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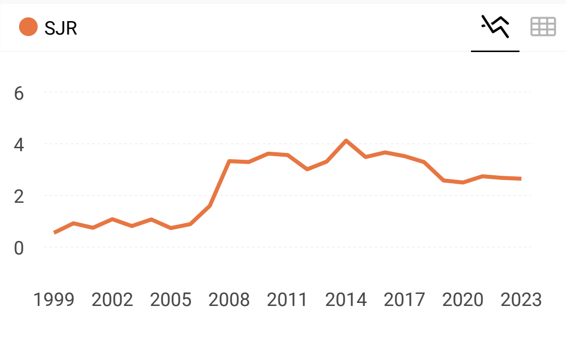

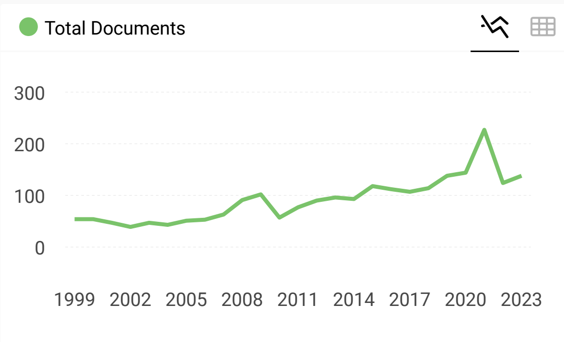

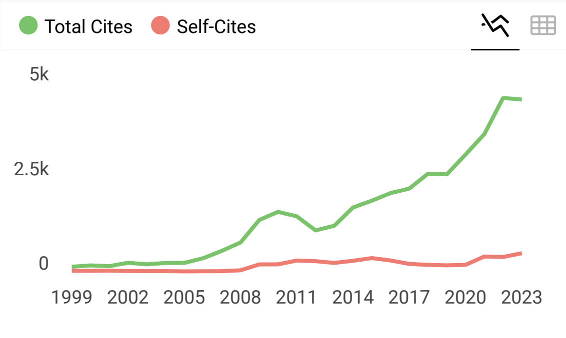

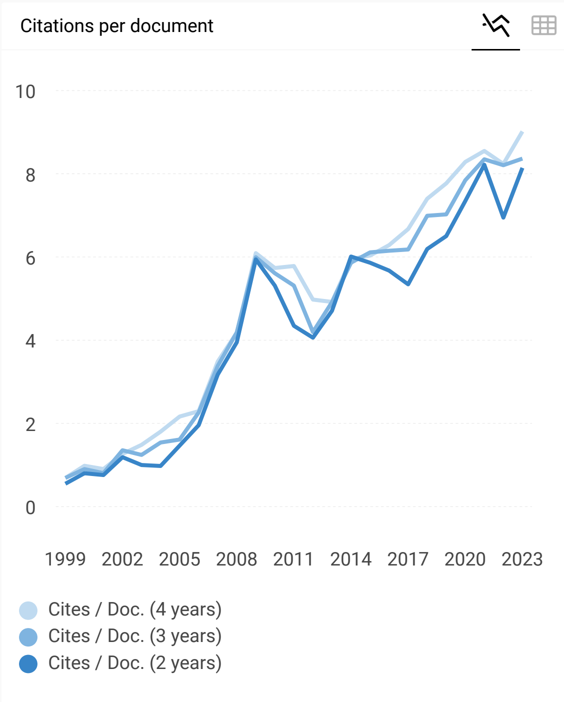

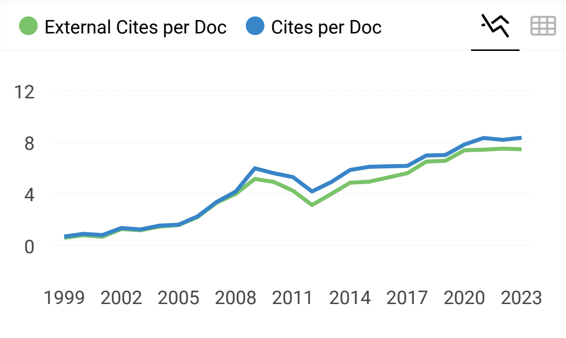

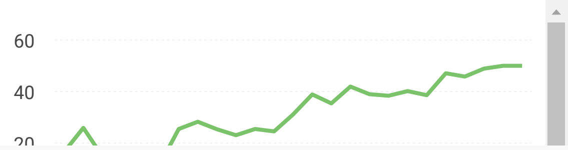

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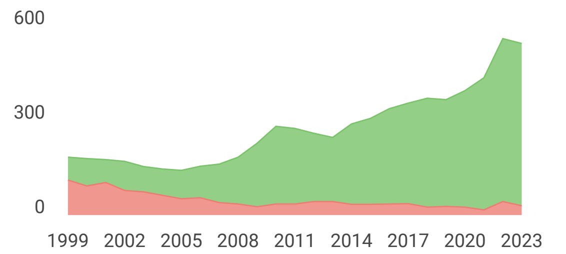

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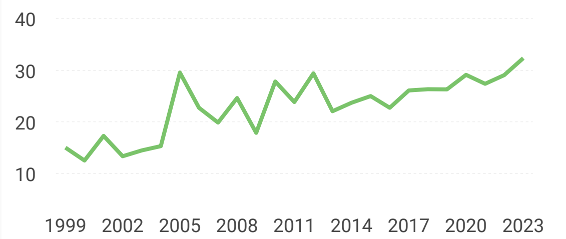

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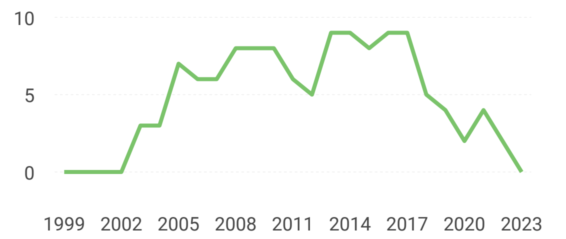

