# Continuous-dynamic modeling of LPG circulation: A preliminary study in a simple system of one filling stationone agent-one store

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**Abstract.** Certain important commodities circulate in the community in certain packaging containers where the empty containers must be returned to the upstream. The existence of these containers is actually beneficial in evaluating the adequacy of the availability of these commodities in the community. The analysis generally uses mathematical methods or discrete simulations. In this paper, continuous modeling is used to study the cyclical movement of LPG tanks from the filling station, to the agent, to the store, to the consumer, back to the store, back to the agent, and back to the filling station. The modeling uses a system dynamics language which even though it assumes the entity is a fluid (continuous material) can be used for the discussion of discrete systems. In fact, the modeling turns out to be simpler to do and has the potential to be extended to more complex systems.

### **INTRODUCTION**

The availability of commodities in the community will always be important for ensuring the economic activity of the community. If the availability is lacking, then these activities will undoubtedly be disrupted, as seen in rising prices which will trigger economic instability. One of the important commodities that can be mentioned here is LPG for household use. This LPG is distributed in tank containers of 3 kg (green) and 12 kg (blue). The filling of gas into the tank is carried out by a filling station (SPBE). The LPG-filled tanks are then distributed to the agents. Furthermore, from gas agents it is distributed to stores for retail sale to household consumers.

The movement of distribution of commodities stored in containers where after the gas is used up the empty container is returned to the earliest echelon of the supply chain can be modeled by focusing on the circulation of the container, in this case, the tanks. The tanks will circulate around (cyclically). Entity circulation like this also occurs in automated production lines that use pallets where workpieces are placed while they are in the system [1]. The total number of pallets will be constant because they will not leave the system. The same thing happens to systems that use kanban [2]. The existence of pallets or kanbans in a production system will limit the number of work-in-processes in the system [3][4].

Various analytical methods have been developed, including mathematical methods and simulation methods. Mathematical methods are generally in the form of mathematical programming such as integer and linear programming [3][5], dynamic programming [4][6] and queueing-based probabilistic modeling [5][7]. Simulation methods are generally based on discrete-event simulation models. There is also a system dynamics method which is actually a mathematical model in the form of a system of differential equations, where it is possible to insert a random simulation into it.

This paper discusses a simple case regarding the circulation of 3 kg LPG cylinders. Here the system is assumed to be composed of only one SPBE, one agent, one shop. The circulation will be driven by retail sales of LPG in store. The time between the arrival of the consumer and the number of tanks purchased by the consumer to the store are random variables. The entity being observed is the tank which is the container for the LPG, which will continue to circulate in a circular manner: from SPBE, to agents, to shops, to consumers (households), back to SPBE. The contribution of this paper is in the basic concept of the model regarding the cyclical circulation of the tanks, if a continuous-dynamic approach is used which is expressed in the system dynamics language using Vensim® PLE [6][8]. The use of a system dynamics language that is simpler than discrete-event simulations will enrich alternative methods of assessing the performance of supply chain systems.

## Description of the system under study

The system studied in this paper is a simplification of a 3 kg LPG supply system to the community. In Indonesia, LPG is distributed in small green tanks. Of course, in reality one SPBE will serve several agents, then each agent will serve several stores, and each store will serve several households. However, this paper discusses a simple system, namely one SPBE only serves one agent, where the agent also only serves one shop, and also the shop only serves one household. Fig.1. shows a sketch of the intended simple supply chain system.



FIGURE 1. Diagram of the cyclic LPG supply chain system studied

In general, every purchase by household consumers is always accompanied by submitting the same number of empty tanks. That way, there will be a gradual return of empty tanks, from consumers to store, from store to agent, and from agent to SPBE. Lotsize of empty tank returns in each stage will be different, where the more upstream the bigger.



