Size, Topology, and Shape Optimization of Truss Structures using Symbiotic Organisms Search

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***Abstract*— Truss structures are common in the building industry. One way to contain construction costs is to implement structural optimization. Optimization has to consider cross-sectional size, area, topology, and node coordinates as design variables. However, each truss structure has numerous complex constraints and variables that make optimizing this structure complex and difficult. The metaheuristic method is efficient and effective in solving large and complex problems. This paper tested three metaheuristic algorithms: particle swarm optimization (PSO), differential evolution (DE), and symbiotic organisms search (SOS). Each algorithm was used to optimize a 10-bar planar truss structure and a 15-bar planar truss structure. SOS was found to have the best optimization results, convergence behavior, and consistency.**

Keywords*— *metaheuristic algorithms, truss structure, optimization

I. Introduction

Truss structure optimization has become one of the "hot" issues in structural engineering for the past decades. A truss structure usually involves interconnected structural members that behave as one single object, where each member is subjected to tension or compression forces only [1]. The most widely studied methods of truss structure optimization are size and topology optimization [2]. Size optimization is used to minimize the cross-sectional area of each member of the truss structure. Topology optimization is used to optimize the number of elements while paying attention to structural stability. A trial-and-error approach is commonly used by engineers to design an optimal truss structure; however, this approach is proven to be time-consuming and cost-inefficient [1].

Truss structure optimization involves many variables and constraints, which makes it more complex and difficult. Additionally, many studies have focused only on sizing and topology, leaving the coordinates of the nodes and the shape of the structure constant. Therefore, current studies are now focusing on finding the best optimization method for truss structure design. By optimizing the size, topology, and shape of the truss structure simultaneously more efficient results can be achieved [3].

The field of metaheuristic algorithms has attracted increased attention from the field of optimization, which uses natural phenomena and randomization concepts to find optimal solutions [3]. Particle swarm optimization (PSO) [4] and differential evolution (DE) [5] are examples of metaheuristic algorithms commonly used to solve many optimization problems. Recently, symbiotic organisms search (SOS) was proposed by Cheng and Prayogo, and has been proven to deliver outstanding performance in structural optimization [6]. This research investigates the performance of SOS in truss design optimization that incorporates size, topology, and shape. The total mass of the truss structure is considered the object of optimization. Additionally, this research uses metaheuristic algorithms, namely, PSO and DE, for comparison purposes.

II. Symbiotic Organisms Search (SOS)

The SOS algorithm was developed by Cheng and Prayogo in 2014 [4]. SOS is a simple and very powerful metaheuristic algorithm, inspired by the interaction between living things known as "symbiosis." SOS applies three forms of symbiosis often seen in nature: mutualism, commensalism, and parasitism. SOS has been used to solve multiple complex and challenging problems since its discovery [7,8].

Mutualism describes the relationship between two organisms that are mutually beneficial to one another such as the relationship between bees and flowers. In the SOS algorithm, if the results of a newer organism are better than the previous organism, then the organism will be replaced by the newer organism. Based on Cheng and Prayogo [6], a mathematical model of the SOS symbiotic mutualism algorithm is found in Eqs. (1)–(3):

$$X_{inew} = X_i + rand(0,1) * (X_{best} - MV * BF_1), \quad (1)$$

$$X_{jnew} = X_j + rand(0,1) * (X_{best} - MV * BF_2), \quad (2)$$

$$MV = \frac{X_i + X_j}{2}, \quad (3)$$

where X_i is organisms that correspond to i-members in the ecosystem; X_j is randomly selected organism from the ecosystem; X_{inew} is new candidate for X_i ; X_{jnew} is new candidate for X_j; BF_1 and BF_2 are random numbers between one and two; and X_{best} is the global best solution.