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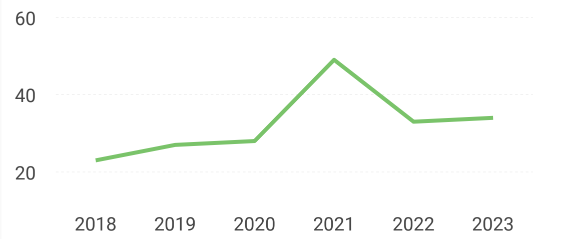

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A production model for deteriorating items with stochastic preventive maintenance time and rework process with FIFO rule

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ABSTRACT

Due to unreliable production facility and stochastic preventive maintenance, deriving an optimal production inventory decision in practice is very complicated. In this paper, we develop a production model for deteriorating items with stochastic preventive maintenance time and rework using the first in first out (FIFO) rule. From our literature search, no study has been done on the above problem. The problem is solved using a simple search procedure; this makes it more practical for use by industries. Two case examples using uniform and exponential distribution preventive maintenance time are applied. Examples and sensitivity analysis are conducted for each case. The results show that rework and preventive maintenance time have significant affected the total cost and the optimal production time. This provides helpful managerial insights to help management in making smart decisions.

1. Introduction

Due to rapid globalization in the last decade, many manufacturers face tight competition in the market place. Nowadays, consumers know where to find goods with cheaper price, better quality and faster delivery. Rework is a popular strategy by many companies to reduce their production cost and to maintain high product quality. From our literature search, no research has considered simultaneously the deteriorating inventory, stochastic preventive time (due to unforeseen circumstances such as finding the unplanned parts that need to be replaced), lost sales possibility and the first in first out (FIFO) rule. In real-life, most manufacturers try to reduce lost sales due to shortage. However, due to the stochastic nature of the model, shortages may occur. By considering lost sales cost in our modeling, we penalize the undesired effect of shortage. By optimizing the model, we seek to find the tradeoff between too much or too little inventory. Shortage should be kept as low as possible especially if the shortage cost is high. However, if the shortage cost is so large that it approaches infinity (for example, shortage of blood for emergency transfusion will result in human death, a very high cost indeed!), then blood in hospital should never be running out. The condition is also application for critical parts in industry where the shortage is extremely costly. Due to the characteristic of our model, it will optimize to no shortage model when shortage cost is very high. Therefore, considering lost sale during shortage gives us wider options in our replenishment decisions, thus a more general solution.

Schrady [1] was one of the earliest researchers in production models who considered rework processes. Chung and Hou [2] considered imperfect process, rework and shortages in their EPQ model. Teunter [3] developed an optimal production and rework lot-size quantity models for two lot sizing policies. Recently, Widyadana and Wee [4] simplified the solution methods by introducing an algebraic approach. Buscher and Lindner [5] developed an EPQ model which addresses lot size of production, rework and shipment. Chiu et al. [6] developed EPQ models which addressed random breakdown of production machines and rework. A similar model considering service level constraints with rework was developed by Chiu et al. [7]. Liu et al. [8] analyzed the number of production and rework setups used in one cycle; as well as their sequence and optimal production quantity in each setup. Cardenas-Barron [9] developed an EPQ model with rework by using a planned backorder. Sana [10] proposed an extended production inventory model with rework by considering variable product reliability factors, variable unit production costs and a dynamic production rate. The effect of scrap, rework and stochastic machine breakdown under an abort/resume (AR) policy was considered by Chiu [11]. Similar research has been conducted by Chiu et al. [12]. However none of the above research considered deteriorating items in their models.

Deteriorating items are items that lose their utility with time due to decay, damage or spoilage. Some examples of deteriorating items are found in semiconductor, pharmaceutical, chemical and foods. The rework process is commonly applied to products that have a

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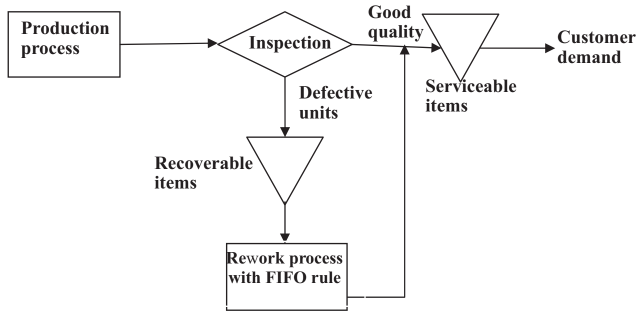

Fig. 1. Production system with rework.

deteriorating characteristic, however few researchers give attention to this area. Flapper and Teunter [13] introduced a logistic planning model with a deteriorating recoverable product and Inderfuth et al. [14] also devised a similar model. Inderfuth et al. [15] developed a production inventory model with rework and defectives items which began to deteriorate while waiting for rework. An EPQ model for deteriorating time with random machine breakdowns and fixed repair time was developed by Lin and Gong [16]. Wee et al. [17] developed an inventory deteriorating model with imperfect quality and assumed that the poor quality items are not recoverable. Later Chang and Ho [18] derived the closed-form solution to solve Wee's model. Some intensive research for deteriorating inventory models which addresses remanufacturing have been done by Wee and Chung [19], Yang et al. [20] and Chung and Wee [21]. Unlike most of the researchers who assumed deterioration only occurred in recoverable items, we consider deterioration occurs for both the serviceable and the recoverable items. To make it more practical, our model also considers the stochastic preventive maintenance time.