FIGURE 2.. Conceptual model of LPG tank cyclic circulation system

## Modeling

The real system above contains discrete entities in the form of LPG tanks (both filled and empty). The LPG tank will circulate around (cyclically). The continuous-dynamic approach is carried out by considering the entity as a fluid (i.e. a continuous material). It may seem strange at first glance but this approach has been applied to discrete case modeling as can be seen in [9][10][11]. More clearly, [12] apply the basic nature of system dynamics to modeling multi-channel subsystems with different lots size by describing the production rate as a flow variable and the amount of stock in the warehouse as a level variable. With the same concept, Fig.2. shows a conceptual model based on a continuous approach to the cases discussed in this paper.

Figure 3 displays the equivalent of the conceptual model in Fig.2. in the form of a system dynamics model. Basically, it can be stated that there are three important mechanisms described in Fig.3. First, is the gas filling process. The filling process is carried out using the following mechanisms: (target stock R, reproduction point r), where R is the target stock and r is the reproduction point (Fig.4.), which adopts the control mechanism of the thermostat system. When the number of filled tanks in stock is equal to or below r, the filling process must begin, and stop when the number of filled tanks has reached R. Some important equations in Fig.4. are as follows:

```
"process #1-filling the cylinder with gas"= (1)
    IF THEN ELSE("stock 0 (empty cylinders at SPBE" < 0:OR: "stock 0
    (empty cylinders at SPBE" = 0, 0, IF THEN ELSE(process 1 start < 1,
    0, daily filling capacity ))/daily filling time
Units: Tanks/Hour</pre>
```



-The total number of cylinders equal to the sum of stock on all level variables

-Purchases to the agent are assumed to always accompanied by the delivery of empty cylinders

FIGURE 3. Stock and flow diagram of the LPG distribution system in a simplified system: One SPBE-One Agent-One Store





```
process 1 start=
                                                                                                                                  (2)
        IF THEN ELSE (proses started :AND:"stock 1 (cylinders filled with
        gas at SPBE" <= target stock R ,1, IF THEN ELSE("stock 1
         (cylinders filled with gas at SPBE" <= reproduction point r,1,0))
Units: Dmnl
proses started=
                                                                                                                                  (3)
        DELAY FIXED (process 1 start,0,0)
Units: Dmnl
                                     order quantity
Q agent
                                                                                                    order quantity
Q store
        stock 1
                                                    stock 2
                                                                                                                           stock 3
      (cylinders filled
                                                    (filled
                                                                                                                           (filled
       with gas at
                                                                                                 X
                                                   cylinders a
                                                                                                                          cylinders at
        SPBE)
                             purchased arrival
                                                                                            purchased arrival
                                                   the agent)
                                                                                                                          the Store)
                                at Agent
                                                                                               at Stor
                                                                reorder point
                                                                                   <Time>
  reorder point
                     <Time>
                                                                   R store
    R agent
                                                                               order submission
                 order submission
                                                                                                                       average of
                                                                                   to agent
                    to SPBE
                                                     average of
delivery time
from SPBE to
                                                                                                                     delivery time from
                                                                                                                      agent to store
                                                         agent
                                                                                         accumulated delivery time
                           accumulated
delivery time
from SPBE to
                                                                                                                       X
       delivery time from
                                                                                         from agent to store
                                         delivery
                                               time from
                                                                      delivery time from
                                                                                                     delivery time from
                              agent
        SPBE to agent start
                                        SPBE to agent stop
                                                                      agent to store start
                                                                                                     agent to store stop
    <TIME STEP>
                                                                  <TIME STEP>
```

FIGURE 5. Stock and flow diagram for ordering and shipping LPG filled tanks

The second process is ordering and shipping LPG by agents to SPBE, or by store to agent (Fig.5). The process involves batch procurement of entities which requires leadtime. The agent itself control the stock in their warehouses with the following mechanisms: (reorder point RAgent, order quantity QAgent). Once the agent decides to order LPG (to SPBE), the SPBE will immediately deliver the LPG. Delivery requires leadtime before the order finally reaches the agent. With the same mechanism, the store will submit an order to the agent with the following mechanism: (reorder point RStore, order quantity QStore). Some of the important equations in Fig.5 are as follows:

```
purchased arrival at Agent=
                                                                        (5)
    IF THEN ELSE ("stock 1 (cylinders filled with gas at SPBE" < order
    quantity Q agent, 0, order quantity Q agent*order submission to
    SPBE*IF THEN ELSE(delivery time from SPBE to agent stop <= 0, 0,
    PULSE(Time, 1)))
Units: Tanks/Hour
delivery time from SPBE to agent start=
                                                                        (6)
    IF THEN ELSE (order submission to SPBE = 1, TIME STEP, 0)
Units: 1/Hours
delivery time from SPBE to agent stop=
                                                                        (7)
    IF THEN ELSE(accumulated delivery time from SPBE to agent <
    average of delivery time from SPBE to agent, 0,
                                                         accumulated
    delivery time from SPBE to agent)
Units: 1/Hours
```

