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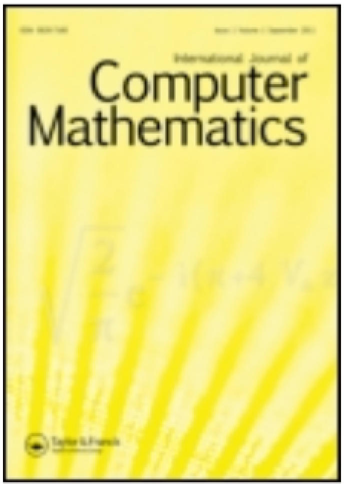

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Vendor–retailer collaboration has an important role in supply chain management. Although vendor–retailer collaboration results in better supply chain profit, collaboration is difficult to realize. This is because most vendors and retailers try to optimize their own profit. This paper applies the Stackelberg game with stochastic demand for the vendor–retailer system. The vendor as a leader determines the product price, and the retailer decides order quantity and frequency of price markdown. This study develops example and sensitivity analyses to illustrate the theory. Results show that the price markdown option has a better total supply chain profit than without a price markdown policy, and the vendor receives more benefit. For different demand variances, the retailer profit is more sensitive than the vendor profit.

Keywords: two-level supply chain; supply chain management; newsboy problem; stackleberg game; genetic algorithm

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1. Introduction

Coordination and collaboration between enterprises can bring great benefits to a supply chain system. Many researchers have concluded that centralized supply chain has a better result. The centralized supply chain can be realized if all members' interests are taken care of with a central decision-maker in the whole of supply chain. But in most supply chains, the supply chain members have conflicting goals and there is no central decision-maker who can control all supply chain members. Vendor and the retailer often make their own decision. They try to optimize their decision by considering the other player's decision. The vendor makes pricing decision by considering the retailer's order quantity. And when the retailer decides the optimal ordering quantity, she will set the vendor price. This situation has typical characteristics of the Stackelberg game.

In a single-period newsboy problem, retailer can predict the expected customer demand from past data. The real customer demand is stochastic and it can be higher or lower than the expected demand. When the real customer demand is lower than the expected demand, retailer will markdown their price to increase their selling units before the selling period time is over. Many researchers investigated price markdown policy, but few simultaneously considered price

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markdown policy with stochastic and price-dependent demand. Moreover, we investigate the effect of retailer reducing price in the Stackelberg game of a vendor–retailer supply chain.

Most fashion products have random demand which is price dependent. Petruzzi and Dada [3] developed the pricing newsboy problem model. In their model, the decision variables are order quantity and price. Multiple discount single-period newsboy problems with price-dependent demand are developed by Khouja [1]. He concluded that multiple discounts lead to increased order quantity.

Some theoretical results of price markdown policy practices in supply chain channels were provided by Tsay et al. [4]. They investigated price markdown policy on retail pricing behaviour. Yu [6] derived the optimal ordering quantity and price for price and time-dependent demand. Urban and Baker [5] developed a mathematical model to determine the price markdowns for different seasons.

In the recent years, some researchers have studied the application of the Stackelberg game in supply chain channels. Zhiyu and Chen [8] studied the Stackelberg game in a vendor– buyer pricing model. They found that greater demand fluctuation resulted in lower vendor's wholesale price, and larger buyer's ordering quantity resulted in greater buyer's expected profit. Zhen et al. [7] compared the methods using independent optimization, joint optimization and the Stackelberg game. They concluded that the Stackelberg game is the most effective and practical one.

The contribution of this paper is to examine price markdown using the Stackelberg game in a vendor–buyer supply chain. The model considers a single vendor–retailer channel. Retailer can markdown his initial selling price, and each price markdown has an equal time period.

This paper is presented in four sections as follows: the first section explains the background of the research and some literature reviews in price markdown, newsboy problem and Stackelberg game. In the second section, we develop a newsboy problem with price markdown for the Stackelberg game. An example and sensitivity analyses are given in Section 3 to illustrate the model. The last section is the conclusions and discusses about future research.

2. Mathematical model

In this section, we develop a Stackelberg game model for the newsboy problem with the price markdown option. The following assumptions are used throughout this study:

1. Demand is stochastic and price dependent.
2. Multiple price markdowns are applicable for all items.
3. The initial price is known.
4. In price markdowns, retailer incurs some advertising costs.

2.1 *From the retailer's perspective*

When price is marked down, the price P has the following relationship:

$$P = W - bx, \tag{1}$$

Table 1. Marked down price scheme.

Number of markdown	Number of prices	Prices used
0	1	$P_0, 0$
1	2	$P_0, P_0/2, 0$
2	3	$P_0, 2P_0/3, P_0/3, 0$
⋮		
m	h	$P_0, (h-1)P_0/h, (h-2)P_0/h, \ldots, P_0/h, 0$

where W is the price-dependent parameter and x is random demand quantity with known distribution, and b is the dependent demand parameter. A single time period has m times price marked down. Table 1 represents the price markdown scheme.

