Predicting IBE Webinar Registrants Using Linearization of Some Sigmoidal Functions

Stefanus Vincent[1, a)], Indriati Bisono[1, b)], Hanijanto Soewandi[2, c)]

[1] *International Business Engineering, Petra Christian University, Surabaya, Indonesia*
[2] *MicroStrategy, Tysons Corner, VA 22182, USA*

[a)]stf.vnc@gmail.com
[b)]Corresponding author: mlindri@petra.ac.id
[c)] hsoewandi@microstrategy.com

Abstract. We present a novel approach to obtain parameters of some Sigmoidal functions such as: Stevens, Gompertz, Logistics, Weibull, Brody, von Bertalaffy, & Ontogenetic. This approach extends the approach relies on iterative process to find "maximum" R^2 and the traditional Ordinary Least Square (OLS) that can be implemented very easily. We demonstrate the approach in predicting IBE Webinar registrants. We also provide a new modification for the stopping rule based on Taylor approximation that can make the algorithm more robust as well as stop faster with little impact to the prediction.

Keywords: *Sigmoid Functions; Ordinary Least Square; Taylor approximation*

INTRODUCTION

This is the second part of our lesson learnt in managing a real event (IBE Webinar) professionally. To help our junior in marketing IBE Webinar, we are asked to provide daily estimate about how many people will register to a particular IBE Webinar at the maximum. This number is very important as our committee member will need to prepare all logistics for the event.

Apparently, predicting event attendance is already widely studied. We found out numerous papers such as: Zhang et.al. (2015), King (2017), Sahin & Ucar (2020), and Nguyen et.al. (2021) on predicting various events (NBA, NFL, Social Event, etc.). Unfortunately, they do not fit our situation. Our webinar event is not the same as NBA, NFL, or Soccer matches where the teams have their own fans. Our speakers may be well known (or comes from well-known companies), but they do not have fan base. The closest that we can find is the paper by Suher (2008) in which he tries to predict various tickets from concerts using Weibull.

While we simply have many unknowns in trying to predict our IBE Webinar, we realize that our situation is very similar to predicting Covid-19 (in which we had some experience – see our Data Science & Innovation web-page: http://dsi.ibe.petra.ac.id/covid19). We have built a closed-loop predictive monitoring BI system for one full year in which we get daily update and our parameter changes. Everyday there will be people registers to our IBE Webinar event, and if our committee is able to make it viral (either internal within our university community or better yet external), we will see an acceleration, and then a steady decline toward a plateau. Pretty much like predicting Covid-19.

When we research on Covid-19 prediction, we found out numerous articles using Bass, Gompertz, & Logistic diffusion models (pretty much like ours). Furthermore, to our surprise, we find out that even Michael Levitt, one of Nobel laureate from Stanford, also use Gompertz function (Levitt *et.al.*, 2020). In fact, his paper made us realize that we could do much simpler in predicting the maximum potential attendance of IBE Webinar, and make a closed-loop predictive monitoring BI system to assist our junior committee. Together with our experience using Software

Reliability Growth Modeling (SRGM) we also realize that using multiple Mathematical models actually can give us more confidence when it reaches a plateau (see Christian *et.al.* 2022).

The rest of our paper is organized as follows. In the next section, we review and derive several linearization of sigmoid functions and pointed out that all have the origin from Stevens' growth model (Stevens, 1951). We then outline a simple algorithm using two-step process to obtain parameters prediction for various Sigmoid models. We illustrate the calculation of our IBE webinar attendance and how we build our closed-loop, predictive-monitoring system.

LITERATURE REVIEW

Basic Deterministic Diffusion/Growth Modeling

Stevens's growth model is perhaps the oldest general form of deterministic diffusion (growth) modelling with three parameters. Mathematically, it can be written as follows:

$$N(t) = m - br^t, 0 < r < 1 \tag{1}$$

If we follow the approach in Levitt, *et.al.* (2020), we can pick $m > \max_t\{N(t)\}$, and rewrite the above equation as:

$$m - N(t) = br^t \tag{2}$$

Taking natural logarithm on both side of the equation will produce the following linear equation:

$$Y(t) \triangleq ln\big(m - N(t)\big) = ln(b) + t \times ln(r) \tag{3}$$

From Stevens growth model, we can generalize further and derive other deterministic diffusion (growth) models, and their linearization (as in next section). But, first, consider the following equation which is the generalized version of von Bertalanffy growth model:

$$N(t) = \alpha\big[1 - e^{-k(1-d)(t-t_0)}\big]^{\frac{1}{(1-d)}} \tag{4}$$

From Equation (4), Figure 1 shows how other diffusion (growth) models are related to the generalized von Bertalanffy growth model (and their transformations).

