Stackelberg Game for Two Level Supply Chain with Price Markdown Option

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Abstract

Vendor-retailer collaboration has an important role in supply chain management. Although vendor-retailer collaboration results in better supply chain profit, collaboration is difficult to realize. This is because most vendors and retailers try to optimize their own profit. This paper applies the Stackelberg game with stochastic demand for the vendor-retailer system. The vendor as a leader determines the product price, and the retailer decides order quantity and frequency of price markdown. This study develops example and sensitivity analyses to illustrate the theory. Results show that the price markdown option has a better total supply chain profit than without a price markdown policy, and the vendor receives more benefit. For different demand variances, the retailer profit is more sensitive than the vendor profit.

Keywords*:* Supply chain management, Newsboy problem, Stackelberg game.

1. Introduction

Coordination and collaboration between enterprises can bring great benefits to a supply chain system. Many researchers have concluded that centralized supply chain has a better result. The centralized supply chain can be realized if all members' interests are taken care of with a central decision-maker in the whole of supply chain. But in most supply chains, the supply chain members have conflicting goals and there is no central decision maker who can control all supply chain members. Vendor and the retailer often make their own decision. They try to optimize their decision by considering the other player's decision. The vendor makes pricing decision by considering the retailer's order quantity. And when the retailer decides the optimal ordering quantity, she will set the vendor price. This situation has typical characteristics of the Stackelberg game.

In a single period newsboy problem, retailer can predict the expected customer demand from past data. The real customer demand is stochastic and it can be higher or lower than the expected demand. When the real customer demand is lower than the expected demand, retailer will markdown their price to increase their selling units before the selling period time is over. Many researchers investigated price markdown policy, but few simultaneously considered price markdown policy with stochastic and price dependent demand. Moreover, we investigate the effect of retailer reducing price in the Stackelberg game of a vendor-retailer supply chain.

Most fashion products have random demand which is price dependent. Petruzzi and Dada (1999) developed pricing newsboy problem model. In their model, the decision variables are order quantity and price. Multiple discount single period newsboy problems with price dependent demand are developed by Khouja (1995). He concluded that multiple discounts lead to increase order quantity.

Some theoretical results of price markdown policy practices in supply chain channels were provided by Tsay *et al.* (2001). They investigated price markdown policy on retail pricing behavior. You (2005) derived the optimal ordering quantity and price for price and time dependent demand. Urban and Baker (1997) developed a mathematical model to determine the price markdowns for different seasons.

In recent years, some researchers have studied the application of Stackelberg game in supply chain channels. Zhiyu and Chen (2007) studied Stackelberg game in a vendor-buyer pricing model. They found that greater demand fluctuation resulted in lower vendor's wholesale price, and larger buyer's ordering quantity resulted in greater the buyer's expected profit. Zhen *et. al*. (2006) compared the methods using independent optimization, joint optimization and Stackelberg game. They concluded that the Stackelberg game is the most effective and practical.

The contribution of this paper is to examine price markdown using Stackelberg game in a vendor-buyer supply chain. The model considers a single vendor-retailer channel. Retailer can markdown his initial selling price and each price markdown has an equal time period.

This paper is presented in four sections as follows: the first section explains the background of the research and some literature reviews in price markdown, newsboy problem and Stackelberg game. In the second section, we develop a newsboy problem with price markdown for Stackelberg game. An example and sensitivity analyses is given in section 3 to illustrate the model. The last section is the conclusions and future research.

2. Mathematical model

In this section, we develop a Stackelberg game model for newsboy problem with price markdown option. The following assumptions are used throughout this study:

1. Demand is stochastic and price dependent
2. Multiple price markdowns are applicable for all items.
3. The initial price is known.
4. In price markdowns, retailer incurs some advertising costs.

2.1. From the retailer's perspective

When price is marked down, the price P has the following relationship:

$$P = W - bx \,, \tag{1}$$

where W is the price dependent parameter and x is random demand quantity with known distribution, and b is the dependent demand parameter. A single time period has m times price marked down. Table 1 represents price markdown scheme.

