3. Bare Bones Particles Swarm Optimization (BBPSO)

Unlike LDWPSO that modifies one parameter, all parameters are erased by BBPSO. Instead of using velocity to update the location, BBPSO uses a Gaussian distribution. The particle's next position is only calculated by its personal best position and swarm global best position. Parameter-free means the algorithm can easily adapt to separate problems [7]:

$$\mu = \frac{pi+gbest}{2}$$

$$\sigma = |pi\text{-}gbest| \tag{4}$$

$$x(i+1)=\begin{cases} N(\mu,\sigma) & if(\omega > 0.5) \\ pi & else \end{cases}$$

where $pi = (p1, p2, ..., pn)$ is the personal best position of each particle, gbest is the best position of the whole swarm, and ω is a random number from 0 to 1.

3. Problem formulation

The objective of this study is to minimize the weight of the truss structure without violating any constraints. Static constraints such as validity, kinematic stability of structure, size, shape, nodal displacement, and element stress are used as constraints in this study. The mathematical formulation of this optimization problem can be performed as follows:

Find,

$$X = \{A_1, A_2, ..., A_m, \xi_1, \xi_2, ..., \xi_n\}$$

To minimize,

$$f(x) = \sum_{i=1}^{m} B_i A_i \rho_i L_i \quad (5)$$

where,

$$B_i = \begin{cases} 0, \text{ if } A_i < \text{Critical Area} \\ 1, \text{ if } A_i \geq \text{Critical Area} \end{cases}$$

Subjected to:

g_1: Check on validity of structure

g_2 : Check on stability of structure

$g_3(X)$: Stress constraints, $|B_i\sigma_i|-|\sigma_i^{max}| \leq 0$

$g_4(X)$: Displacement constraints, $|\delta_i|-|\delta_j^{max}| \leq 0$

$g_5(X)$: Size constraints, $A_i^{Critical} \leq A_i \leq A_i^{Upper}$

$g_6(X)$: Shape constraints, $\xi_j^{Lower} \leq \xi_j \leq \xi_j^{Upper}$

where, $i= 1,2,..,m$ and $j= 1,2,...,n$, and where A_i, ρ_i, L_i and σ_i are cross-sectional area, density, modules of elasticity, length, and stress of element i, respectively. σ_i and ξ_j are real values of nodal displacement and coordinates of node j, respectively. B_i is a topological bit, which is 0 for absence and 1 for presence of element i, respectively. The truss structure is called invalid ($g1$) if during the optimization process loaded or support nodes are being deleted.

4. Material and method

A combination of the direct stiffness method (DSM) and metaheuristics is used for this optimization. Metaheuristics is used to find the optimal size, topology, and shape of the truss structure while DSM is used to run the structural calculation. Before conducting the research, researchers prepared a DSM program for a planar truss, and prepared three metaheuristic algorithms: PSO, LDW-PSO, and BBPSO. The DSM and metaheuristic algorithms were written using MATLAB 2017a and the results of the three algorithms were compared to determine the best performing algorithm. In general, this program randomizes the cross-section area, and iterates using trial and error until it reaches its maximum iteration. A flow chart of the truss optimization process is diagrammed in Figure 1.