Table 1 shows the retailer revenue when multiple price markdowns are applied. From Khouja [2], the following model is suggested:

$$R_B = \begin{cases} P_0 Q & \text{if } P_a \geq P_0, \\ \frac{j}{h}P_0 x_0 + \frac{h-j}{h}P_0 Q + \frac{P_0^2}{bh^2}\sum_{i=0}^{h-1} i - jF & \text{if } \frac{h-j}{h}P_0 \leq P_a < \frac{h-j+1}{h}P_0, \\ P_0 x_0 + \frac{P_0^2}{bh^2}\sum_{i=0}^{h-1} i - (h-1)F & \text{if } P_a < \frac{P_0}{h}, \end{cases} \tag{2}$$

where F is the advertising cost, h is the number of selling price, Q is the retailer ordering quantity, P_0 is the initial retailer price and item price is P_a. The expected profit using for the retailer using (2) is

$$\begin{aligned} \mathrm{E}(\mathrm{BP}(h)) = {} & \int_Q^\infty P_0 Q f(x_0)\,\mathrm{d}x_0 + \sum_{j=1}^{h} \int_{Q-jP_0/hb}^{Q-(j-1)P_0/hb} \left[\begin{aligned} & \frac{j}{h}P_0 x_0 + \frac{h-j}{h}P_0 Q \\ & + \frac{P_0^2}{bh^2}\sum_{i=0}^{j-1} i - jF \end{aligned} \right] f(x_0)\,\mathrm{d}x_0 \\ & + \int_0^{Q-(h-1)P_0/hb} \left[\frac{j}{h}P_0 x_0 + \frac{P_0^2}{bh^2}\sum_{i=0}^{j-1} i - (h-1)F \right] f(x_0)\,\mathrm{d}x_0 - CQ, \end{aligned} \tag{3}$$

where C is the item price offered by the vendor to the retailer. In this paper, we assume x_0 is uniformly distributed with the range $[\alpha, \beta]$. Equation (3) can be rewritten as follows:

$$\begin{aligned} E(BP(h)) = {} & \frac{1}{\beta - \alpha} \left[P_0 \left(\begin{aligned} & Q(\beta - Q) + \frac{P_0 Q(h-1)}{hb} \\ & + \frac{P_0^2}{12b^2h^2}(-2h+1)(h-1) \end{aligned} \right) \right. \\ & + \frac{P_0}{2}\left(\left(Q - \frac{(h-1)P_0}{hb} \right)^2 - \alpha^2 + \frac{F(1-h)}{b} \right) \\ & \left. + \left(\frac{P_0^2 \sum_{i=0}^{h-1} i}{h^2 b} - (h-1)F \right) \left(Q - \frac{(h-1)P_0}{hb} - \alpha \right) \right] - CQ. \end{aligned} \tag{4}$$

2.2 *From the vendor's perspective*

The vendor revenue is the product of the vendor profit margin and the retailer optimal order quantity. One has

$$E(SP) = (C - S)Q, \tag{5}$$

where S is the vendor unit cost.

2.3 *The Stackelberg game*

Here, the vendor acts as a Stackelberg leader and the retailer as the follower. This relationship between the vendor and the retailer is a sequential non-cooperative game. The vendor first decides the product price, and the retailer then decides the order quantity and the frequency of price markdown. The vendor tries to determine product price and maximize her profit after considering the retailer behaviour. The retailer's decision is to optimize the order quantity and the frequency of price markdown so as to maximize the retailer profit at a given vendor price. The vendor–retailer chain can be modelled as Stackelberg game as follows:

$$\begin{array}{ll} \text{Max} & \in E(BP(h)), E(SP) \\ \text{s.t.} & P_0 > C > S. \end{array}$$

Since the vendor acts as the leader, the retailer optimizes her decision first. The retailer optimal markdown frequency $h\,(Q_h^*)$ can be found by differentiating the retailer's expected profit (4) with respect to Q. The optimal retailer order quantity is

$$\frac{\partial E(BP(h))}{\partial Q} = \frac{1}{\beta-\alpha}\left[\begin{array}{c} P_0(\beta-2Q) + \dfrac{P_0^2}{hb}(h-1) \\ +P_0\left(Q - \dfrac{P_0(h-1)}{hb}\right) + \dfrac{P_0^2 0.5h(h-1)}{h^2 b} \\ -(h-1)F - C(\beta-\alpha) \end{array}\right] = 0,$$

$$Q_h^* = \frac{1}{2hbP_0}\left(\begin{array}{c} 2P_0\beta bh + P_0^2(h-1) \\ +2Fhb(1-h) + 2Chb(\alpha-\beta) \end{array}\right). \tag{6}$$

The expected retailer profit is concave if the second-order derivative of the expected retailer profit with respect to Q is less than zero. The second-order derivative of the expected retailer profit can be written as follows:

$$\frac{\partial E^2(BP(h))}{\partial Q^2} = -\frac{P_0}{\beta-\alpha}. \tag{7}$$

Since the value of P_0, β, and α are positive and $\beta > \alpha$, the value of Equation (7) is negative and the expected retailer profit is concave.