Table 1

Table 1 shows the retailer revenue when multiple price markdowns are applied. From Khouja (2000), the following model is suggested:

$$R_B = \begin{cases} P_0 Q & if \;\; P_a \geq P_0, \\ \frac{j}{h} P_0 x_0 + \frac{h-j}{h} P_0 Q + \frac{P_0^2}{bh^2} \sum_{i=0}^{h-1} i - jF & if \;\; \frac{h-j}{h} P_0 \leq P_a < \frac{h-j+1}{h} P_0, \\ P_0 x_0 + \frac{P_0^2}{bh^2} \sum_{i=0}^{h-1} i - (h-1)F & if \;\; P_a < \frac{P_0}{h} \end{cases} \quad (2)$$

where F is the advertising cost, h is the number of selling price, Q is the retailer ordering quantity, P_0 is the initial retailer price and item price is P_a. The expected profit using for the retailer using (2) is:

$$\text{E(BP(h))} = \int_Q^{\infty} \text{P}_0 Q f(x_0) dx_0 +$$

$$\sum_{j=1}^{h} \int_{Q-jP_0/hb}^{Q-(j-1)P_0/hb} \left[\begin{matrix} \frac{j}{h} P_0 x_0 + \frac{h-j}{h} P_0 Q \\ + \frac{P_0^2}{bh^2} \sum_{i=0}^{j-1} i - jF \end{matrix} \right] f(x_0) dx_0$$

$$+ \int_0^{Q-(h-1)P_0/hb} \left[\frac{j}{h} P_0 x_0 + \frac{P_0^2}{bh^2} \sum_{i=0}^{j-1} i - (h-1)F \right] f(x_0) dx_0 - CQ \quad (3)$$

where C is the item price offered by the vendor to the retailer. In this paper, we assume x_0 is uniformly distributed with the range[~,]. Equation (3) can be rewritten as follows:

$$E(BP(h)) = \frac{1}{\;\; - \sim} \left[P_0 \left(\begin{matrix} Q(\;\; - Q) + \frac{P_0 Q(h-1)}{hb} \\ + \frac{P_0^2}{12b^2h^2}(-2h+1)(h-1) \end{matrix} \right) \right.$$

$$\left. + \frac{P_0}{2} \left(\left(Q - \frac{(h-1)P_0}{hb} \right)^2 - \sim^2 + \frac{F(1-h)}{b} \right) + \left(\frac{P_0^2 \sum_{i=0}^{h-1} i}{h^2 b} - (h-1)F \right) \left(Q - \frac{(h-1)P_0}{hb} - \sim \right) \right]$$

$$- CQ \quad (4)$$

2.2. From the vendor's perspective

The vendor revenue is the product of the vendor profit margin and the retailer optimal order quantity. One has:

$$E(SP) = (C - S)Q \tag{5}$$

where S is vendor unit cost.

2.3. The Stackelberg game

Here, the vendor acts as a Stackelberg leader and the retailer as the follower. This relationship between the vendor and the retailer is a sequential non-cooperative game. The vendor first decides the product price, and the retailer then decides the order quantity and the frequency of price markdown. The vendor tries to determine product price and maximize her profit after considering the retailer behavior. The retailer's decision is to optimize the order quantity and the frequency of price markdown so as to maximize the retailer profit at a given vendor price. The vendor-retailer chain can be modeled as Stackelberg game as follows:

Max *E(BP(h)), E(SP)*

st. *P0 > C > S*

Since the vendor act as the leader the retailer optimizes her decision first. The retailer optimal markdown frequency h (Q^*_h) can be found by differentiating the retailer's expected profit (4) with respect to Q. The optimal retailer order quantity is:

$$\frac{\partial E(BP(h))}{\partial Q} = \frac{1}{\quad - \sim} \begin{bmatrix} P_0(\quad - 2Q) + \frac{P_0^2}{hb}(h-1) \\ + P_0(Q - \frac{P_0(h-1)}{hb}) + \frac{P_0^2\, 0.5h(h-1)}{h^2 b} \\ -(h-1)F - C(\quad - \sim) \end{bmatrix} = 0$$

$$Q_h^* = \frac{1}{2hbP_0}\begin{pmatrix} 2P_0 \quad bh + P_0^{\,2}(h-1) \\ + 2Fhb(1-h) + 2Chb(\sim - \quad) \end{pmatrix} \tag{6}$$

The expected retailer profit is concave if the second order derivative of the expected retailer profit with respect to Q is less than zero. The second order derivative of the expected retailer profit can be written as:

$$\frac{\partial E^2(BP(h))}{\partial Q^2} = -\frac{P_0}{\quad - \sim} \tag{7}$$

Since value of P_0, , and ~ are positive and > ~ , the value of (7) is negative and the expected retailer profit is concave.