Salameh and Jaber [22] were one of the first authors who considered lot-sizing problems with regular maintenance interruptions. Abboud et al. [23] developed an economic lot sizing model by considering machine unavailability due to preventive maintenance and shortage. Later the model was extended by Chung et al. [24] who considered deteriorating items with stochastic machine unavailability time and shortage. Sheu and Chen [25] developed an EPQ model taking into account the level of preventive maintenance for an imperfect production system. They assumed that the constant renewal of the production system will reduce the preventive maintenance level. Production-inventory models for a deteriorating production process and deteriorating items were developed by Alfares et al. [26]. They also considered some realistic aspects in their model such as varying demand, varying production rate, inspection and maintenance. Gharbi et al. [27] developed a production inventory model by considering preventive maintenance, machine failure and backorders. Backordering occurs if demands during repair time exceed the total inventory. Tsou and Chen [28] shown that preventive maintenance has an effect on the production system. Chakraborty et al. [29] developed production inventory models by considering deteriorating production process, random breakdown machine and preventive maintenance. The effect of perfect and imperfect preventive maintenance time in an EPQ model was introduced by Liao et al. [30]. Karamatsoukis and Kyriakidis [31] developed a production-inventory model with a predetermined buffer, idle periods, and preventive maintenance. The effect of stochastic maintenance and corrective time in an EPQ model for deteriorating items are considered by Widyadana and Wee [32]. Berthaut et al. [33] analyzed production inventory control policy by simultaneously considered corrective cost, maintenance cost, holding cost and backlog cost. Wee and Widyadana [34] assumed rework using the last in first out (LIFO) rule. Lev and Weiss [35] developed optimal inventory policy for both finite and infinite horizon models with changes in any or all costs. Yang et al. [36] considered products risks due to rapid technological innovation Wee and Wang [37], assumed an EPQ model with partial backordering and phase-dependent backordering rate. All of the above researchers have developed procedures for computing their respective optimal policies.

From the authors' literature search, there is no model that considers stochastic preventive maintenance time, deteriorating items and rework process using the first in first out (FIFO) rule simultaneously. In this paper, a lot sizing model for deteriorated items with rework and stochastic maintenance time is developed. Both the serviceable and the recoverable items are assumed to deteriorate. The rework production system is shown in Fig. 1. In this system, items are produced before inspection. Good quality items are sold to the customer immediately and defective items are kept for rework. We assume all recoverable items can be reworked to an "as new" condition. We maintained the machine at the end of the production time; and the maintenance time is assumed stochastic. When there is no serviceable inventory, the rework process starts as long as there is available machine. Shortages will occur when the demand is greater than the existing stock during machine maintenance time, and shortage items are assumed to be lost sales. Two different preventive maintenance time distributions, the uniform distribution and the exponential distribution, are considered. This paper is divided into four sections. Section 1 is the literature review and the motivation for the study. The model development is explained in Section 2. Section 3 shows a numerical example and the sensitivity analysis of the study. Finally, conclusion and future research are given in Section 4.

2. Model development

The assumptions:

1. The good quality rate and rework rate must be greater than the demand rate.
2. The stringent preventive maintenance results in negligible machine breakdown during production and rework period.
3. Production, rework and demand rate are constant.
4. Deteriorating rate is constant.
5. There is no repair or replacement for deteriorated items.
6. Defective items are generated during the production and the rework period. The defective items appear during the rework process are rejected.
7. All shortages are lost.

Notations

I_{1a}	serviceable inventory level in a production time
I_{2a}	serviceable inventory level in a production down time
I_{3a}	serviceable inventory level in a rework up time
I_{3r}	serviceable inventory level from rework up time
I_{4r}	serviceable inventory level from the rework process in a rework down time
I_{r1}	recoverable inventory level in a production time
I_{r3}	recoverable inventory level in a rework up time
TI_{1a}	total serviceable inventory in a production time
TI_{2a}	total serviceable inventory in a production down time
TI_{3a}	total serviceable inventory in a rework up time
TI_{3r}	total serviceable inventory from a rework up time
TI_{4r}	total serviceable inventory from rework process in a rework down time
TTI_{r1}	total recoverable inventory in a production time
TTI_{r3}	total recoverable inventory in a rework up time
T_{1a}	production time
T_{2a}	production down time
T_{3r}	rework up time
T_{4r}	rework down time
T_{sb}	total production down time
T_{1aub}	production time when the total production down time is equal to the upper bound of the uniform distribution parameter
I_m	inventory level of serviceable items at the end of production time
I_{Mr}	maximum inventory level of recoverable items in a production time
I_{Ms}	maximum inventory level of serviceable items in a production time
I_w	total recoverable inventory
D_i	total deteriorated items
p	production rate
p_1	rework process rate
d	demand rate
θ	deteriorating rate
x	product defect rate
x_1	product scrap rate
K_s	production setup cost
C_p	production unit cost
C_r	rework unit cost
C_{in}	inspection unit cost
h	serviceable items holding cost
h_1	recoverable items holding cost
π	deteriorating cost
S_c	scrap cost
S	lost sales cost
TC	total cost
T	total replenishment time
T_3	lost sales time
TCT	total cost per unit time for lost sales model
TCT_{NL}	total cost per unit time for without lost sales model
TCT_U	total cost per unit time for lost sales model with uniform distribution preventive maintenance time
TCT_E	total cost per unit time for lost sales model with exponential distribution preventive maintenance time

The inventory level of serviceable items is illustrated in Fig. 2. The production system is designed using the first in first out (FIFO) rule. Production is performed during the T_{1a} time period. During production period, there are x defective products at each unit time; and the defective products can be reworked. Machine maintenance starts after the predetermined production time. The rework process is planned on the T_{3r} time period. Since the maintenance time is stochastic, there is a possibility that rework process may not be able to start on time; and lost sales may occur during the T_3 period.

