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Inventory Models for Multi Items Stock-Dependent Demand and Stock-dependent Holding Rate with Capacity Constraint

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Abstract. Inventory models for stock-dependent demand have been developed continuously in recent years. In this paper, we develop inventory dependent model for multi-items by considering stock-dependent demand and stock-dependent holding rate for a rack that has a specific capacity. Two models are developed. The first model is an individual order quantity for each item and the second model is a joint replenishment model. The non-linear model with constraint is solved using an evolutionary algorithm. A numerical example and sensitivity analysis are conducted to show how the models work and to get some management insights. The sensitivity analysis shows that there is no dominant model. However, the joint replenishment order outperforms the individual order when the rack capacity is tight and there is a high ordering cost

INTRODUCTION

Inventory control issue has been discussed widely not only in manufacture but also in warehouses and racks at retailers. There are some interesting topics related to the application of inventory control at retailer racks such as inventory control with stock-dependent demand where customer demand depends on the number of stocks on the racks. Higher stock will drive customers' attention to check items and buy the items. Some stock-dependent demand inventory models have been developed. The first inventory model by considering stock-dependent demand was developed by Baker and Urban [1]. Later many inventory models related with stock dependent demand was developed such as deteriorating inventory models with price dependent demand [2], inventory models by considering the time value of money and inflation [3], and maximization return investment [4]. One interesting extension of the inventory model with stock-dependent demand is in consideration of non-linear holding cost, where the holding cost depends on time.

The first model of inventory model with stock-dependent demand and non-linear holding cost was developed by Giri and Chauduri [5]. A later, similar model was developed by Chang [6]. Both of them considering stock-dependent demand and deteriorating inventory models. More developed models for an inventory model with stock-dependent demand and non-linear holding cost were introduced by Pando *et al.* [7], Pando *et al.* [8], Yang [9], and Chaudury *et al.* [10].

In this paper, we develop Yang [9] model by introducing multi-items and rack capacity. Yang's [9] model is used as the basic model of this paper since this paper discusses non-deteriorated inventory model that is different than Chaudhuri's model [10]. The model that was developed by Yang [9] is similar to Pando *et al.* [7] and did not consider cost budget as Pando [8], therefore we use Yang's [9] model in this paper. This model is related to practice conditions since most racks at retailers consist of more than one item and each rack has its capacity. This model can be applied to

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support smart racks at supermarkets or convenience stores where the rack is designed to inform the stock level timely. Therefore, every time the stock level on the rack can be observed. When the stock level on the rack reach reorders point, then the rack should be replenished with a determined order quantity. Since the rack can be used to store many items, it is more efficient to fill in the racks once for many items compare to one item for each replenishment activity. Since the rack has its capacity, therefore the total units stored on the rack cannot exceed rack capacity. The model will be solved using Evolutionary Algorithm since the model is a non-linear model with constraints.

MODEL DEVELOPMENT

The model in this paper uses some notations as below:

K_j = ordering cost for item *j*
$H_{j,t}$ = holding cost for item *j* at time *t*
$I_{j,t}$ = inventory for item *j* at time *t*
Q_j = ordering unit for each *j*
p_j = purchase cost for each *j*
s_j = sales price for each *j*
λ_j, β_j = Stock dependent parameters for item *j*
γ_j = holding rate parameter for item *j*
TP = total profit for individual order
TPT = total profit per unit time for individual order
TPJ = total profit for a joint replenishment order
TPTJ = total profit per unit time for a joint replenishment order
CAP = rack capacity

The model has some assumptions as follows:

1. Lead time is constant
2. Shortage is not allowed
3. Nonlinear holding is assumed to the follows holding cost model:
$$H_{j,t} = h(I_{j,t})^{\gamma_j}$$
4. The demand is a stock level function at time *t* as follows:
$$D_j(t) = D_j[I_t] = \begin{cases} \lambda_j (I_{j.t})^{\beta_j}, I_t > 0 \\ \lambda_j, \quad\quad I_t \leq 0 \end{cases}$$

The inventory level *I(t)* using the assumptions for each $t \in [0, T)$ is:

$$\frac{d}{dt} I(t) = -\lambda [I(t)]^{\beta} \quad (1)$$

With condition of $I_1(0) = Q$ and $I_2(T) = 0$, one has

$$I_{j,t} = (Q^{1-\beta_j} - (1-\beta_j)\lambda_j t)^{\frac{1}{1-\beta_j}}, \quad 0 \leq t \leq T$$

$$T_j = \frac{{Q_j}^{1-\beta_j}}{(1-\beta_j)\lambda_j} \text{ and} \quad (2)$$

$$Q_j = (T_j(1-\beta_j)\lambda_j)^{1/(1-\beta_j)} \quad (3)$$

The model consists of two costs which are total ordering cost and total inventory cost. Using the modification model from Yang [9], we get the ordering cost per cycle is *K* and the total inventory holding cost per cycle for each item is:

$$H_j = h_j \int_0^t (I_{j,t})^{\gamma_j} dt = h \int_0^t (Q_j^{1-\beta_j} - (1-\beta_j)\lambda_j t)^{\gamma_j/(1-\beta_j)} dt$$

$$H_j = \frac{h}{\lambda_j(\gamma_j + 1 - \beta_j)} Q_j^{\gamma_j + 1 - \beta_j} \quad (4)$$