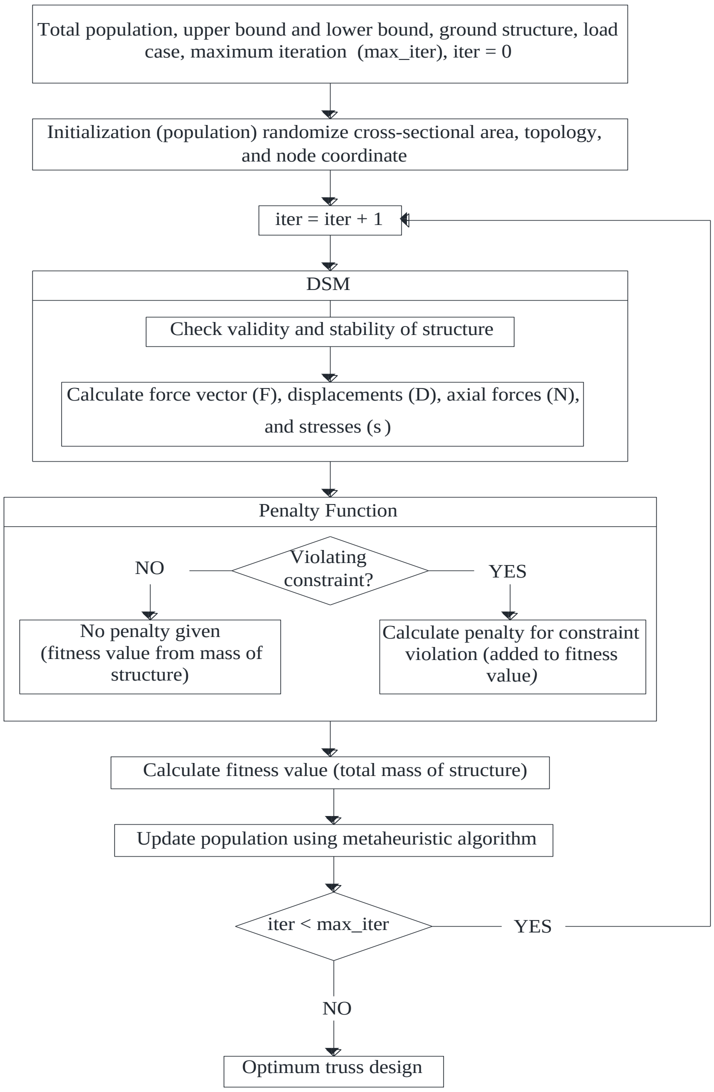

Figure 1. Flow chart for truss optimization

Upper bound and lower bound are used as size and shape constraints. Displacement of each node as well as the axial force and stress of each element from DSM are also used as constraints for this optimization. Whenever a solution violates the constraints, a penalty is given to the solution. This study used two types of penalty. When there are stability and validity constraint violations, the fitness value will be given a dead penalty. Unlike stability and validity constraints, when displacement and stress constraints are violated, a penalty value will be given accordingly. Fpenalty multiplied to the total mass of the structure using Equation (6)–(8) [2]:

$$F_{penalty}=(1+\varepsilon_1 \times C)^{\varepsilon_2}, \tag{6}$$

$$C=\sum_{i=1}^{q} C_i, \tag{7}$$

$$C_i=\left|1-\frac{p_i}{p_i^*}\right|. \tag{8}$$

p_i is a level of violation that is violated against the p_i^* limit, q is the number of constraints used, and ε_1 and ε_2 are parameters set by the researcher. This study refers to [2] on the values of ε_1 and ε_2 being 3.

Then, the results of the $F_{penalty}$ will be multiplied by the total mass of the structure to obtain the fitness value.

5. Test problems and results

This paper compares the performance of three PSO variants using 2 planar truss structure problems. All problems are optimized using shape, topology and size considerations. Each algorithm was run 30 times and with 50 populations. The structures were analyzed using DSM. Cognitive (C1) and social (C2) parameters for PSO and LDWPSO were set to 2. Inertia weight (W) for PSO was set to 0.8 while the LDWPSOs inertia weight linearly decreased from 0.9 to 0.1 with respect to iterations. Algorithms and structural analyses were coded in MATLAB 2017a.

5.1. Planar 10-bar truss structure

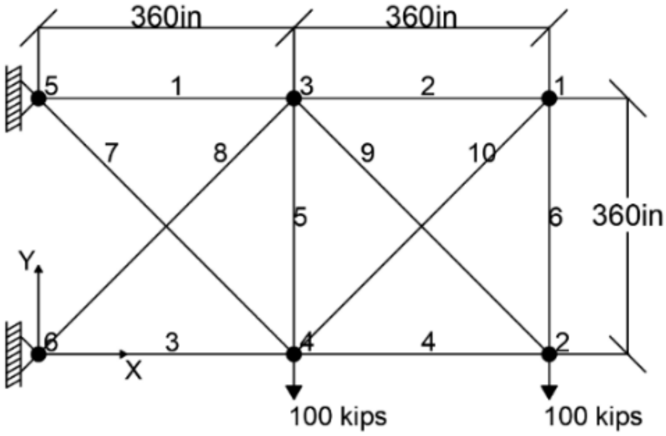

Figure 2. Ground structure for 10-bar truss structure.