When the retailer has decided on the optimal number of order, then the vendor will react to the retailer decision by determining the optimal vendor price. The optimal vendor decision can be built by substituting the retailer optimal order quantity (6) into the vendor expected profit (5). Differentiating the equation with respect to C, one has:

$$E(SP) = (C - S)\frac{1}{2hbP_0}\begin{pmatrix} 2P_0 \quad bh + P_0^{\ 2}(h-1) \\ + 2Fhb(1-h) + 2Chb(\sim - \quad) \end{pmatrix}$$

$$\frac{\partial E(SP)}{\partial C} = \frac{1}{2hbP_0}\begin{pmatrix} 2P_0 \quad bh + P_0^{\ 2}(h-1) \\ + 2Fhb(1-h) + (4C - 2S)hb(\sim - \quad) \end{pmatrix}$$

$$C^* = \frac{1}{4hb(\quad - \sim)}\begin{pmatrix} 2P_0 \quad bh + P_0^{\ 2}(h-1) \\ + 2Fhb(1-h) + 2Shb(\quad - \sim) \end{pmatrix} \qquad (8)$$

The second order derivative of the expected vendor profit is:

$$\frac{\partial E^2(SP)}{\partial^2 C} = \frac{1}{P_0}(\sim - \quad) \qquad (9)$$

Since $\alpha < \beta$, and the value of P_0, , and ~ are positive, (9) is always negative and therefore the expected vendor profit is concave.

Using equations above, we develop an algorithm to solve the Stackelberg problem as follows:

Step 1: Set t = 1 and h_t = 1,

Step 2: Substitute h_t to equations (6) and (8) to get the optimal ordering quantity (Q^*_h) and the optimal vendor price (C^*).

Step 3: Substitute Q^*_h and C^* to equations (4) and (5) to derive the buyer expected profit ($E(BP(h_t))$) and the supplier expected profit E(SP).

Step 3: Set h_{t+1}=h+1 and repeat step 2 and 3. If $E(BP(h_{t+1})) < E(BP(h_t))$ then STOP, the optimal expected profit has been derived.

3. Numerical example and sensitivity analysis

This section illustrates the Stackelberg game model with price markdown. Consider W be uniformly distributed with [275, 775] and b = 0.05. It implies that x_0 is uniformly distributed with ~ = 5000 and = 15000 for P_0 = \$25, F = \$100 and S = \$8. This study illustrates two scenarios. In the first scenario, the retailer determines the optimal ordering quantity (Q^*) and the frequency of price markdown. Assuming the retailer price markdown for a maximum of 9 times, the result of the numerical example is shown in Table 2.

Table 2

Table 2 shows the optimal total supply chain profit is obtained when the retailer markdown the initial price seven times. When the retailer increases the price markdown frequency, then the optimal order quantity increases. This condition is similar to the decentralized situation. However in Stackelberg game, the vendor has an opportunity to increase the unit price.

Since the retailer incurs less profit when the selling price is markdowned, there is no reason for the retailer to markdown price. Therefore, the retailer establishes the optimal order quantity without price markdown, and the vendor determines the vendor unit price based on the retailer's order quantity. When the vendor has determined the vendor's unit price, the retailer can optimize the expected profit by determining the frequency of price markdown. When this scenario is applied, one has the optimal profit as shown in Table 3. The total profits are derived when vendor selling price is equal to 22.75.

Table 3

The optimal supply chain profit is achieved when the frequency of price change is equal to 8. This solution is similar to the first scenario. The optimal order quantity is also larger than the first scenario. The expected supply chain profit in the second scenario is 1.4% larger than the first scenario.

The sensitivity analyzes are conducted by investigating the effect of different demand variance with equal demand mean. Table 4 and 5 show the results.

Table 4

Table 5

Table 4 and 5 show that the frequencies of price markdown are not sensitive to difference demand variances. Table 4 shows that the vendor price tends to increase when demand variance is decreasing. It shows that the vendor is taking on the retailer risk when demand variances are wide. Since the retailer price is lower, bigger quantity is ordered. Table 5 shows the total supply chain profit is bigger when demand variance is smaller. The retailer has bigger benefit for wider demand variance and the vendor has bigger benefit for smaller demand variance. This result is similar as Zhiyu and Chen (2007).

Figure 1 shows the retailer profits decrease rapidly and the vendor profits increase smoothly when demand variance decrease. It can be said that the retailer profit is more sensitive to demand variance than the vendor profit. This circumstance occurs because the retailer handles more risk than the vendor.

Figure 1

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4. Conclusion

This study develops a price markdown supply chain model using the Stackelberg game. Example and sensitivity analysis show that the frequency of price markdown affected the optimal supply chain profit. The current investigation introduced two scenarios of Stackelberg game theory. In the first scenario, the price markdown option gave more benefit to the vendor than to the retailer. In the second scenario, the price markdown option increased both the vendor and retailer benefit. Sensitivity analysis shows results similar to the results by Zhiyu and Chen (2007). For different values of demand variance, the retailer profit is more sensitive than the vendor profit.