Commensalism describes the relationship between two organisms in which only one benefits while the other does not gain any advantage or disadvantage. The relationship between remora fish with sharks is one example of commensalism. In the SOS algorithm, organism i (X_i) will interact with organism k (X_k), where X_k is taken randomly and $k \neq i$. This interaction will only renew organism i. The formula for X_{inew} in this symbiosis is shown as Eq. (4):

$$X_{inew}= X_i+rand\left(-1,1\right)*(X_{best}-X_k). \quad (4)$$

Parasitism describes the relationship between two organisms that benefits one organism while the other is harmed. The relationship between Anopheles mosquitoes and humans is an example of symbiotic parasitism. Anopheles mosquitoes carry plasmodium parasite into the human body, which can cause malaria. The organism X_i is given a similar role as the Anopheles mosquito through an artificial parasite or "parasite vector." Furthermore, the fitness value of the parasite vector will be compared with the fitness value of the X_j organism. If the fitness value of the parasite vector is better, then the position of organism X_j will be replaced by the parasite vector.

III. PROBLEM FORMULATION

The objective of this study is to minimize the weight of the truss structure without violating any constraints. The constraints used in this study are static constraints and include nodal displacement, element stress, validity, and kinematic stability of structure. The mathematical formulation of this optimization problem can be performed as follows:

Find, $X= \{A_1, A_2, \ldots, A_m, \xi_1, \xi_2, \ldots, \xi_n\}$ (5)

To minimize, $f(x)= \sum_{i=1}^{m} B_i A_i \rho_i L_i$

where, $B_i=\begin{cases} 0, \text{ if } A_i < \text{Critical Area} \\ 1, \text{ if } A_i \geq \text{Critical Area} \end{cases}$

Subjected to:

g_1: Check on validity of structure

g_2 : Check on stability of structure

g_3(X): Stress constraints, $|B_i\sigma_i|-|\sigma_i^{max}| \leq 0$

g_4(X): Displacement constraints, $|\delta_i|-\left|\delta_j^{max}\right| \leq 0$

g_5(X): Size constraints, $A_i^{Critical} \leq A_i \leq A_i^{Upper}$

g_6(X): Shape constraints, $\xi_j^{Lower} \leq \xi_j \leq \xi_j^{Upper}$

where, $i= 1,2,..,m$ and $j= 1,2,\ldots,n$,

and where A_i, ρ_i, L_i and σ_i are cross-sectional area, density, modules of elasticity, length, and stress of element i, respectively. σ_i and ξ_j are real values of nodal displacement and coordinates of node j, respectively. B_i is a topological bit, which is 0 for absence and 1 for presence of element i, respectively. The truss structure is called invalid (g_1) if during the optimization process there are loaded or support nodes being deleted.

IV. METHODOLOGY

The combination of direct stiffness method (DSM) and metaheuristics is used for this optimization. Metaheuristics is used to find the optimal size, topology, and shape of the truss structure while DSM is used to run the structural calculation. DSM as well as the metaheuristic algorithms was written using MATLAB 2017a. A flow chart of the truss optimization process is presented as Fig. 1.