When the retailer has decided on the optimal number of order, then the vendor will react to the retailer decision by determining the optimal vendor price. The optimal vendor decision can be built by substituting the retailer optimal order quantity (6) into the vendor expected profit (5). Differentiating the equation with respect to C, one has:

$$E(SP) = (C-S)\frac{1}{2hbP_0}\left(\begin{array}{c} 2P_0\beta bh + P_0^2(h-1) \\ +2Fhb(1-h) + 2Chb(\alpha-\beta) \end{array}\right),$$

$$\frac{\partial E(SP)}{\partial C} = \frac{1}{2hbP_0}\left(\begin{array}{c} 2P_0\beta bh + P_0^2(h-1) \\ +2Fhb(1-h) + (4C-2S)hb(\alpha-\beta) \end{array}\right),$$

$$C^* = \frac{1}{4hb(\beta-\alpha)}\left(\begin{array}{c} 2P_0\beta bh + P_0^2(h-1) \\ +2Fhb(1-h) + 2Shb(\beta-\alpha) \end{array}\right). \tag{8}$$

The second-order derivative of the expected vendor profit is

$$\frac{\partial E^2(SP)}{\partial^2 C} = \frac{1}{P_0}(\alpha-\beta). \tag{9}$$

Since $\alpha < \beta$, and the value of P_0, β, and α are positive, Equation (9) is always negative and therefore the expected vendor profit is concave.

Using equations above, we develop an algorithm to solve the Stackelberg problem as follows:

Step 1: Set $t = 1$ and $h_t = 1$.

Step 2: Substitute h_t in Equations (6) and (8) to get the optimal ordering quantity (Q_h^*) and the optimal vendor price (C^*).

Step 3: Substitute Q_h^* and C^* in Equations (4) and (5) to derive the buyer expected profit ($E(\mathrm{BP}(h_t))$) and the supplier expected profit $E(\mathrm{SP})$.

Step 3: Set $h_{t+1} = h + 1$ and repeat step 2 and 3. If $E(\mathrm{BP}(h_{t+1})) < E(\mathrm{BP}(h_t))$ then STOP, the optimal expected profit has been derived.

3. Numerical example and sensitivity analysis

This section illustrates the Stackelberg game model with price markdown. Consider W be uniformly distributed with [275, 775] and $b = 0.05$. It implies that x_0 is uniformly distributed with $\alpha =$ 5[illegible]00 and $\beta =$ 15,000 for $P_0 = \$25$, $F = \$100$ and $S = \$8$. This study illustrates two scenarios. In the first scenario, the retailer determines the optimal ordering quantity (Q^*) and the frequency of price markdown. Assuming the retailer price markdown for a maximum of 9 times, the result of the numerical example is shown in Table 2.

Table 2 shows that the optimal total supply chain profit is obtained when the retailer markdowns the initial price seven times. When the retailer increases the price markdown frequency, then the optimal order quantity increases. This condition is similar to the decentralized situation. However, in the Stackelberg game, the vendor has an opportunity to increase the unit price.

Table 2. Optimal price, order quantity and profit (first scenario).

h	C	Q	Retailer profit	Vendor profit	Total profit
1	22.75	5900	12,263	87,025	99,288
2	22.90	5961	11,608	88,819	100,427
3	22.95	5979	11,401	89,381	100,782
4	22.97	5988	11,308	89,633	100,941
5	22.98	5992	11,261	89,760	101,021
6	22.99	5994	11,237	89,825	101,062
7	22.99	5995	11,226	89,854	101,081
8	**22.99**	**5995**	**11,224**	**89,861**	**101,085**
9	22.99	5995	11,227	89,853	101,080

Note: Bold values denote optimal solution.

Table 3. Optimal price, order quantity and profit (second scenario).

h	Q	Retailer profit	Vendor profit	Total profit
1	5900	12,263	87,025	99,288
2	6021	12,514	88,810	101,324
3	6059	12,595	89,365	101,960
4	6076	12,631	89,614	102,245
5	6084	12,650	89,739	102,389
6	6088	12,659	89,803	102,462
7	6090	12,664	89,832	102,495
8	**6091**	**12,665**	**89,839**	**102,503**
9	6090	12,664	89,831	102,495

Note: Bold values denote optimal solution.