The number of inventory in a production time for the serviceable items can be formulated as

$$\frac{dI_1(t_1)}{dt_1}+\theta I_1(t_1)=p-d-x \quad 0\le t_1\le T_{1a} \tag{1}$$

The number of inventory level in a production down time is represented by the following equation:

$$\frac{dI_2(t_2)}{dt_2}+\theta I_2(t_2)=-d \quad 0\le t_2\le T_{2a} \tag{2}$$

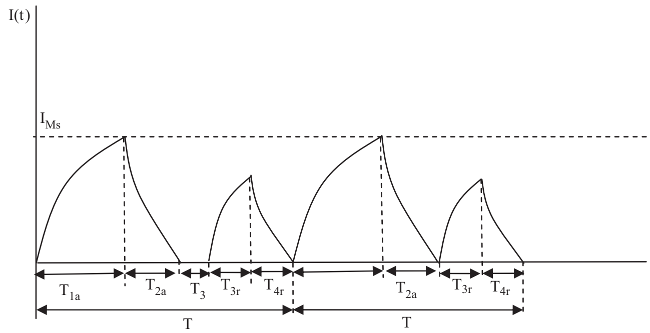

Fig. 2. Serviceable inventory level.

since $I_1(0)=0$, the inventory level in the production period is

$$I_1(t_1)=\frac{p-d-x}{\theta}\left(1-e^{-\theta t_1}\right) \qquad 0\le t_1\le T_{1a} \tag{3}$$

The total inventory in the production time is:

$$TI_1(t_1)=\int_{t_1=0}^{T_{1a}}\frac{p-d-x}{\theta}\left(1-e^{-\theta t_1}\right)dt_1$$

$$TI_1=\frac{(p-d-x)\left(e^{-\theta T_{1a}}-1+\theta T_{1a}\right)}{\theta^2} \tag{4}$$

for a small θT_{1a} value, using Taylor series approximation (Yang and Wee [38], Lo et al. [39]), (4) can be simplified as:

$$TI_1=\frac{(p-d-x)T_{1a}^2}{2} \tag{5}$$

Similarly, the total inventory in a production down time can be formulated as

$$TI_2=\frac{dT_{2a}^2}{2} \tag{6}$$

Since $I_1=I_2$ when $t_1=T_{1a}$ and $t_2=0$, one has:

$$\frac{p-d-x}{\theta}\left(1-e^{-\theta T_{1a}}\right)=\frac{d}{\theta}\left(e^{\theta T_{2a}}-1\right) \tag{7}$$

using the Taylor series approximation, (6) can be rewritten as:

$$(p-d-x)\left(T_{1a}-\frac{1}{2}\theta T_{1a}^2\right)=d\left(T_{2a}+\frac{1}{2}\theta T_{2a}{}^2\right)$$

Since $\theta T_{2a}/2$ is a small number, T_{2a} can be approximated in terms of T_{1a} as

$$T_{2a}\cong\frac{(p-d-x)\left(T_{1a}-1/2\theta T_{1a}^2\right)}{d} \tag{8}$$

The total scraped items depend on the scraped rate and production up time; the total scraped items can be reduced by maintaining production facility regularly. Some papers assumed constant scrap rate (see for example Chiu et al. [6] and Liu et al., [8]). However in practice, scrap rate is not constant. Chiu et al. [12] more realistically assumed the scrap rate as a function of production time. When a machine has been maintained, its performance will initially be as good as new, until after sometime when it will deteriorate with time. The total inventory of rework up time, rework down time and the rework down time can respectively be modeled as follows:

$$TI_{3r}=\frac{(p_1-d-x_1T_{1a})T_{3r}^2}{2} \tag{9}$$

$$TI_{4r}=\frac{dT_{4r}^2}{2} \tag{10}$$

$$T_{4r}\cong\frac{(p_1-d-x_1T_{1a})\left(T_{3r}-1/2\theta T_{3r}^2\right)}{d} \tag{11}$$

The inventory level of recoverable items is shown in Fig. 3. The inventory level of recoverable items in a production time can be modeled as

$$\frac{dI_{r1}(t_{r1})}{dt_{r1}}+\theta I_{r1}(t_{r1})=x \quad 0\le t_{r1}\le T_{1a} \tag{12}$$

since $I_{r1}(0)=0$, the inventory level in the production period is

$$I_{r1}(t_{r1})=\frac{x}{\theta}\left(1-e^{-\theta t_{r1}}\right) \quad 0\le t_{r1}\le T_{1a} \tag{13}$$

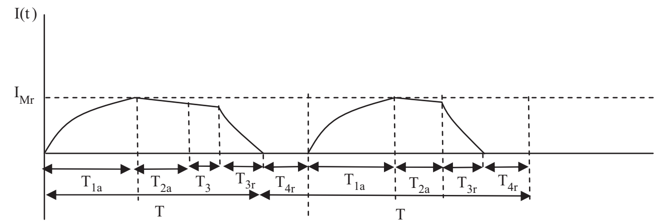

Fig. 3. Recoverable inventory level.

Using Taylor series approximation, the total inventory of recoverable items in a production time is

$$TTI_{r1}=\frac{xT_{1a}^2}{2} \tag{14}$$

The initial recoverable inventory level in the production time is equal to I_{Mr}; and it can be modeled as

$$I_{Mr}=\frac{x}{\theta}\left(1-e^{-\theta T_{1a}}\right) \tag{15}$$

using Taylor series approximation, (15) can be reformulated as

$$I_{Mr}=x\left(T_{1a}-\frac{\theta T_{1a}^2}{2}\right) \tag{16}$$

The inventory level of recoverable items in the production down time can be modeled as

$$\frac{dI_{r2}(t_{r2})}{dt_{r2}}+\theta I_{r2}(t_{r2})=0 \quad 0\le t_{r2}\le T_{2a}+T_3 \tag{17}$$

using (16) and (17), the inventory level of recoverable items in the production down time can be modeled as

$$I_{r2}(t_{r2})=I_{Mr}e^{-\theta\, t_{r2}} \quad 0\le t_{r2}\le T_{2a}+T_3 \tag{18}$$

The total inventory of recoverable items in the production down time can be modeled as

$$TI_{r2}=\int_{t_{r2}=0}^{T_{2a}+T_3} I_{Mr}e^{-\theta t_{r2}}dt_{r2} \tag{19}$$

using Taylor series approximation, one has:

$$TI_{r2}=I_{Mr}\left(T_{2a}+T_3-\frac{\theta(T_{2a}+T_3)^2}{2}\right) \tag{20}$$