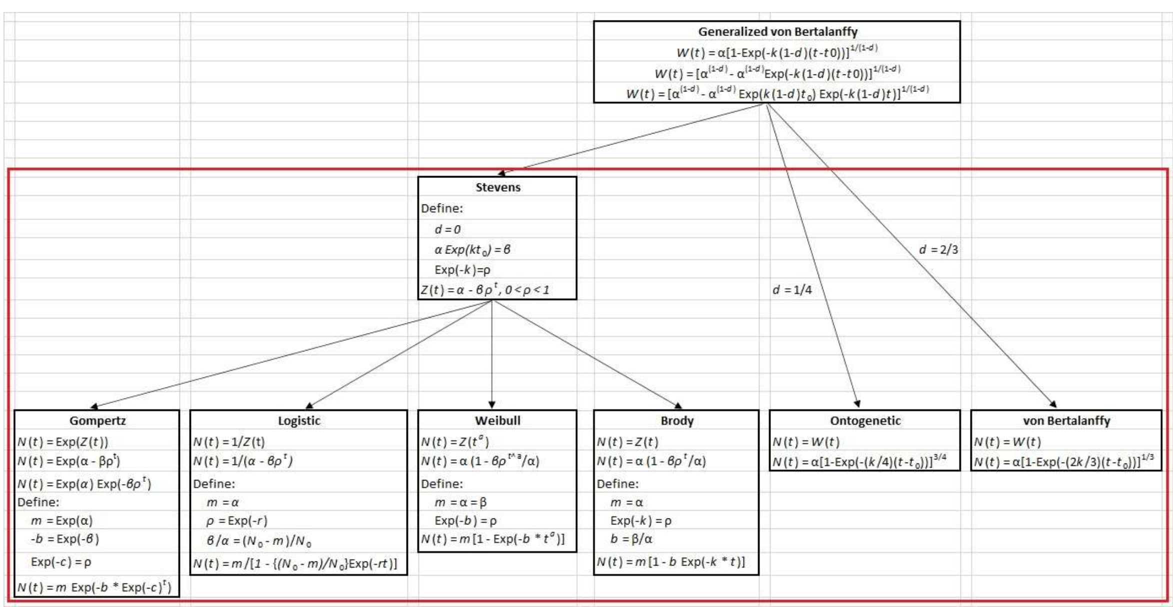

FIGURE 1. Relationship among Sigmoid Functions that we consider in this paper

RESEARCH METHOD

Data Collection

The data is collected from the daily registrations from the last IBE Webinar in 2021 (see Tamara *et.al.* 2022 for detail). We repeat the data here to make this paper self-contained. It should also be noted that we ran two email campaigns, on 11/15/2021 and on 11/20/2021. Hence, we mark the data with different color to indicate this fact.

Table 1. Daily data from the last IBE Webinar in 2021

Date	Day (t)	Registrants	1st Cumulative Registrants $N(t)$	2nd Cumulative Registrants $N(t)$
11/15/2021	1	110	110	
11/16/2021	2	129	239	
11/17/2021	3	54	293	
11/18/2021	4	44	337	
11/19/2021	5	16	353	
11/20/2021	6 | 1	24	377	24
11/21/2021	7 | 2	30	407	54
11/22/2021	8 | 3	36	443	90
11/23/2021	9 | 4	44	487	134
11/24/2021	10 | 5	33	520	167
11/25/2021	11 | 6	20	540	187
11/26/2021	12 | 7	18	558	205
11/27/2021	13 | 8	22	580	
Total			**580**	

Mathematical Model & Data Processing

To provide prediction on how many registrants we would be able to capture from our email campaigns, we will use several Sigmoid functions (Stevens, Gompertz, Logistic, Weibull, von Bertalanffy, Brody, & Ontogenetic Growth). Instead of using non-linear least square (NLS), we design a curve-fitting algorithm that uses Ordinary Least Square (OLS). First, we demonstrate that all of these Sigmoid functions can be made linear using simple Algebraic manipulation.

It is imperative to explain our rationale here as why we do not simply rely on NLS. Imagine that someday (like in day 5 in Table 1) we already know that there are 353 people registered for our IBE Webinar event, an NLS using data from day 1 to day 5 may provide a parameter m (asymptotic value of the numbers of people that will register) to be less than 353 (say m = 350). Obviously, this type of prediction does not settle well with many people. NLS has this characteristic. Let's first state our lemma here to provide a foundation to the algorithm.