The third process is the return of the tanks to their upstream echelons (Fig.6.). The return of the tanks in real terms, as shown in Fig.2, occurred in stages. Consumers (households) buy in retail to the store by paying the amount of the purchase, while delivering the empty tank to the store. If the empty tanks have accumulated up to a certain amount, the store then sends them to the agent using a small fleet (shown by truck capacity from store to agent) to the empty tank warehouse at the agent's location.

At the agent's place, after a certain amount has been collected (which is indicated by the Truck capacity from agent to SPBE), the empty tanks are sent to the SPBE. This has been seen in the stock and flow diagram (Fig.3), the cut is shown in Fig.6. Here, it is assumed that the truck is always available when an empty tank delivery has to be made, so that there is no delay.

```
"process #2 (empty cylinder collection at Store)"=
                                                                        (8)
    IF THEN ELSE ("stock 3 (filled cylinders at the Store) "<"retail-
    demand size", 0, min("retail-demand size","stock 3 (filled
    cylinders at the Store)") *PULSE TRAIN(1, 1, "inter-arrival demand
    to store", FINAL TIME))
Units: Tanks/Hours
"process #3 (delivery of empty cylinder from the Store to Agent)"=
                                                                        (9)
    IF THEN ELSE ("stock 4 (empty cylinders at the store) "<Truck
    capacity from store to agent, 0, IF THEN ELSE (returning time of
    empty cylinders to agent stop <=0, 0, Truck capacity from store
    to agent))
Units: Tanks/Hours
"process #4 (delivery of empty cylinder from the agent to SPBE)"=
                                                                       (10)
    IF THEN ELSE ("stock 5 (empty cylinders at the agent)"<Truck
    capacity from agent to SPBE, 0, IF THEN ELSE (returning time of
    empty cylinders to SPBE stop <=0, 0, Truck capacity from agent
    to SPBE))
Units: Tanks/Hour
```

It can be seen that the time required for the delivery of the filled tank from the SPBE to the agent, from the agent to the store, and the return of the empty cylinder from the store to the agent, from the agent to the SPBE are accommodated in the model through the variable level accumulated delivery time. However, the existence of the entity during the trip has not been accommodated. Then the tanks are only visible on the level variables: stock 0, stock 1, stock 2, stock 3, stock 4, stock 5



FIGURE 6.. Stock and flow diagrams for the process of returning empty tanks

The input variables that must be given to the model include two types, namely nature variables and decision variables. The nature variable reflects the natural conditions in the system, in this case it includes: daily filling capacity, daily filling time, average of delivery time from SPBE to agent, average of delivery time from agent to store, average of delivery time from agent to store, average returning time of empty cylinders to SPBE, average inter-arrival demand to store, average retail-demand size.

TABLE 1. List of nat	ure variables and exa	amples of their values
TINDED II DISC OF Hat	are fullaoites and one	ampies of men varaes

No	Variable name	Type of variable	Value	Unit
1	daily filling capacity	Nature	1000	Tanks/Day
2	daily filling time	Nature	8	Hours/Day
3	average of delivery	Nature	3	Hours
	time from SPBE to Agent			
4	average of delivery	Nature	1	Hours
	time from Agent to			
	Store			
5	average returning time	Nature	1	Hours
	of empty cylinders to			
	agent			
6	average returning time	Nature	3	Hours
	of empty cylinders to			
	SPBE			
7	average inter-arrival	Nature	RANDOM	Hours
	demand to store		UNIFORM	
			(1, 3, seed#)	
8	average retail-demand	Nature	RANDOM	Tanks
	size		UNIFORM	
			(1, 2, seed#)	

Some of the decision variables are: target stock R, reproduction point r, Truck capacity from agent to SPBE, reorder point RAgent, order quantity QAgent, reorder point RStore, order quantity QStore, and Truck capacity from store to agent, and the total number of cylinder tanks circulating in the system.