Inventory level of recoverable item at the end of the production down time and the maintenance time is

$$I_{Er}=I_{Mr}e^{-\theta(T_{2a}+T_3)} \tag{21}$$

using Taylor series approximation, one has:

$$I_{Er}=I_{Mr}\left(1-\theta(T_{2a}+T_3)-\frac{(\theta(T_{2a}+T_3))^2}{2}\right) \tag{22}$$

substitute I_{Mr} from (17), one has:

$$I_{Er}=x\left(T_{1a}-\frac{\theta T_{1a}^2}{2}\right)\left(1-\theta(T_{2a}+T_3)-\frac{(\theta(T_{2a}+T_3))^2}{2}\right) \tag{23}$$

The inventory level of the recoverable item in the rework up time can be formulated as

$$\frac{dI_{r3}(t_{r3})}{dt_{r3}}+\theta I_{r3}(t_{r3})=-p_1 \quad 0\le t_{r3}\le T_{3r} \tag{24}$$

one has:

$$I_{r3}(t_{r3})=\frac{p_1}{\theta}\left(e^{\theta(T_{3r}-t_{r3})}-1\right) \tag{25}$$

The total inventory of the recoverable items in the rework up time can be formulated as

$$TI_{r3}(t_{r3})=\int_{t_{r3}=0}^{T_{3r}}\frac{p_1}{\theta}\left(e^{\theta(T_{3r}-t_{r3})}-1\right)dt_{r3} \tag{26}$$

using Taylor series approximation, one has:

$$TI_{r3}=\frac{p_1T_{3r}^2}{2} \tag{27}$$

when $t_{r3}=0$, the number of recoverable inventory is equal to I_{Er}. Eq. (25) can be reformulated as

$$\frac{p_1}{\theta}\left(e^{\theta T_{3r}}-1\right)=I_{Er} \tag{28}$$

Since $\theta T_{3r}<<1$, using Taylor series approximation results in

$$T_{3r}=\frac{I_{Er}}{p_1} \tag{29}$$

substituting I_{Er} from (23) into (29), one has:

$$T_{3r}=\frac{1}{p_1}\left(x\left(T_{1a}-\frac{\theta T_{1a}^2}{2}\right)\left(1-\theta(T_{2a}+T_3)-\frac{(\theta(T_{2a}+T_3))^2}{2}\right)\right) \tag{30}$$

The total recoverable inventory can be formulated as

$$TRI=TI_{r1}+TI_{r2}+TI_{r3}$$

$$TRI=\frac{xT_{1a}^2}{2}+I_{Mr}\left(T_{2a}+T_3-\frac{\theta(T_{2a}+T_3)^2}{2}\right)+\frac{p_1T_{3r}^2}{2} \tag{31}$$

The number of deteriorating item is equal to the number of total items produced minus the number of total demands and the number of total rejected items. The number of deteriorating item can be formulated as

$$Di=(p-x)T_{1a}+(p_1-x_1T_1)T_{3r}-d(T_{1a}+T_{2a}+T_{3r}+T_{4r})-x_1T_{1a}T_{3r} \tag{32}$$

lost sales will occur if the maintenance time is longer than the production down time. Lost sales time can be modeled as

$$T_3=\int_{t=T_{2a}}^{\infty}(t-T_{2a})f(t)dt \tag{33}$$

The total cost consists of the setup cost, the production cost, the inspection cost, the rework cost, the maintenance cost, the serviceable inventory cost, the recoverable inventory cost, the deteriorating cost, the rejected cost and the lost sales cost. The total cost per unit time can be modeled as follows:

$$TCT=\frac{K_s+(C_p+C_{in})pT_{1a}+(C_r+C_{in})p_1T_{3r}+M\int_{t=T_{2a}}^{\infty}tf(t)dt+h(TI_1+TI_2+TI_3+TI_4)+h_1TRI+\pi Di+S_cx_1T_{3r}+SdT_3}{T_{1a}+T_{2a}+T_{3r}+T_{4r}+T_3} \tag{34}$$

The optimal solution must satisfy the following condition:

$$\frac{dTCT(T_{1a})}{dT_{1a}}=0 \tag{35}$$

2.1. Uniform distribution maintenance time

Assume that the maintenance time t, is a random variable that is uniformly distributed over the interval [0, b]. The probability density function, $f(t)$, is given as

$$f(t)=\begin{cases}1/b, & 0\le t\le b\\ 0, & \text{otherwise}\end{cases}$$

The expected shortage time can be written as

$$E(T_3)=\frac{1}{b}\int_{t=T_{2a}}^{b}(t-T_2a)dt$$

$$E(T_3)=\frac{\left(b-\frac{(p-d)T_{1a}(1-0.5T_{1a})}{d}\right)^2}{2b} \tag{36}$$

The total cost per unit time for uniform distribution maintenance time can be modeled as

$$E(TCT_U)=\frac{K_s+(C_p+C_{in})pT_{1a}+(C_r+C_{in})p_1T_{3r}+\frac{Mb}{2}+h(TI_1+TI_2+TI_3+TI_4)+h_1TRI+\pi Di+S_cx_1T_{1a}T_{3r}+\frac{Sd\left(b-\frac{(p-d)T_{1a}(1-0.5T_{1a})}{d}\right)^2}{2b}}{T_{1a}+T_{2a}+T_{3r}+T_{4r}+\frac{\left(b-\frac{(p-d)T_{1a}(1-0.5T_{1a})}{d}\right)^2}{2b}} \tag{37}$$

Substituting all the time variables in (37) in terms of T_{1a}, the optimal T_{1a} can be derived when the following equation is satisfied.

$$\frac{dTCT_U(T_{1a})}{dT_{1a}}=0 \tag{38}$$

Lost sales will not occur if the production down time is longer than the maximum maintenance time (b). If this condition occurs, then (37) can be rewritten as

$$E(TCT_{NL})=\frac{K_s+(C_p+C_{in})pT_{1a}+(C_r+C_{in})pT_{3r}+h(TI_1+TI_2+TI_3+TI_4)+h_1TRI+\pi Di+S_cx_1T_{1a}T_{3r}}{T_1+T_2+T_3+T_4} \tag{39}$$

where:

$$TRI=\frac{xT_{1a}^2}{2}+I_{Mr}\left(T_{2a}-\frac{\theta(T_{2a})^2}{2}\right)+\frac{p_1T_{3r}^2}{2} \tag{40}$$

and

$$T_{3r}=\frac{1}{p_1}\left(x\left(T_{1a}-\frac{\theta T_{1a}^2}{2}\right)\left(1-\theta T_{2a}-\frac{(\theta T_{2a})^2}{2}\right)\right) \tag{41}$$