Lemma 1:

If we know the asymptotic value m, then all these Sigmoid Functions (Gompertz, Logistic, Weibull, Brody, von Bertalanffy, and Ontogenetic Growth) can be made linear in the following forms:

Gompertz: $N(t) = me^{-be^{-ct}}$ to become: $Y(t) \triangleq ln\left(ln\left(\frac{m}{N(t)}\right)\right) = ln(b) - ct$ (5)

Logistic: $N(t) = \frac{m}{1+\left(\frac{m-N_0}{N_0}\right)exp(-rt)}$ to become: $Y(t) \triangleq ln\left(\frac{m-N(t)}{N(t)}\right) = ln\left(\frac{m-N_0}{N_0}\right) - rt$ (6)

Weibull: $N(t) = m\left(1-e^{-bt^a}\right)$ to become: $Y(t) \triangleq ln\left(-ln\left(1-\frac{N(t)}{m}\right)\right) = ln(b) + a\,ln(t)$ (7)

Brody: $N(t) = m(1-be^{-kt})$ to become: $Y(t) \triangleq ln\left(1-\frac{N(t)}{m}\right) = ln\,b - kt$ (8)

von Bertalanffy (Essington *et.al.*, 2001) with $d = \frac{2}{3}$: $N(t) = m\left(1-e^{-k(t-t_0)}\right)^3$ to become:

$$Y(t) \triangleq ln\left\{1-\left(\frac{N(t)}{m}\right)^{\frac{1}{3}}\right\} = -kt + kt_0 \quad (9)$$

Ontogenetic Growth (von Bertalanffy with $d = \frac{1}{4}$): $N(t) = m\left(1 - e^{-k(t-t_0)}\right)^{4/3}$ to become:

$$Y(t) \triangleq ln\left\{1-\left(\frac{N(t)}{m}\right)^{\frac{3}{4}}\right\} = -kt + kt_0 \quad (10)$$

Proof:

Straight forward Algebraic manipulation similar to Stevens' model. Hence, they are omitted. □

Notice that the value of m in all equations (3) & (5) – (10) requires $m > \max_t\{N(t)\}$, otherwise the natural logarithm for the transformation is not defined. This will guarantee that our asymptotic value, m will be larger than the maximum known registrants to date. Hence, it will provide a reasonable parameter prediction for certain types of problems like predicting IBE Webinar registrations (or Covid-19 pandemic, or Software Reliability Growth Model).

Furthermore, carefully examining equations (3) & (5) – (9), ones will notice that the linearization suggests that we can iterate over m value that gives the best correlation coefficient, ρ (or coefficient of determination, R^2) between $Y(t)$ and *t*. Unfortunately, we could not prove that correlation coefficient (or coefficient of determination) is concave over m. In fact, we can create a counter example that we continue to increase m and getting better ρ or R^2, *e.g.*, readers can easily verify themselves with several data points that are linear or convex – in these scenarios, the value of R^2 will continue to increase as we increase m – see Table 2 in which $Y(t) \triangleq ln\big(m - N(t)\big)$. Therefore, Levitt *et.al.* (2020) suggested to stop when correlation is satisfactory high without giving too much detail. We think this stopping condition can be further improved.

Table 2. Linear data points in which $R^2 \to 1$ (monotonically) as $m \to \infty$ for Stevens' model

R^2	m	Day (t)	1	2	3	4	5	6	7	CV	mCV
		$N(t)$	8	16	24	32	40	48	56		
0.880642	60	$Y(t)$	3.951	3.784	3.584	3.332	2.996	2.485	1.386	29.10%	21.85%
0.955847	70	$Y(t)$	4.127	3.989	3.829	3.638	3.401	3.091	2.639	14.95%	12.42%
0.975443	80	$Y(t)$	4.277	4.159	4.025	3.871	3.689	3.466	3.178	10.29%	8.95%
0.984096	90	$Y(t)$	4.407	4.304	4.190	4.060	3.912	3.738	3.526	7.84%	7.00%
0.988785	100	$Y(t)$	4.522	4.431	4.331	4.220	4.094	3.951	3.784	6.31%	5.74%
0.991640	110	$Y(t)$	4.625	4.543	4.454	4.357	4.248	4.127	3.989	5.27%	4.86%
0.993515	120	$Y(t)$	4.718	4.644	4.564	4.477	4.382	4.277	4.159	4.51%	4.20%
0.998279	200	$Y(t)$	5.257	5.215	5.170	5.124	5.075	5.024	4.970	2.02%	1.95%
0.999329	300	$Y(t)$	5.677	5.649	5.620	5.591	5.561	5.529	5.497	1.16%	1.13%
0.999645	400	$Y(t)$	5.971	5.951	5.930	5.908	5.886	5.864	5.841	0.80%	0.78%
0.999781	500	$Y(t)$	6.198	6.182	6.165	6.148	6.131	6.114	6.096	0.60%	0.59%
0.999907	750	$Y(t)$	6.609	6.599	6.588	6.576	6.565	6.554	6.542	0.37%	0.36%
0.999949	1000	$Y(t)$	6.900	6.892	6.883	6.875	6.867	6.859	6.850	0.26%	0.26%
0.999968	1250	$Y(t)$	7.124	7.118	7.112	7.105	7.098	7.092	7.085	0.20%	0.20%