The difference of eq. (4) compare to common inventory cost are stock dependent parameters (λ_j, β_j) and the holding rate parameter (γ_j) since stock level is depend on the stock dependent demand dan the holding cost depend on the stock level. The holding rate parameter (γ_j) only affect the total holding cost. The total inventory during the *t* period depends on demand parameters and the dynamic holding cost. The total purchase cost per cycle is equal to purchase cost per unit times ordering quantity (Q). Therefore, each item total purchase cost is given by:

$$P_j = p_j Q \tag{5}$$

The total sales revenue is equal to unit sales times ordering quantity and the total sales revenue for each item and each cycle can be modeled as follows:

$$S_j = s_j Q \tag{6}$$

Therefore, the total profit per cycle is equal to total revenue minus ordering cost, holding cost, and purchase cost:

$$TP = \sum_j s_j Q_j - \sum_j K_j - \sum_j \frac{h_j}{\lambda_j(\gamma_j+1-\beta_j)} Q_j^{\gamma_j+1-\beta_j} - \sum_j p_j Q_j \tag{7}$$

The total profit per period is the equal total profit per replenishment time (T) as follows:

$$TPT = \sum_j \frac{s_j Q_j}{T_j} - \sum_j \frac{K_j}{T_j} - \sum_j \frac{\frac{h_j}{\lambda_j(\gamma_j+1-\beta_j)} Q_j^{\gamma_j+1-\beta_j}}{T_j} - \frac{\sum_j p_j Q_j}{T_j} \tag{8}$$

Substitute T to Q using (3), one has:

$$TPT = \sum_j \frac{s_j Q_j}{\left(\frac{Q_j^{1-\beta_j}}{(1-\beta_j)\lambda_j}\right)} - \sum_j \frac{K_j}{\left(\frac{Q_j^{1-\beta_j}}{(1-\beta_j)\lambda_j}\right)} - \sum_j \frac{\frac{h_j}{\lambda_j(\gamma_j+1-\beta_j)} Q_j^{\gamma_j+1-\beta_j}}{\left(\frac{Q_j^{1-\beta_j}}{(1-\beta_j)\lambda_j}\right)} - \frac{\sum_j p_j Q_j}{\left(\frac{Q_j^{1-\beta_j}}{(1-\beta_j)\lambda_j}\right)} \tag{9}$$

With the constraint of the rack, capacity should be more than the total ordering quantity and it can be modeled as follows:

$$\sum_j Q_j \leq CAP \tag{10}$$

For multi-items inventory models, joint replenishment can be used as an alternative to getting the optimal solution. The model is rather different from equation (9) since there is only an indifferent period for all items. Instead of finding replenishment time for each item (T_j), in this model we only find one replenishment period (T). The total profit for the joint replenishment model per period can be derived by substituting equation (3) to equation (9), one has:

$$TPTJ = \sum_j \frac{s_j\left((T(1-\beta_j)\lambda_j)^{1/(1-\beta_j)}\right)}{T} - \frac{K}{T} - \sum_j \frac{\frac{h_j}{\lambda_j(\gamma_j+1-\beta_j)}\left((T(1-\beta_j)\lambda_j)^{1/(1-\beta_j)}\right)^{\gamma_j+1-\beta_j}}{T} - \frac{\sum_j p_j\left((T(1-\beta_j)\lambda_j)^{1/(1-\beta_j)}\right)}{T} \tag{11}$$

A NUMERICAL EXAMPLE

In this section, we illustrate the model above using data from Pando *et al.* [4] and some additional data for capacity and items number 2 and 3. Data for items 2 and 3 are generated using higher parameters than item 1 to show items that have higher demand and value than item 1. The detailed data for the numerical example is shown in Table 1. The rack capacity is 50.

TABLE 1. Data

Parameter	Item 1	Item 2	Item 3
λ	1,2	1,8	2,4
β	0.1	0.2	0.3
K	10	10	10
h	0.5	0.6	0.7
γ	0,95	0,9	0,88
s	62	70	85
p	50	55	60

The solutions are found using an evolutionary algorithm in excel since the model is a non-linear model with a constraint. The result for the individual and joint replenishment solution is shown in Table 2.