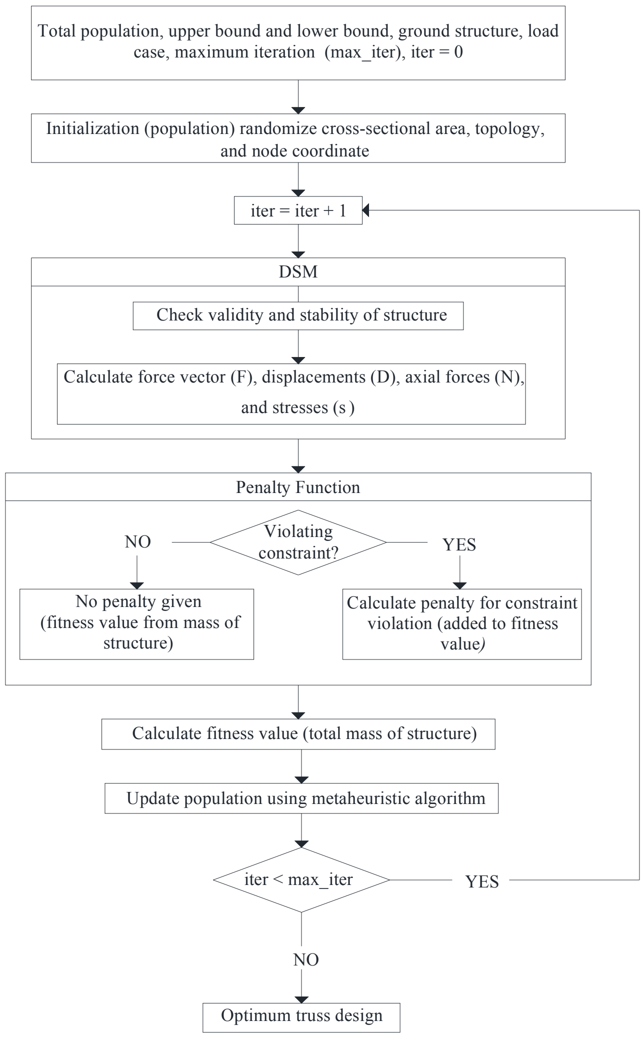

Fig. 1. Flow chart for truss optimization

DSM outputs are the displacement of each node as well as the axial force and stress of each element. These outputs are used as constraints for this optimization. Whenever a solution violates the constraint, a penalty is given to the solution. In this study, stability is reviewed in two ways. The structure is unstable when the rank of the global stiffness matrix is not same as the size of global stiffness matrix or the global stiffness matrix is not definitely positive. When there are constraint violations, a penalty value will be added to the total mass of the structure using Eqs. (6)–(8) [2]:

$$F_{penalty}=(1+\varepsilon_1 \times C)^{\varepsilon_2}, \quad (6)$$

$$C= \sum_{i=1}^{q} C_i, \quad (7)$$

$$C_i= \left|1-\frac{p_i}{p_i^*}\right|. \quad (8)$$

p_i is a level of violation that is violated against the p_1^* limit, q is the number of constraints used, and ε_1 and ε_2 are parameters set by the researcher. This study refers to [2] on the values of ε_1 and ε_2 being 3. Then, the results of the $F_{penalty}$ will be multiplied by the total mass of the structure to obtain the fitness value.

V. Test Problem and Results

This paper compares three metaheuristic algorithm performances using planar and spatial bar structure problems. Each structure has their load case and discrete variables, which will be described next. The goal is to minimize the weight of the structure while not violating the constraints. All algorithms were run 30 times and with 50 populations. Structures are analyzed using a direct stiffness method. Algorithms and structural analyses were coded in MATLAB 2017a. Cognitive (C_1) and social (C_2) parameters for PSO were set to 2 and inertia weight (W) was set to 0.8.

A. Planar 10-bar truss structure

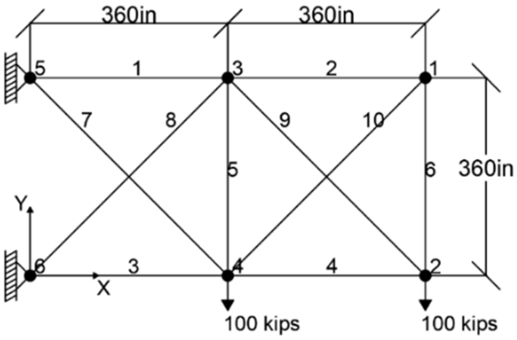

Fig. 2. Ground structure of 10-bar truss

This 10-bar structure has six nodes and twelve degrees of freedom due to X and Y directions as shown in Fig. 2. The material density is 0.1 lb/in^3 and elastic modulus 107 psi. The stress limits for compression/tension is 25,000 psi and displacement should be not more than ±2 in. There are 13 design variables in this problem: ten cross-section area variables and three geometric variables. For geometric variables, nodes 1, 3, and 5 could move between 180 and 1000 inches in Y direction. The cross-sectional areas available are:

D = [0.1, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0, 6.5, 7.0, 7.5, 8.0, 8.5, 9.0, 9.5, 10.0, 10.5, 11.0, 11.5, 12.0, 12.5, 13.0, 13.5, 14.0, 14.5, 15.0, 15.5, 16.0, 16.5, 17.0, 17.5, 18.0, 18.5, 19.0, 19.5, 20.0, 20.5, 21.0, 21.5, 22.0, 22.5, 23.0, 23.5, 24.0, 24.5, 25.0, 25.5, 26.0, 26.5, 27.0, 27.5, 28.0, 28.5, 29.0, 29.5, 30.0, 30.5, 31.0, 31.5] (in^2).

Table I reports that SOS had the best result and the smallest standard deviation. The stopping criterion of all algorithms is set to 15,000 structural analyses. PSO, DE, and SOS obtain minimum weights of 2749.171 lb, 2940.873 lb, and 2705.169 lb, respectively. Figure 3 shows the iteration process of 10-bar truss structure optimization. In terms of consistency, SOS had the best convergence behavior as shown in Fig. 4.

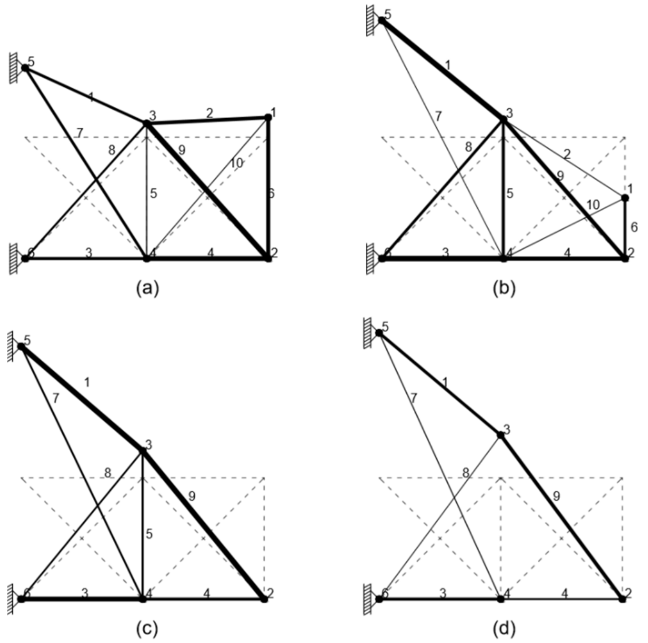

Fig. 3. Iteration of 10-bar truss structure: (a) iteration number 5; (b) iteration number 15; (c) iteration number 25; and (d) iteration number 75

TABLE I. Final designs of sizing, shape, and topology for the 10-bar truss

Variable	**GA** [9]	**PSO**	**DE**	**SOS**
A_1	11.5	11.5	11.5	11.5
A_2	0	0	0	0
A_3	11.5	11.5	11.5	11.5
A_4	5.74	7.22	11.5	7.22
A_5	0	0	0	0
A_6	0	0	0	0
A_7	5.74	5.74	5.74	5.74
A_8	3.84	3.13	4.18	2.88
A_9	13.5	13.5	11.5	13.5
A_{10}	0	0	0	0
y_1	-	-	-	-
y_2	485.5	486.76	505.39	486.66
y_5	789.73	780.6457	760.57	789.9996
Best (lb)	2723.05	2749.171	2940.87	2705.17
Average (lb)	-	3118.027	3084.24	2848.52
Stdev (lb)	-	260.0294	100.17	85.03
Max stress (ksi)	19.1463	19.1849	19.27	19.15
Max displacement (inch)	1.999996	2	1.995376	2
No. of analyses	-	15,000	15,000	15,000
Constraint violations	None	2.44E-11	None	None