0.999978	1500	$Y(t)$	7.308	7.302	7.297	7.292	7.286	7.281	7.275	0.16%	0.16%
0.999984	1750	$Y(t)$	7.463	7.458	7.454	7.449	7.444	7.440	7.435	0.14%	0.13%
0.999988	2000	$Y(t)$	7.597	7.593	7.589	7.585	7.581	7.577	7.573	0.12%	0.12%
0.999990	2250	$Y(t)$	7.715	7.712	7.708	7.704	7.701	7.697	7.693	0.10%	0.10%
0.999992	2500	$Y(t)$	7.821	7.818	7.814	7.811	7.808	7.805	7.801	0.09%	0.09%

Second, it is important to point out that all Sigmoid functions in Lemma 1 have an asymptotic value *m*. If we let $m \to \infty$, then practically the transformed variable $Y(t)$ will be a horizontal line (regardless how the data points actually look like). Hence, there is no variation.

In experimental labs statistics, we know that we can accept multiple measures as the same when its variation is low. The most common measure for this is coefficient of variation (CV). In fact, it is generally accepted that CV less than 5% should be a good enough measure. If needed, we can further tighten this by proposing a CV less than 2.5% (or any acceptable value). Unfortunately, the value of $Y(t)$ can be negative in some transformations. To make it dimensionless, we choose to modify coefficient of variation by replacing the average of $Y(t)$ with $\ln(m)$ to make it dimensionless. Hence, we can have the following Lemma 2 and 3.

Lemma 2:

The first term of Taylor series for modified coefficient of variation, $mCV(m) = \frac{StDev(Y(t))}{\ln(m)}$ is monotonically decreasing with respect to *m* for all transformations in (3) & (5) – (10).

Proof:

We will use the Taylor approximation of variance of a function that is twice differentiable to prove our lemma (see: [09]) for Stevens model. We leave the rest as an exercise to our readers. The first two terms approximation for variance of a function of random variable *x* is given by:

$$VAR\big(f(x)\big) \approx \left(\frac{df}{dx}(\mu)\right)^2 \sigma^2 - \frac{1}{4}\left(\frac{d^2}{x^2}(\mu)\right)^2 \sigma^4 \quad (11)$$

where: $\mu = E[x]$ = the expected value of *x*. Notice that for a given data points $\{x_i\}$, μ and $VAR(x) = \sigma^2$ are just two constant values that are positive. In our case: $x = N(t)$ and $f(x) = Y(N(t))$.

For Stevens, $Y(t) \triangleq ln\big(m - N(t)\big)$, we have the first term of Taylor series for $VAR\big(Y(t)\big)$ as:

$$VAR\big(Y(t)\big) \approx \left(-\frac{1}{(m-\mu)}\right)^2 \sigma^2 = \frac{\sigma^2}{(m-\mu)^2} \quad (12)$$

Substituting Equation (11) above to our definition of $mCV(m)$ we have:

$$mCV(m) \approx \frac{StDev\big(Y(t)\big)}{\ln(m)} = \frac{\sigma}{(m-\mu)\ln(m)} \quad (13)$$

And taking the derivative with respect to *m* yields:

$$\frac{dmCV(m)}{dm} = -\frac{\sigma\big(m\ln(m) + (m-\mu)\big)}{m\ln^2(m)(m-\mu)^2} < 0 \quad (14)$$

since $(m-\mu) > 0$ (recall we choose $m > \max_t\{N(t)\}$). Hence, $mCV(m)$ is monotonically decreasing function with respect to *m*. □

Lemma 3:

If $m > \mu + \sigma$, then the first two-terms of Taylor series for $mCV(m)$ is monotonically decreasing.