TABLE 2. The optimal order quantity

	Q_1^*	Q_2^*	Q_3^*
Individual	5,30	9,77	34,93
Joint replenishment	7,99	14,83	27,18

Table 2 shows both solutions try to maximize the capacity, however, there are different optimal quantities for each item. The revenue and cost for individual and joint replenishment are shown in Table 3.

TABLE 3. Revenue and cost per period

	Individual	**Joint Replenishment**
Revenue	363,86	361,11
Purchasing cost	271,65	270,69
Ordering cost	4,20	1,46
Inventory cost	5,75	6,19
Total Profit	82,26	82,75

Table 3 shows that the joint replenishment profit is higher than the individual order quantity. However, the difference is not significant in this case, the total profit of joint replenishment is only 0.6% higher than the individual item replenishment method. Since individual order quantity can optimize every item inventory, therefore inventory cost of individual order quantity is less than the joint replenishment order. To know deeper on the difference between individual replenishment order dan joint replenishment model, a sensitivity analysis with different value of rack capacity and ordering cost are derived. The rack capacity and ordering cost parameters are a change from -40% of initial value to 40% of initial value and set the other parameter same as the initial value. The result of the total profit for different values of capacity is shown in Fig. 1. The figure shows that for tight capacity, joint replenishment order profit is better than the individual order. However, the total profit of individual replenishment is better than joint replenishment for loose capacity.

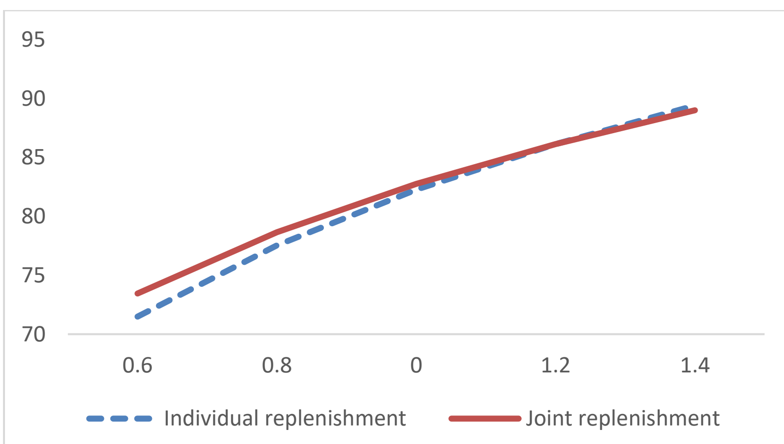

FIGURE 1. Capacity sensitivity analysis

The sensitivity analysis for the difference value of ordering cost is shown in Fig. 2. Figure 2 shows that the total profit for joint replenishment orders is less than individual orders for small ordering costs. The total profit difference of joint replenishment and individual replenishment is bigger when ordering costs increase.

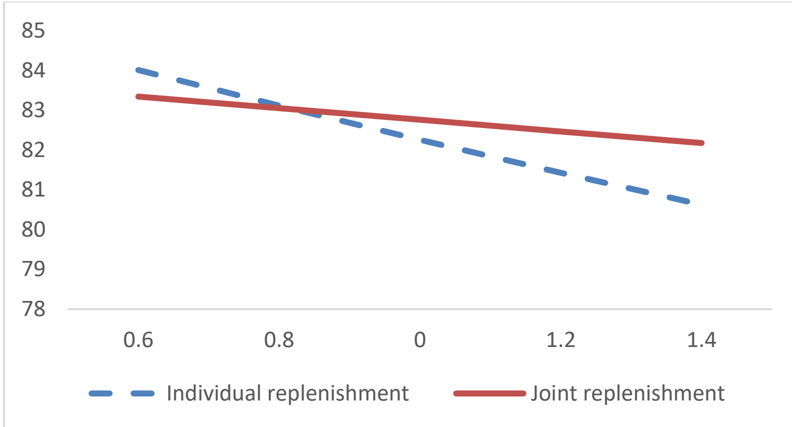

FIGURE 2 Total profit for different value of ordering cost

The sensitivity analysis shows that there is no dominant between individual order and joint replenishment order. Therefore, management should analyst carefully before they decide to choose a method. Joint replenishment order tends to be better than individual order when rack capacity is tight and ordering cost is high.

CONCLUSION

In this paper, multi-items inventory models with stock-depend demand and stock-dependent holding rates are developed. The first inventory model is an individual order and the second inventory model is a joint replenishment order. The models are nonlinear programming with constraints and an evolutionary algorithm method is used to solve the problem. A numerical example is shown to show how the models work and sensitivity analysis for different values of ordering cost and rack capacity is conducted to give some management insights. The sensitivity analysis shows that in this case, there is no dominant model between joint replenishment and individual replenishment. However joint replenishment tends to give better results in tight rack capacity and high ordering costs. The results give some interesting insight; therefore, the model can be improved by allowing shortage and lead times.

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