This study shows that the price markdown option can benefit both players. Future research can consider a more general model with varying demand distributions or fuzzy demand.

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Table 1 Marked down price scheme

Number of markdown	Number of prices	Prices Used
0	1	P_0,0
1	2	P_0, P_0/2, 0
2	3	P_0, 2P_0/3, P_0/3, 0
…		
m	h	P_0,(h-1)P_0/h, (h-2)P_0/h, …, P_0/h, 0

Table 2 Optimal price, order quantity and profit (1st scenario)

h	*C*	*Q*	Retailer profit	Vendor profit	Total profit
1	22.75	5900	12263	87025	99288
2	22.90	5961	11608	88819	100427
3	22.95	5979	11401	89381	100782
4	22.97	5988	11308	89633	100941
5	22.98	5992	11261	89760	101021
6	22.99	5994	11237	89825	101062
7	22.99	5995	11226	89854	101081
8	**22.99**	**5995**	**11224**	**89861**	**101085**
9	22.99	5995	11227	89853	101080

Table 3 Optimal price, order quantity and profit (2nd scenario)

h	*Q*	Retailer profit	Vendor profit	Total profit
1	5900	12263	87025	99288
2	6021	12514	88810	101324
3	6059	12595	89365	101960
4	6076	12631	89614	102245
5	6084	12650	89739	102389
6	6088	12659	89803	102462
7	6090	12664	89832	102495
8	**6091**	**12665**	**89839**	**102503**
9	6090	12664	89831	102495

Table 4 Optimal price and order quantity for difference demand variance

	~		*h*	*C*	*Q*
1	1000	19000	8	17.19	6811
2	2000	18000	8	18.06	6631
3	3000	17000	8	19.18	6451
4	4000	16000	8	20.67	6271
4	5000	15000	8	22.75	6091

Table 5 Optimal profit for difference demand variance

	~		*h*	Retailer profit	Vendor profit	Total profit
1	1000	19000	8	31213	62621	93834

2	2000	18000	8	30583	66722	97305
3	3000	17000	8	28044	72110	100154
4	4000	16000	8	22644	79430	102074
5	5000	15000	8	12665	89839	102503

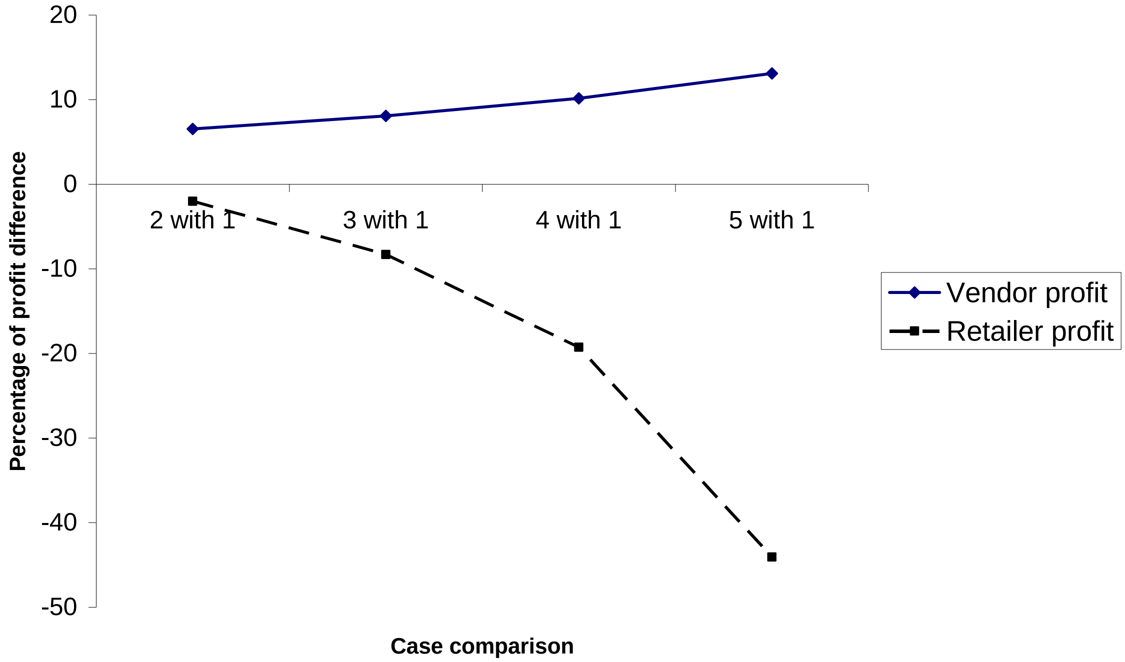

Fig.1. Percentage of profit increase and decrease for difference demand variance