Proof:

For Stevens, $Y(t) \triangleq ln\big(m - N(t)\big)$, we have:

$$VAR\big(Y(t)\big) \approx \left(-\frac{1}{(m-\mu)}\right)^2 \sigma^2 - \frac{1}{4}\left(-\frac{1}{(m-\mu)^2}\right)^2 \sigma^4 = \frac{\sigma^2}{(m-\mu)^2} - \frac{\sigma^4}{4(m-\mu)^4} \quad (15)$$

Substituting Equation (14) above to our definition of $mCV(m)$ we have:

$$mCV(m) \approx \frac{StDev\big(Y(t)\big)}{\ln(m)} = \frac{\sigma}{(m-\mu)\ln(m)} - \frac{\sigma^2}{2(m-\mu)^2\ln(m)} \quad (16)$$

Therefore,

$$\frac{dmCV(m)}{dm} = -\frac{\sigma(m\ln(m)+(m-\mu))}{m\ln^2(m)(m-\mu)^2} + \frac{\sigma^2(2m\ln(m)+(m-\mu))}{2m\ln^2(m)(m-\mu)^3} = \frac{\sigma m\ln(m)(2\sigma-2(m-\mu))+\sigma(m-\mu)(\sigma-(m-\mu))}{2m\ln^2(m)(m-\mu)^3} = \frac{2\sigma m\ln(m)((\mu+\sigma)-m)+\sigma(m-\mu)((\mu+\sigma)-m)}{2m\ln^2(m)(m-\mu)^3} < 0 \quad (17)$$

Clearly Equation (17) is less than 0 if $m > \mu + \sigma$. □

Therefore, we can use this simple but practical rule as our stopping criteria. In Table 2, we illustrate the values of both *CV* and *mCV*. It should be clear that if we are given a series of data which are linear {8, 16, 24, 32, 40, 56}, then using $mCV(Y(t)) < \alpha = 5\%$, we can say that practically there is no point to go beyond *m* = 110 for Stevens' transformation. Hence, we can stop increasing the value of *m* until more evidence from new data comes in. This 1st step effort to find the best *m* can be illustrated in Figure 2.

One can think our approach as follows: we try to increase the value of asymptotic *m* as much as possible so that the line becomes as linear as possible (maximize R^2), but we do not want to displace the line too much. The new proposal using *mCV* and α is an attempt to help the effort so that we do not push the line upward too far.

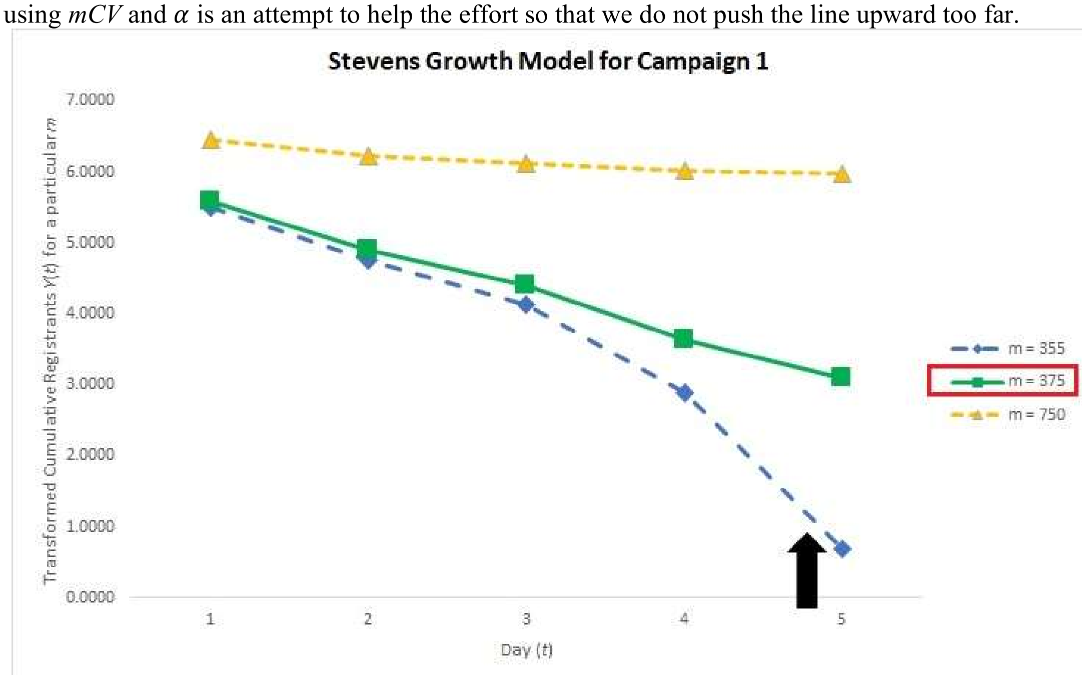

Figure 2. Impact of increasing *m* values on *Y*(*t*) and *t*

Once we decide on the asymptotic value *m*, we can simply run an ordinary least square (OLS) since all Sigmoid functions have been transformed into a linear equation using (3) & (5) – (10). Hence, our algorithm can be written as a two-stage process as follows:

Curve Fitting Algorithm for Linearized Sigmoid Functions

Stage 1:

- Step 1: Initialize $R_0^2 = 0$, $m = \max_t\{N(t)\} + 1$
- Step 2: Transform *N*(*t*) to *Y*(*t*) following equations (3) – (8)
- Step 3: Calculate R_i^2 between *Y*(*t*) and *t*.
- Step 4: If $R_0^2 > R_i^2$ or $mCV(Y(t)) < \alpha$, then STOP (go to Stage 2 – we found *m*)
- Else, set $R_0^2 = R_i^2$ and $m = m + \Delta m$, go to Step 2.

Stage 2: Run OLS according to equations (3) – (9) using *m* from Stage 1.

Please note that in the above algorithm, we can replace $mCV(Y(t))$ with its first order Taylor series approximation to guarantee the stopping rule. Now, we are ready to apply the above algorithm to the IBE Webinar registrants.

FINDINGS and DISCUSSION

We apply the above algorithm to the first 5 days of IBE Webinar data above with $m_0 = 355$ and $\Delta m = 5$. The step-by-step algorithms for Stage 1 above are illustrated for Stevens and Gompertz models in the Tables 3a and Table

3b. The optimal asymptotic value, *m*, for all Sigmoid models & other parameters are summarized in table 4. Notice how the *CV* and *mCV* behaves for Gompertz (CV is negative because the mean is negative).

Table 3a. Stage 1 calculation for Stevens model for Campaign 1

5	0.9969	***m***	**Day (*t*)**	1	2	3	4	5	***CV***	***mCV***
Index	**R^2**		***N*(*t*)**	110	239	293	337	353		
0	0.9295	355	*Y*(*t*)	5.5013	4.7536	4.1271	2.8904	0.6931	0.5239	0.3206
0	0.9797	360	*Y*(*t*)	5.5215	4.7958	4.2047	3.1355	1.9459	0.3590	0.2391
0	0.9920	365	*Y*(*t*)	5.5413	4.8363	4.2767	3.3322	2.4849	0.2953	0.2049
0	0.9960	370	*Y*(*t*)	5.5607	4.8752	4.3438	3.4965	2.8332	0.2564	0.1831
1	0.9969	375	*Y*(*t*)	5.5797	4.9127	4.4067	3.6376	3.0910	0.2289	0.1671
0	0.9963	380	*Y*(*t*)	5.5984	4.9488	4.4659	3.7612	3.2958	0.2079	0.1545
0	0.9950	385	*Y*(*t*)	5.6168	4.9836	4.5218	3.8712	3.4657	0.1911	0.1442
0	0.9934	390	*Y*(*t*)	5.6348	5.0173	4.5747	3.9703	3.6109	0.1772	0.1355
0	0.9915	395	*Y*(*t*)	5.6525	5.0499	4.6250	4.0604	3.7377	0.1655	0.1280
0	0.9895	400	*Y*(*t*)	5.6699	5.0814	4.6728	4.1431	3.8501	0.1554	0.1214

Table 3b. Stage 1 calculation for Gompertz model for Campaign 1

3	0.9964	***m***	**Day (*t*)**	1	2	3	4	5	***CV***	***mCV***
Index	**R^2**		***N*(*t*)**	110	239	293	337	353		
0	0.9556	355	*Y*(*t*)	0.1584	-0.9272	-1.6505	-2.9558	-5.1761	-0.9732	0.3498
0	0.9920	360	*Y*(*t*)	0.1703	-0.8925	-1.5802	-2.7178	-3.9304	-0.8892	0.2704
1	0.9964	365	*Y*(*t*)	0.1818	-0.8594	-1.5154	-2.5281	-3.3983	-0.8612	0.2370
0	0.9948	370	*Y*(*t*)	0.1931	-0.8277	-1.4553	-2.3706	-3.0569	-0.8480	0.2156
0	0.9914	375	*Y*(*t*)	0.2041	-0.7975	-1.3994	-2.2364	-2.8058	-0.8418	0.1998
0	0.9872	380	*Y*(*t*)	0.2149	-0.7685	-1.3471	-2.1195	-2.6077	-0.8399	0.1874
0	0.9829	385	*Y*(*t*)	0.2254	-0.7407	-1.2980	-2.0162	-2.4444	-0.8408	0.1772
0	0.9786	390	*Y*(*t*)	0.2356	-0.7140	-1.2519	-1.9237	-2.3058	-0.8438	0.1686
0	0.9744	395	*Y*(*t*)	0.2456	-0.6883	-1.2083	-1.8401	-2.1855	-0.8485	0.1611
0	0.9704	400	*Y*(*t*)	0.2554	-0.6636	-1.1670	-1.7639	-2.0795	-0.8546	0.1546