This structure is very popular in truss optimization and was previously studied by Miguel [2] and Rahami [8]. The 10-bar structure has a total of six nodes with three fixed nodes and three moving nodes as shown in Fig. 2. It has 12 degrees of freedom due to X and Y directions. The material density is 0.1 lb/in3 and elastic modulus 107 psi. The stress limit for compression/tension is 25,000 psi and displacement should be no more than ±2 in. This problem has 13 variables: Ten cross-section area variables and three geometric variables. A shape constraint for this problem was that nodes 1, 3, and 5 could move in the Y direction only between 180 and 1000 inches. The cross-sectional areas available were:

D = [0.1, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0, 6.5, 7.0, 7.5, 8.0, 8.5, 9.0, 9.5, 10.0, 10.5, 11.0, 11.5, 12.0, 12.5, 13.0, 13.5, 14.0, 14.5, 15.0, 15.5, 16.0, 16.5, 17.0, 17.5, 18.0, 18.5, 19.0, 19.5, 20.0, 20.5, 21.0, 21.5, 22.0, 22.5, 23.0, 23.5, 24.0, 24.5, 25.0, 25.5, 26.0, 26.5, 27.0, 27.5, 28.0, 28.5, 29.0, 29.5, 30.0, 30.5, 31.0, 31.5] (in^2).

Table 1 shows that BBPSO and LDWPSO have the most optimal weight (2705.1667 lb), while PSO cannot obtain such a minimum weight. There is a great gap between BBPSO and the other two algorithms in terms of consistency. The average and standard deviation for BBPSO are far lower than PSO or LDWPSO. BBPSO is also superior in terms of convergence behavior shown in Figure 3. Shape, topology, and size changes can be seen in Figure 4. From previous study, genetic algorithm (GA) [8] obtains larger best result than PSO variants used in this study. However, PSO has small constraints violation.

Table 1. Final design of sizing, shape, and topology for 10-bar truss.

Variables	[8]	PSO	LDWPSO	BBPSO
A1	11.5	11.5	11.5	11.5
A2	0	0	0	0
A3	11.5	11.5	11.5	11.5
A4	5.74	7.22	7.22	7.22
A5	0	0	0	0
A6	0	0	0	0
A7	5.74	5.74	5.74	5.74
A8	3.83	3.13	2.88	2.88
A9	13.5	13.5	13.5	13.5
A10	0	0	0	0
Y1	0	201.4377	180	180
Y3	506.4203	486.7639	486.6606	486.68129
Y5	789.7306	780.6457	790	789.99058
Best (lb)	2723.05	2708.614	2705.167	2705.167
Average (lb)	-	2973.832	2923.337	2804.739
Stdev (lb)	-	222.036	201.069	92.222
Max Stress (ksi)	19.1463	19.185	19.145	19.145
Max Displacement (inch)	1.999996	2	2	2
No. of analyses	-	50000	50000	50000
Constraint violation	None	2.44E-11	None	None