<b>TABLE 2.</b> List of decision variables and examples of their values						
No	Variable name	Type of variable	Value	Unit		
1	target stock R	Decision	500	Tanks		
2	reproduction point r	Decision	100	Tanks		
3	reorder point R <sub>Agent</sub>	Decision	25	Tanks		
4	order quantity $Q_{Agent}$	Decision	100	Tanks		

5	reorder point R <sub>Store</sub>	Decision	5	Tanks
6	order quantity $Q_{\text{store}}$	Decision	25	Tanks
7	Truck capacity from	Decision	100	Tanks
	agent to SPBE			
8	Truck capacity from	Decision	25	Tanks
	store to agent			
9	total number of	Decision	1000	Tanks
	cylinder tanks			
	circulating			

The tank (empty or filled) circulates cyclically, so the total number of cylinder tanks circulating in the system will always remain constant. However, in the model there needs to be a check to make sure.



FIGURE 7. Stock and flow diagram pieces for the process of checking the total number of cylinder tanks circulating in the system

## **Numerical Example**

It is interesting to see the configuration of the combination of decision variables under certain combinations of nature that can ensure the availability of a minimum amount of tank circulation. Or, specifically, how does the total number of cylinder tanks influence the decision variable on the guarantee of LPG availability. An example of a combination of nature and decision is given in Table 1. and Table 2. Meanwhile, Fig.8 shows an example of the effect given by a combination of these decisions. It can be seen that the availability of LPG is guaranteed by the absence of negative stock in all level variables: stock 0, stock 1, stock 2, stock 3, stock 4, stock 5.





**FIGURE 8.** Example of run results for a certain combination of decisions. Seen there is no negative stock at time = 0 - 600

Under these conditions it can be checked that the number of tubes in the system is constant (Fig.9).



"Check: total cylinders"

FIGURE 9. Total number of cylinders in the system is constant at time = 0 - 600 (1000 tanks)

## **RESULTS AND DISCUSSION**

The structured continuous-dynamic model can be used to perform what-if analysis, that is, for a certain combination of nature, how the best combination of decisions can be chosen so that the availability of LP in the system is not negative. A negative stock indicates poor system performance, namely a shortage of LPG availability in the system or community. In this case, it means that the wrong combination of decisions has been chosen. For example, if the total number of circulating cylinder tanks is set to 700, there will be a negative number of empty tank stock at the SPBE at time = 0 to 600 (Fig.10). With the model that has been obtained, it is possible to find the minimum total number of cylinder tanks circulating. For example, by trial and error, the minimum number is 925.



FIGURE 10. An example if the total number of cylinder tanks circulating is set to 700 then stock 0 become negative at time = 549 - 600, indicating that the LPG is not fully provided to the community.

To better describe the real system conditions, the number of agents and the number of stores served by each agent can be added to the model easily. However, a better systematic model is needed, for example by using the concept of a submodel. This paper does not discuss this extension because it emphasizes only the basic concept.

## CONCLUSION

The problem of the circulation of the basic commodities of a society where the commodities are packaged in a cyclical circulation can be modeled using the continuous-dynamic method in the language of system dynamics. The advantage of this method over other methods such as mathematical methods based on queuing theory or discrete-event simulation methods is its simplicity. In this paper, the studied system is still simplified and the model obtained is still basic, so it does not fully reflect the real system because some assumptions have been taken to simplify the problem. This paper shows that the use of a system dynamics language can provide an alternative way of system modeling that is simpler than discrete-event simulation. It is hoped that this will enrich various alternative methods that can be used for supply chain system analysis and design.

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