Since the retailer incurs less profit when the selling price is markdowned, there is no reason for the retailer to markdown price. Therefore, the retailer establishes the optimal order quantity without price markdown, and the vendor determines the vendor unit price based on the retailer's order quantity. When the vendor has determined the vendor's unit price, the retailer can optimize the expected profit by determining the frequency of price markdown. When this scenario is applied, one has the optimal profit as shown in Table 3. The total profits are derived when vendor selling price is equal to 22.75.

The optimal supply chain profit is achieved when the frequency of price change is equal to 8. This solution is similar to the first scenario. The optimal order quantity is also larger than the first scenario. The expected supply chain profit in the second scenario is 1.4% larger than the first scenario.

Table 4. Optimal price and order quantity for difference demand variance.

	α	β	h	C	Q
1	1000	19,000	8	17.19	6811
2	2000	18,000	8	18.06	6631
3	3000	17,000	8	19.18	6451
4	4000	16,000	8	20.67	6271
4	5000	15,000	8	22.75	6091

Table 5. Optimal profit for difference demand variance.

	α	β	h	Retailer profit	Vendor profit	Total profit
1	1000	19,000	8	31,213	62,621	93,834
2	2000	18,000	8	30,583	66,722	97,305
3	3000	17,000	8	28,044	72,110	100,154
4	4000	16,000	8	22,644	79,430	102,074
5	5000	15,000	8	12,665	89,839	102,503

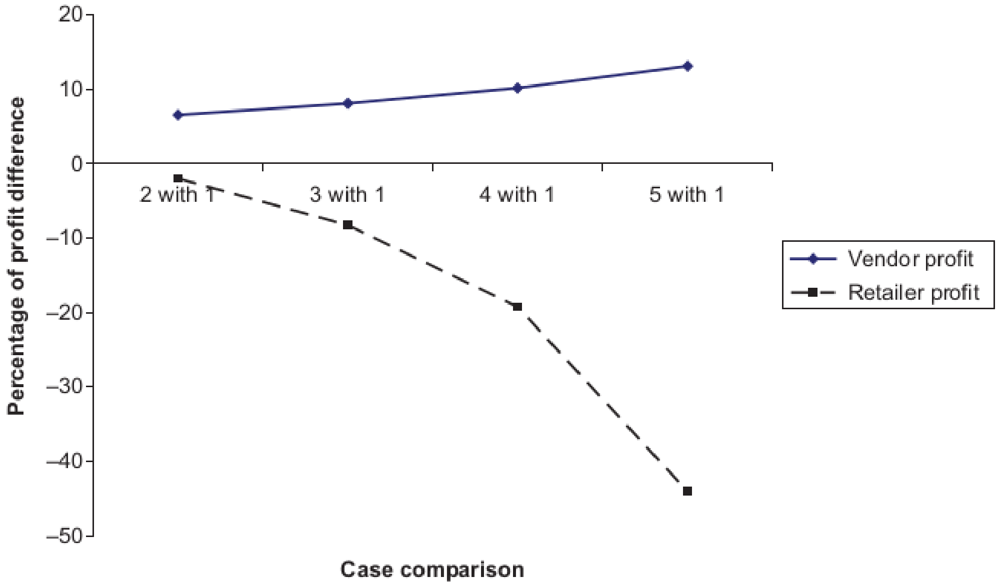

Figure 1. Percentage of profit increase and decrease for difference demand variance.

The sensitivity analyses are conducted by investigating the effect of different demand variances with equal demand mean. Tables 4 and 5 show the results.

Tables 4 and 5 show that the frequencies of price markdown are not sensitive to difference demand variances. Table 4 shows that the vendor price tends to increase when demand variance is decreasing. It shows that the vendor is taking on the retailer risk when demand variances are wide. Since the retailer price is lower, bigger quantity is ordered. Table 5 shows that the total supply chain profit is bigger when demand variance is smaller. The retailer has bigger benefit for wider demand variance and the vendor has bigger benefit for smaller demand variance. This result is similar as Zhiyu and Chen [8].

Figure 1 shows that the retailer profits decrease rapidly and the vendor profits increase smoothly when demand variance decreases. It can be said that the retailer profit is more sensitive to demand variance than the vendor profit. This circumstance occurs because the retailer handles more risk than the vendor.

4. Conclusion

This study develops a price markdown supply chain model using the Stackelberg game. Example and sensitivity analysis show that the frequency of price markdown affected the optimal supply chain profit. The current investigation introduced two scenarios of the Stackelberg game theory. In the first scenario, the price markdown option gave more benefit to the vendor than to the retailer. In the second scenario, the price markdown option increased both the vendor and retailer benefit. Sensitivity analysis shows results similar to the results by Zhiyu and Chen [8]. For different values of demand variance, the retailer profit is more sensitive than the vendor profit.

This study shows that the price markdown option can benefit both players. Future research can consider a more general model with varying demand distributions or fuzzy demand.

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