Table 4. Final result from Stage 1 & Stage 2 for all Models for Campaign 1

Model	**Campaign 1 Parameters**			***mCV***
	m	***b*, t₀, *N*₀**	***c*, *r*, *a*, *k***	
Stevens	375	493.3799	0.5351	16.71%
Gompertz	365	2.7867	0.8829	23.70%
Logistic	360	48.5757	1.1487	30.96%
Weibull	360	0.3703	1.4511	17.11%
Brody	375	1.3157	0.6252	16.71%
von Bertalanffy	370	-0.5052	0.7611	20.39%
Ontogenetic	380	0.6165	0.5449	14.53%
Average	369			
StDev	8			

Notice that from the value of mCV in table 6 above (all values > 15%), we can see that for all models, the algorithm is terminated because R^2 is reaching the maximum value.

To better understand the stopping rule that we propose, we illustrate the calculation for Stevens' model as in Table 5 below for 7 days in doing campaign 2. Notice that for Stevens model with $\alpha = 5\%$ (or $\alpha = 10\%$) our algorithm produces $m = 310$ (or $m = 220$), while the maximum R^2 is obtained when $m = 325$. Obviously, other parameters (b & r) of Stevens model will also be different for different values of m.

The complete results of all Sigmoid models for Campaign 2 are given in Table 6 below (we listed some models if the algorithm stops at different values of α) as well as its prediction for day 8 (the day that the IBE Webinar event takes place). If we take simple average from all predictions (that are not marked – these are the result if we implement $\alpha = 5\%$), we have the value of 578 registrants with standard deviation of 12 registrants. Notice how close the predictions for day 8 with the actual data that we have in table 2 above (recall that we had 580 registrants).

Table 5. Stage 1 calculation for Stevens' model for Campaign 2 with $\alpha = 5\%$, 10%, max R^2

9	0.9935		**Day (*t*)**	1	2	3	4	5	6	7	
Index	**R^2**	***m***	***N*(*t*)**	24	54	90	134	167	187	205	***mCV***
0	0.8940	210	*Y*(*t*)	5.2257	5.0499	4.7875	4.3307	3.7612	3.1355	1.6094	0.1107
0	0.9349	215	*Y*(*t*)	5.2523	5.0814	4.8283	4.3944	3.8712	3.3322	2.3026	0.1039
0	0.9545	220	*Y*(*t*)	5.2781	5.1120	4.8675	4.4543	3.9703	3.4965	2.7081	0.0980
		…									
0	0.9933	305	*Y*(*t*)	5.6384	5.5255	5.3706	5.1417	4.9273	4.7707	4.6052	0.0504
0	0.9934	310	*Y*(*t*)	5.6560	5.5452	5.3936	5.1705	4.9628	4.8122	4.6540	0.0490
0	0.9935	315	*Y*(*t*)	5.6733	5.5645	5.4161	5.1985	4.9972	4.8520	4.7005	0.0476
0	0.9935	320	*Y*(*t*)	5.6904	5.5835	5.4381	5.2257	5.0304	4.8903	4.7449	0.0464
1	0.9935	325	*Y*(*t*)	5.7071	5.6021	5.4596	5.2523	5.0626	4.9273	4.7875	0.0452
0	0.9935	330	*Y*(*t*)	5.7236	5.6204	5.4806	5.2781	5.0938	4.9628	4.8283	0.0440

Table 6. Several Sigmoid functions parameters and prediction for day 8

Model	Campaign 2 Parameters			mCV	Prediction for day 8
	m	***b*, t0, *N0***	***c*, *r*, *a*, *k***		
Stevens with α = 10%	220	387.8998	0.6552	9.80%	560
Stevens with α = 5%	310	354.6102	0.8394	4.90%	576
Stevens with max R2	325	365.2464	0.8514	4.52%	578
Gompertz	235	3.7518	0.4716	13.80%	569
Logistic	215	0.1248	0.8158	24.28%	568
Weibull	280	0.0858	1.4235	16.35%	580
Brody with α = 10%	210	1.7632	0.4228	9.80%	560
Brody with α = 5%	310	1.1439	0.1751	4.90%	576
Brody with max R2	325	1.1238	0.1609	4.52%	578
von Bertalanffy	250	-0.5801	0.3633	10.56%	572
Ontogenetic with α = 10%	230	0.8954	0.3853	9.92%	572
Ontogenetic with max R2	285	0.4374	0.2337	6.68%	601
Average all Models with α = 5% or max R2	270				578
StDev all Models with α = 5% or max R2	37				12