The optimal solution is derived when the following equation is satisfied:

$$\frac{dTCT_{NL}(T_{1a})}{dT_{1a}}=0 \tag{42}$$

for the case with uniform distribution and lost sales, we develop a solution procedure as follows:

Step 1 Calculate (42) and (8); set $T_{sb}=T_{2a}$.
Step 2 If $T_{sb}<b$, the solution is not feasible; go to step 3. Otherwise, optimal solution is derived.
Step 3 Set $T_{2a}=b$, find T_{1aub} using (8), and calculate $TCT_{NL}(T_{1aub})$ from (39).
Step 4 Find T_{1a} by solving (38), and T_{2a} by solving (8); then set $T_{sc}=T_{2a}$.
Step 5 If $T_{sc}\geq b$, then $T^*_{1a}=T_{1aub}$. If $T_{sc}<b$, then calculate $TCT_u(T_{1a})$ using (39).
Step 6 If $TCT_{NL}(T_{1aub})<TCT_U(T_{1a})$, then $T^*_{1a}=T_{1aub}$, otherwise $T^*_{1a}=T_{1a}$.

2.2. Exponential distribution maintenance time

For the second case, the unavailability machine time is a random variable that is distributed exponentially. Exponential probability density function with mean $1/\lambda$, is given as

$$f(t)=\lambda e^{-\lambda t} \quad for \;\; \lambda>0$$

The expected machine unavailability time is

$$E(T_3)=\int_{t=T_{2a}}^{\infty}(t-T_{2a})\lambda e^{-\lambda t}dt$$

$$E(T_3)=\frac{e^{-\lambda T_{2a}}}{\lambda} \tag{43}$$

substituting T_{2a}, one has:

$$E(T_3)=\frac{e^{-\lambda\left(\frac{(p-d-x)\left(T_{1a}-\frac{1}{2}\theta T_{1a}^2\right)}{d}\right)}}{\lambda} \tag{44}$$

substituting (44) into (34) and do some simplification, one has:

$$E(TCT_E)=\frac{K_s+(C_p+C_{in})pT_{1a}+(C_r+C_{in})p_1T_{3r}+\frac{M}{\lambda}}{T_{1a}+T_{2a}+T_{3r}+T_{4r}+(e^{-\lambda(((p-d-x)(T_{1a}-1/2\theta T_{1a}^2))/d)})/\lambda}$$
$$+\frac{h(TI_1+TI_2+TI_3+TI_4)+h_1TRI+\pi Di+S_cx_1T_{1a}T_{3r}+(Sde^{-\lambda(((p-d-x)(T_{1a}-0.5T_{1a}^2))/d)})/\lambda}{T_{1a}+T_{2a}+T_{3r}+T_{4r}+(e^{-\lambda(((p-d-x)(T_{1a}-0.5T_{1a}^2))/d)})/\lambda} \tag{45}$$

The optimal solution is derived when:

$$\frac{dTCT_E(T_{1a})}{dT_{1a}}=0 \tag{46}$$

Since a closed form solution cannot be derived, the problem is solved using a simple search approach such as bisection or Newton's search method. The cost functions (38), (42) and (46) are nonlinear equations and the second derivative with respect to T_{1a} is extremely complicated. This means the optimal solution can be guaranteed only in certain conditions as shown in the appendix.

3. Numerical examples and sensitivity analysis

In this section, a numerical example and sensitivity analysis are given to illustrate this model. Let K_s=\$400 per production setup, p=10,000 units per unit time, p_1=9000 units per unit time, d=6000 units per unit time, x=500 units per unit time, x_1=400 units per unit time h=\$5 per unit per unit time, h_1=\$3 per unit per unit time, S=\$15 per unit, π=\$5 per unit, S_c=\$15 per unit, C_p=\$10 per unit, C_r=\$4 per unit, C_{in}=\$0.5 per unit, M=\$500 per unit time and θ=0.05. The total cost per unit time for varying T_{1a} is shown in Fig. 4.

Fig. 4 shows that the total cost per unit time is convex for small values of T_{1a}. The optimal total cost is equal to \$70320.31 and ${T_{1a}}^*$=0.3593.

The sensitivity analysis is performed by changing each of the parameters by −20%, −10%, +10% and +20%. One parameter is taken at a time and the remaining parameters are kept constant. The optimal production time ${T_{1a}}^*$ and the optimal total cost per unit time for varying parameters are shown in Table 1.

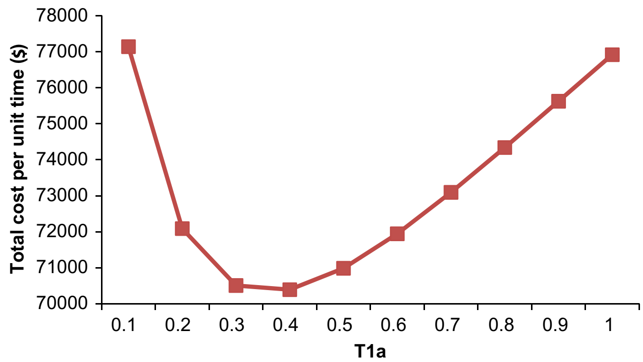

Fig. 4. Variation of total cost per unit time vs. changing production time, T_{1a}.

Table 1
Sensitivity analysis of $T_{1a}{}^{*}$ and total cost per unit time: for uniform distribution.