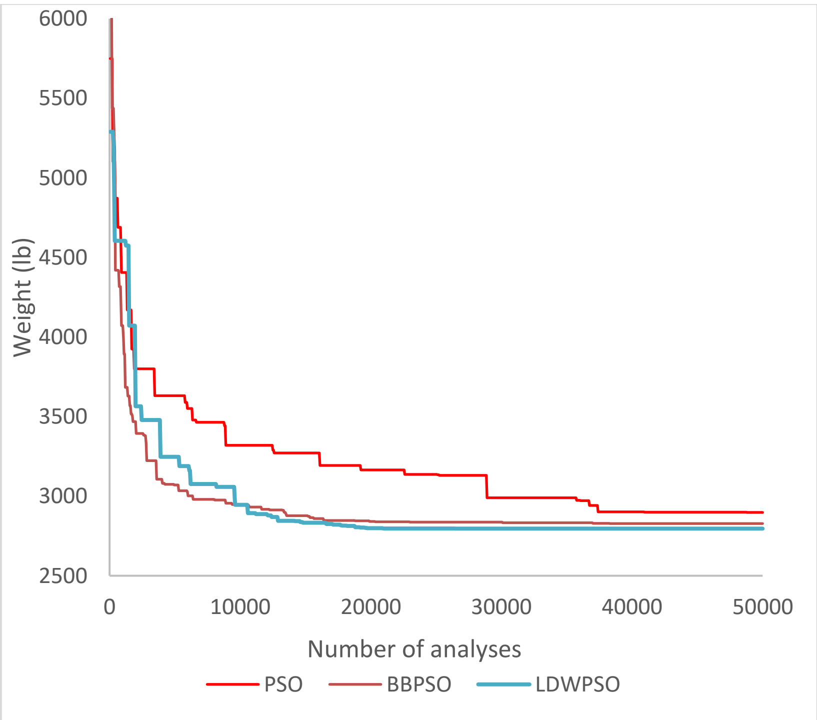

Figure 3. Convergence behavior for 10-bar truss structure.

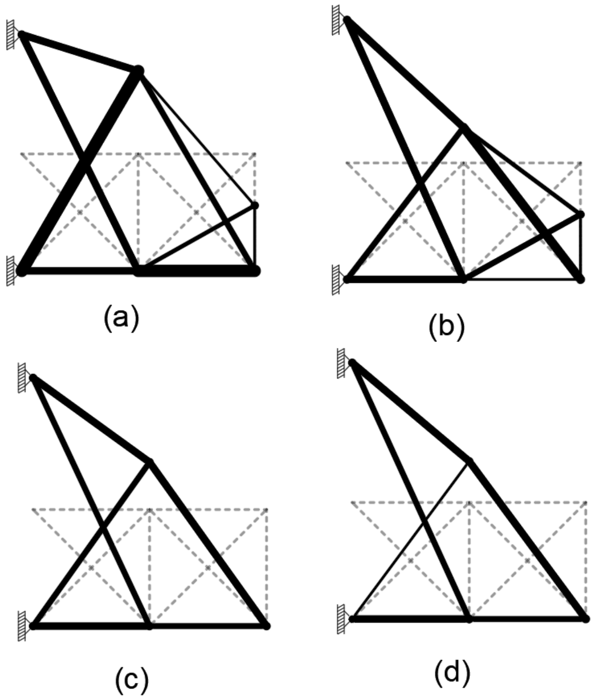

Figure 4. Iteration for 10-bar truss structure (a) first iteration, (b)10th iteration, (c) 100th iteration, (d) final design.