From Table 6, we can see that our asymptotic parameter (*m*) for the 2nd campaign has higher variation (as measured by standard deviation) across models (37 registrants) compare to variation across models for the 1st campaign (see Table 4 – the standard deviation is 8 registrants). This means that our data tells us that there are more uncertainties in the 2nd campaign since data may not reach to a plateau yet, but we have to end the registration since our IBE Webinar event happens on day 8 of the 2nd campaign.

CONCLUSION, LIMITATION, & FURTHER RESEARCH

It is important to understand that the application of Sigmoid functions is enormous, *e.g.*, software growth reliability model, predicting product life cycle, tumor growth, fishery, *etc*. We propose a novel yet simple approach to get parameters of several Sigmoid functions in two stages. We enhance the stopping criteria so that we don't have to run into cases that R^2 increasing indefinitely. We also illustrate the proposal with real life experience when we help to manage our IBE Webinar registration successfully.

While we are able to prove some conditions, we have not been able to demonstrate how our stopping criteria (*mCV*) is related to R^2. We still treat them as two unrelated variables in our algorithm. This is worth investigating further.

In this particular case, we know exactly when we have the 2nd campaign. However, in real life, we don't know when a second (or n^{th}) wave started by looking at data, *e.g.*, Covid-19 pandemic prediction for 2nd wave, imperfect debugging in software reliability growth model, *etc.* are examples of this problem. We believe this area is worth studying further.

REFERENCES

X. Zhang, J. Zhao and G. Cao (2015), "Who Will Attend? Predicting Event Attendance in Event-Based Social Network," 2015 16th IEEE International Conference on Mobile Data Management, pp. 74 – 83, doi: 10.1109/MDM.2015.23.

King, B.E. (2017), "Predicting National Basketball Association Game Attendance Using Random Forests," Journal of Computer Science and Information Technology, Vol. 5 No. 1, pp. 1 – 14.

Şahin, M., & Uçar, M. (2021), "Prediction of Sports Attendance: A Comparative Analysis," Proceedings of the Institution of Mechanical Engineers, Part P: Journal of Sports Engineering and Technology, https://doi.org/10.1177/1754337120983135

Jeremy K. Nguyen, Adam Karg, Abbas Valadkhani & Heath McDonald (2021), “Predicting Individual Event Attendance with Machine Learning: a ‘Step-Forward’ Approach,” Applied Economics, DOI: 10.1080/00036846.2021.2003747

Suher, J., 2008. Forecasting Event Ticket Sales.

West, G. B., Brown, J. H., & Enquist, B. J. (2001), “A general model for ontogenetic growth,” *Nature*, 413(6856), pp. 628 – 631. https://doi.org/10.1038/35098076.

Levitt, M., Scaiewicz, A., & Zonta, F. (2020), “Predicting the Trajectory of Any COVID19 Epidemic From the Best Straight Line” *medRxiv : The Preprint Server for Health Sciences*, 2020.06.26.20140814. https://doi.org/10.1101/2020.06.26.20140814.

Shiffler, R.E. and Harsha, P.D. (1980), “Upper and Lower Bounds for the Sample Standard Deviation,” *Teaching Statistics*, Vol 2, pp. 84 – 86, https://doi.org/10.1111/j.1467-9639.1980.tb00398.x

Alex Christian, Christopher Wibowo, Indriati Bisono, Hanijanto Soewandi (2022), “When to Stop Testing – a Proposal on How to Use SRGM to Build Confidence in Dashboard Testing at PT AGIT”, to appear in *International Conference on Industrial and Systems Engineering, Technology, Innovation, and Management 2022*.

Tamara Regina, Feliciya Yanson, Nova Sepadyati, and Karina Agustin (2022), “Marketing an IBE Webinar Part 1: Lesson Learnt from a Real Experiment”, to appear in *International Conference on Industrial and Systems Engineering, Technology, Innovation, and Management 2022*.

Stevens, S.S., 1951. Mathematics, measurement, and psychophysics.

Timothy E Essington, James F Kitchell, and Carl J Walters (2001), "The von Bertalanffy growth function, bioenergetics, and the consumption rates of fish," *Canadian Journal of Fisheries and Aquatic Sciences*, Vol. 58(11), pp. 2129 – 2138. https://doi.org/10.1139/f01-151