Parameter	−20% changed		−10% changed		+10% changed		+20% changed	
	$T_{1a}{}^{*}$	Total cost	$T_{1a}{}^{*}$	Total cost	$T_{1a}{}^{*}$	Total cost	$T_{1a}{}^{*}$	Total cost
K_s	0.356	70191.26	0.3576	70255.91	0.3609	70384.47	0.3626	70448.39
p	0.5657	70478.77	0.4395	70390.07	0.3038	70264.46	0.2631	70218.82
d	0.2485	56587.60	0.2993	63461.23	0.4314	77157.99	0.5202	83964.03
h	0.3814	69728.11	0.3699	70029.30	0.3493	70601.78	0.3399	70874.26
p_1	0.385	70342.96	0.3589	70330.41	0.3596	70312.03	0.3598	70305.10
h_1	0.3651	70159.00	0.3622	70240.03	0.3564	70399.87	0.3537	70478.70
π	0.3598	70306.72	0.3595	70313.52	0.3590	70327.10	0.3588	70333.89
θ	0.3612	70253.42	0.3603	70286.90	0.3583	70353.66	0.3573	70386.94
x	0.3629	69993.37	0.3611	70156.88	0.3574	70483.67	0.3556	70646.95
x_1	0.3600	70300.87	0.3596	70310.60	0.3589	70330.02	0.3586	70339.72
S	0.3598	70306.76	0.3595	70313.54	0.3590	70327.08	0.3588	70333.85
b	0.3000	69478.98	0.3297	69894.91	0.3886	70751.37	0.4176	71185.32
C_p	0.4085	58646.08	0.3864	64510.75	0.3284	76044.82	0.2792	81626.19
C_r	0.3604	70094.06	0.3598	70207.20	0.3587	70433.4	0.3582	70546.57
C_{in}	0.3624	69714.23	0.3609	70017.38	0.3577	70623.02	0.3561	70925.49
M	0.3585	70288.14	0.3589	70304.24	0.3597	70336.38	0.3601	70352.42
S_c	0.3598	70306.76	0.3595	70313.54	0.3590	70327.08	0.3588	70333.85

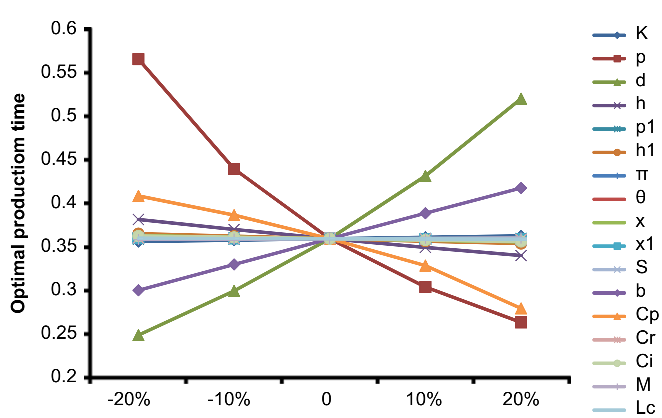

Fig. 5. Optimal production time T_{1a}^{*} for varying parameters: uniform distribution case.

Table 1 shows that the optimal production time ($T_{1a}{}^{*}$) increases as the parameter K_s, d, p_1, b, M, *and* S_c increase, and decreases as the other parameters increase. Parameters b, M, and S_c affect the lost sales cost. Thus, a manufacturer has to increases the production time when these parameters increase. The table also shows that the total cost per unit time decreases as the parameters p and p_1 increase, and decrease as the other parameters increase. The result shows that a manufacturer can reduce his total cost by optimizing the optimal production rate (p) and rework rate (p_1).

The optimal production period for the varying parameters is shown in Fig. 5. The figure shows that the optimal production period is sensitive to changes in parameters p and d, moderately sensitive to changes in parameters C_p, S_c, b and h and insensitive to changes in the other parameters. The production rate (p) and the demand rate (d) are the main parameters in our model. In practice, production and demand quantity have direct correlation to the production and inventory costs.

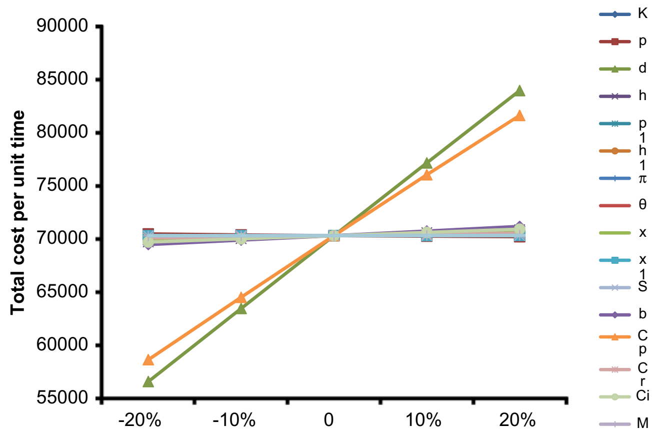

Fig. 6. Total cost per unit time for varying parameters: uniform distribution case.

Table 2
Sensitivity analysis of T_{1a}* and total cost per unit time: for exponential distribution.

Parameter	−20% changed		−10% changed		+10% changed		+20% changed	
	T_{1a}*	Total cost	T_{1a}*	Total cost	T_{1a}*	Total cost	T_{1a}*	Total cost
K_s	0.2125	68050.61	0.2143	68162.22	0.2181	68382.75	0.2201	68491.65
p	0.4149	68700.64	0.2820	68396.54	0.1763	68209.06	0.1493	68171.13
d	0.1444	55051.16	0.1757	61656.37	0.2706	74915.31	0.3477	81609.77
h	0.2238	67910.09	0.2198	68093.18	0.2128	68449.62	0.2097	68623.49
p_1	0.2159	68286.93	0.2160	68279.17	0.2163	68267.81	0.2164	68263.53
h_1	0.2178	68189.34	0.2172	68222.86	0.2152	68322.77	0.2143	68372.37
π	0.2163	68264.49	0.2163	68268.71	0.2161	68277.15	0.2160	68281.36
θ	0.2168	68232.39	0.2165	68252.67	0.2159	68293.18	0.2156	68313.42
x	0.2139	67930.7	0.2150	68101.38	0.2174	68445.39	0.2186	68618.78
x_1	0.2164	68260.9	0.2163	68266.92	0.2161	68278.94	0.2159	68284.95
S	0.2163	68264.44	0.2163	68268.69	0.2161	68277.17	0.2160	68281.41
λ	0.2458	68589.11	0.2296	68411.71	0.2049	68163.29	0.1952	68076.11
C_p	0.2284	56333.35	0.2228	62307.83	0.2078	74223.93	0.1962	80150.86
C_r	0.2164	68037.37	0.2163	68155.16	0.2160	68390.71	0.2159	68508.48
C_{in}	0.2169	67647.50	0.2166	67960.24	0.2158	68585.59	0.2154	68898.21
M	0.2161	68266.81	0.2161	68269.87	0.2162	68275.99	0.2163	68279.06
S_c	0.1783	67981.93	0.2031	68166.69	0.2252	68349.22	0.2322	68409.23