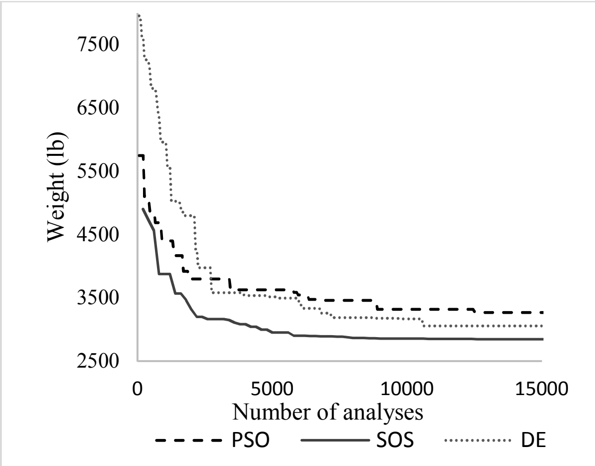

Fig. 4. Convergence behavior for the size, topology, and shape for 10-bar truss optimization

B. Planar 15-bar truss structure

The ground structure illustrated in Fig. 5 shows a vertical load of 10 kips applied on node 8. The allowable stress is 25 ksi and the material properties (modulus of elasticity and weight density) are the same as in the previous examples. The *x*- and *y*- coordinates of nodes 2, 3, 6, and 7, and the *y*-coordinates of nodes 4 and 8 are taken as design variables. However, nodes 6 and 7 are constrained to have the same *x*-coordinates as nodes 2 and 3, respectively. Thus, the problem includes 15 size and eight shape variables ($x_2 = x_6$, $x_3 = x_7$, y_2, y_3, y_4, y_6, y_7, y_8).

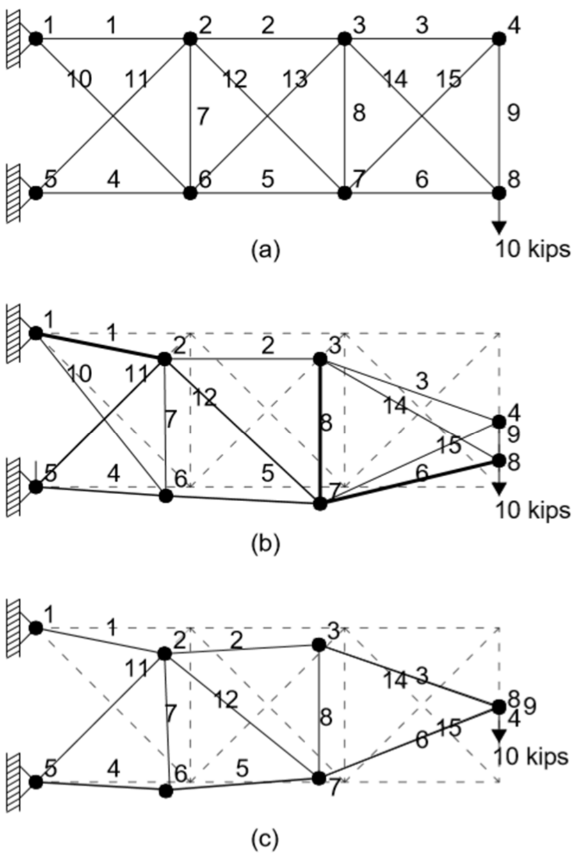

Fig. 5. (a) Ground structure of 15-bar truss, (b) iteration number 15, and (c) iteration number 250

The cross-sectional areas are chosen from:

D = [0.111, 0.141, 0.174, 0.220, 0.270, 0.287, 0.347, 0.440, 0.539, 0.954, 1.081, 1.174, 1.333, 1.488, 1.764, 2.142, 2.697, 2.800, 3.131, 3.565, 3.813, 4.805, 5.952, 6.572, 7.192, 8.525, 9.300, 10.850, 13.330, 14.290, 17.170, 19.180] (in^2).