5.2. Planar 39-bar truss structure

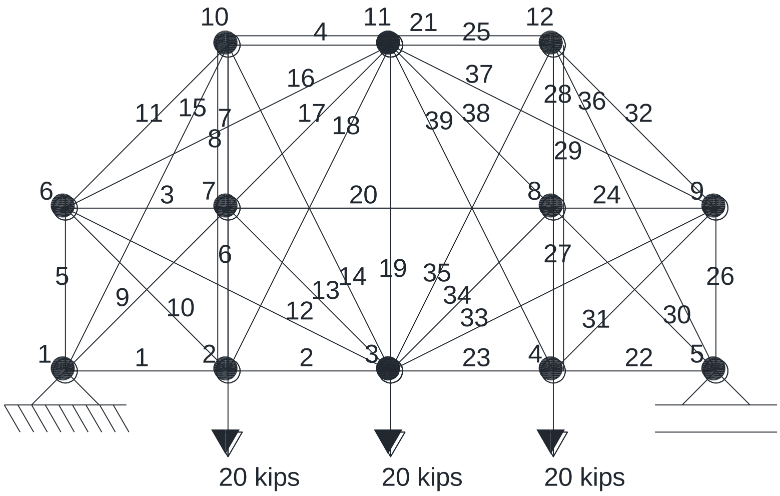

Figure 5. Ground structure for 39-bar truss structure

The ground structure illustrated in Figure 5 shows a vertical load of 20 kips applied on nodes 2,3 and 4. The allowable stress is 20 ksi and allowable displacement is ±2 in. This structure has been studied before by Miguel [2], Deb [9], and Tejani[10]. The material properties (modulus of elasticity and weight density) are the same as in the previous examples. Members of the structure are grouped into 21 groups for symmetrical reasons. For shape constraint, all loading and support nodes are fixed. All nodes can move from 120 to -120 in an x and y direction from its original position, except for node 11 which can only move in a y direction. Nodes move symmetrically, which means there are only seven shape constraints: $x_6 = -x_9$, $y_6 = y_9$, $x_7 = -x_8$, $y_7 = y_8$, $x_{10} = -x_{12}$, $y_{10} = y_{12}$, and y_{11}. This is a continuous problem with a max sectional area of 2.25 $inch^2$ and a minimum of 0.05 $inch^2$.

Table 2. Final design of sizing, shape, and topology for 39-bar truss

Variables	[10]	PSO	LDWPSO	BBPSO
A1,A22	0.1905	0.050001	0.8547715	0.182017
A2,A23	0.9157	1.013031	0.9500338	1.0127308
A3,A24	0	0	0	0
A4,A25	1.4694	0	0.7066295	0
A5,A26	0	0	0.0655538	0
A6,A27	0	0.0503	0	0.0501156
A7,A28	0	1.118042	0.050144	1.1588633
A8,A29	0	2.25	1.0052822	1.2771902
A9,A30	1.2353	0	0	0
A10,A31	0.9966	0	0	0
A11,A32	0	0	2.25	0
A12,A33	0	0	0	0
A13,A34	0.5099	0.501794	2.25	0.5163225
A14,A35	0	0	0	0
A15,A36	0	2.25	1.6655047	1.511016
A16,A37	0	0	0.0917872	0
A17,A38	0	0	0	0
A18,A39	0	0	0	0
A19	1.0159	0	1.0003739	0
A20	15.6136	2.25	2.2489131	1.1418484
A21	143.9449	0	0.402704	0
x6	0	120	120	230.5454
y6	0	0	185.35876	148.74514
x7	192.6985	239.9501	239.99901	185.869
y7	236.2853	240	0	330.34409
x10	0	0	102.93241	134.56055
y10	0.1905	120	181.53663	-120
y11	0.9157	120	290.88376	-120
Best (lb)	190.1088	242.678	230.390	187.896
Average (lb)	211.3174	329.740	311.734	213.512
Stdev (lb)	10.8810	55.580	50.879865	20.068
Max Stress (ksi)	19.9998	19.999	19.999	19.999
Max Displacement (inch)	1.7658	1.4756	1.7418	1.377
No. of analyses	50000	50000	50000	50000
Constraint violation	None	None	None	None

From Table 2, BBPSO is the best algorithm of the three that have been tested. BBPSO gains minimum weight of structure (187.89617 lb) with the lowest average and standard deviation from three PSO variants. With PSO and LDWPSO also showing similar results from previous problem. LDWPSO has the second best result (230.38976 lb) and PSO has the worst result (242.6785 lb). BBPSO has a 63.89% less standard deviation than PSO. Furthermore, BBPSO also shows exceptional convergence behavior in Figure 6. Iteration for the 39-bar truss structure can be seen in Figure 7. In the 100th iteration, BBPSO has found its optimum shape and topology while still optimizing the sectional area. PVS from Tejani[10] has better result than PSO and LDWPSO with 190.1088 lb. BBPSO still has better result and average than PVS. However, PVS has smaller standard deviation (10.8810 lb) than BBPSO.