The side constraints for the configuration variables are 100 in $\leq x_2 \leq$ 140 in, 220 in $\leq x_3 \leq$ 260 in, 100 in $\leq y_2 \leq$ 140 in, 100 in $\leq y_3 \leq$ 140 in, 50 in $\leq y_4 \leq$ 90 in, −20 in $\leq y_6 \leq$ 20 in, −20 in $\leq y_7 \leq$ 20 in, 20 in $\leq y_8 \leq$ 60 in.

Table II shows that SOS had the best result and the smallest standard deviation. Figure 5 shows the iteration process of 15-bar truss structure optimization. In terms of consistency, SOS had the best convergence behavior as shown in Fig. 6.

TABLE II. FINAL DESIGNS OF SIZING, SHAPE, AND TOPOLOGY FOR THE 15-BAR TRUSS

Variables	PSO	DE	SOS
A_1	1.174	0.954	1.333
A_2	0.44	0.954	0.539
A_3	0	0	0.27
A_4	1.174	1.333	0.954
A_5	0.954	0.539	0.954
A_6	0.44	0.539	0.347
A_7	0	0.141	0.141
A_8	0.347	0.22	0.22
A_9	0	0	8.525
A_{10}	0.141	0.347	0
A_{11}	0.347	0	0.347
A_{12}	0.954	0	0.539
A_{13}	0	0.539	0
A_{14}	0.44	0.539	0.347
A_{15}	0	0	0.27
x_2	100	139.5696	105.8613
x_3	220	260	221.0399
y_2	100	107.224	100.4678
y_3	140	100	106.7655
y_4	50	63.3698	58.9022
y_6	-14.7454	12.8818	-6.5144
y_7	-19.9961	20	3.4627
y_8	60	60	58.9067
Best (lb)	84.0683	78.8838	76.9757
Average (lb)	99.9911	84.0552	80.8648
Stdev (lb)	15.1098	3.2419	2.4049
Max stress (ksi)	24.3588	24.9776	24.9998
No. of analyses	50,000	50,000	50,000
Constraint violations	None	None	None

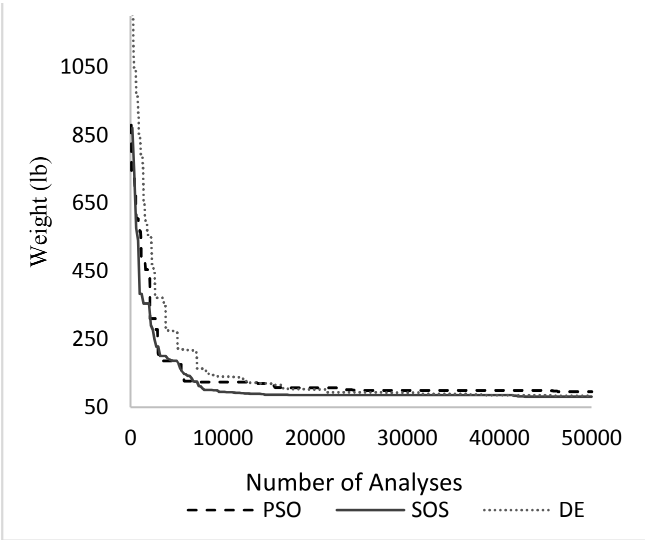

Fig. 6. Convergence behavior for the size, shape, and topology for 15-bar truss optimization

VI. Conclusion

This paper compared the optimization performance of three metaheuristic algorithms, namely, PSO, DE, and SOS, by reviewing two case studies. With the same number of analyses for each algorithm, the result showed that of the three algorithms tested, SOS performed best in terms of optimization result, convergence behavior, and consistency. SOS also had no constraint violations in either the 10-bar or 15-bar problem. In terms of optimization result, DE performed worst on the 10-bar problem and PSO performed worst on the 15-bar problem. In terms of consistency, DE has better performance than PSO on both problems.

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