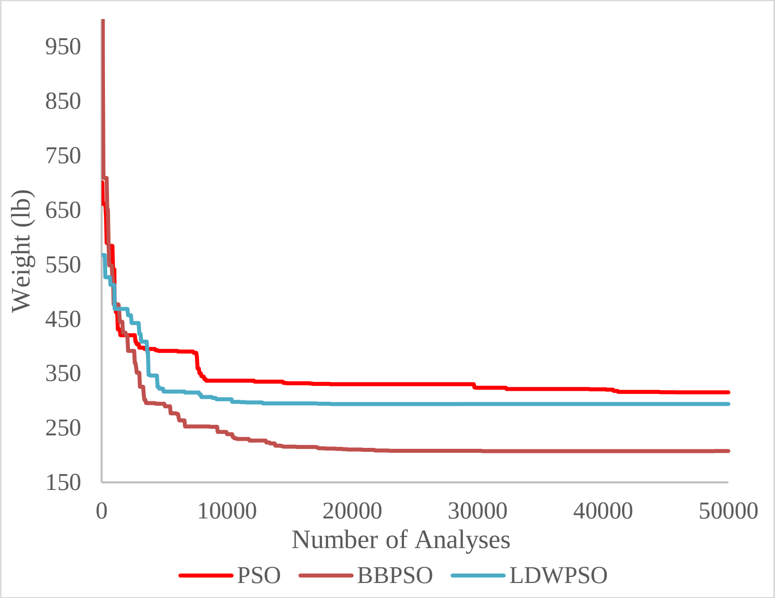

Figure 6. Convergence behavior for 39-bar truss structure.

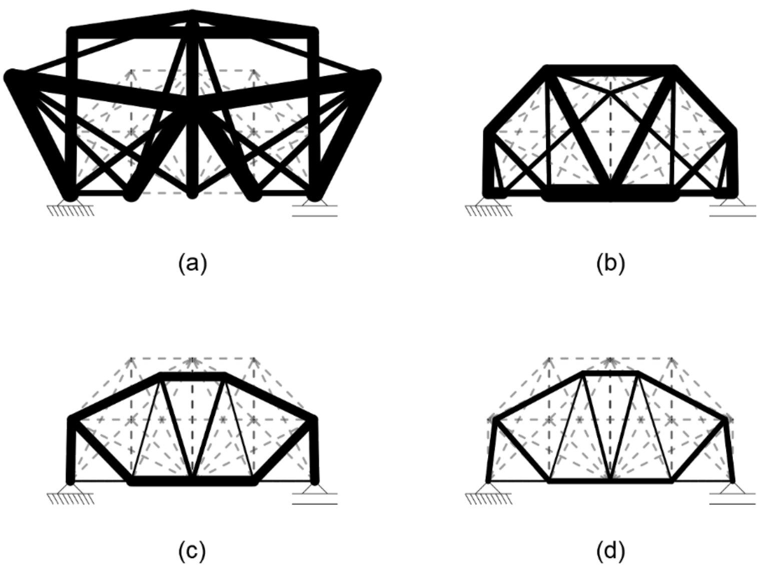

Figure 7. Iteration for 39-bar truss structure (a) first iteration, (b)10th iteration, (c) 100th iteration, (d) final design.

6. Conclusion

In this paper, the three PSO variants (PSO, LDWPSO, and BBPSO) are tested using two planar truss structures. Every benchmark problem is optimized using shape, topology, and size considerations. Static constraints such as stresses, displacements, stability, and validity are used. Optimized shape, topology and size simultaneously deliver a high increase in the number of constraints and variables, thus making the problem more complex and difficult. The results show that the BBPSO algorithm ranks first in achieving lighter trusses, followed by the LDWPSO and PSO algorithms. The BBPSO also outperforms other algorithms in terms of consistency and convergence behavior, followed by LDWPSO and PSO. Even from the previous studies, BBPSO is superior from GA in 10-bar truss problem and GA in 39-bar truss problem. LDWPSO that modified the inertia weight parameter has better result than original PSO, while BBPSO that eliminate the parameters outperform PSO and LDWPSO. It can be concluded that BBPSO is the best PSO variants that has been tested and the performance of PSO can be improved by modifying the parameters.

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