Fig. 6 shows that the demand parameter (d) and the production unit cost (C_p) have the biggest effect on the total cost per unit time. Thus, reducing the production unit cost will result in a significant saving.

From the working paper by Yang et al. [40] considering deteriorating items with shortage lost sale, it is noted that the profit for FIFO policy is either equal or greater than LIFO policy for a complete lost sale policy. However, in our research, we do not try to explain why one rule becomes better than the other in our paper since it is beyond the scope our research. From common sense, one may adopt FIFO policy for the sake of achieving fairness and freshness. For more detail, see Abboud et al. [23].

The optimal production time and the total cost per unit time are shown in Table 2. It shows that the optimal production time (T_{1a}*) is increasing as K_s, d, p_1, x, M, and S_c parameters increase. Fig. 7 shows the total cost per unit time for varying parameters. The total cost per unit time is sensitive to changes in parameters p and d, moderately sensitive to changes in parameters λ, S_c, and C_p, and insensitive to changes in the other parameters. The total cost per unit time for the exponential distribution case is also significantly influenced by the demand (d) and production unit cost (C_p) parameters. The optimal total cost sensitivity analysis for exponential distribution preventive maintenance time is similar to the uniform distribution preventive maintenance time as shown in Fig. 8.

4. Conclusion

In this study, we have developed a deteriorating production inventory model suitable for manufacturers with stochastic preventive maintenance time and rework process using FIFO rule. The cases for uniform and exponential distribution are conducted to illustrate the model. The sensitivity analysis is carried out to show the robust behavior of the model facing expected variations of the system parameters. Though the sensitivity analysis is not sufficient to validate the model, it is a very common practice by many others inventory researchers to show the robustness of the inventory model. The sensitivity analysis from our study shows that the optimal production period in sensitive to production rate and demand rate. Since demand rate is uncontrollable, to minimize the lost sales cost, a manufacturer should increase the production time whenever the preventive maintenance time and cost increase. However, it is not easy to solve production up time and production rate simultaneously using analytical solution because the machine unavailability time is stochastic. Therefore, the problem is solved using a simple search procedure. For future research, one may try to solve the problem using meta-heuristics methods.

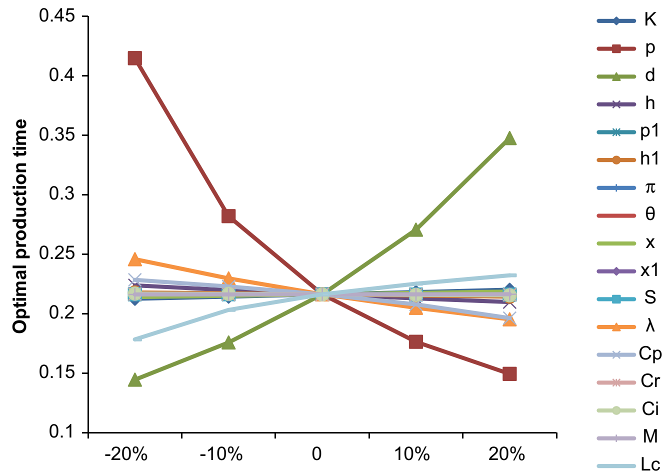

Fig. 7. Optimal production time $T_{1a}{}^{*}$ for varying parameters: exponential distribution case.

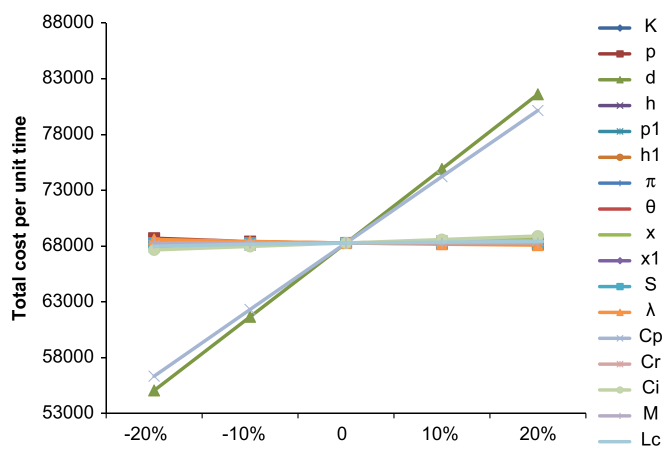

Fig. 8. Total cost per unit time for varying parameters: exponential distribution case.

Appendix

Convexity test of the uniform distribution maintenance time

Simplifying second order of the total cost with respect to T_{1a} and set $T_{1a}=0$, one has:

$$\frac{\partial^2 TCT(T_{1a})}{\partial {T_{1a}}^2} = \frac{-2(C_r+C_{in})x\theta(Y/d-2b(p-d)/d)+h_s\left(Y+Y^2/d+(d+Y_1)Y_1x^2\left(1-\theta b^2\right)^2/dp_1^2\right)+h_r\left(2x(2+(Y/d))+x^2\left(1-\theta b^2\right)^2/p_1\right)}{b^2}$$
$$+\frac{D_c\theta(Y+Y_1x^2/p_1^2)/d-2Sx_1x\theta(Y/d-2b(p-d)/d)/p_1+2S_c(p-d)^2/d}{b^2}-\frac{2\left((C_p+C_{in})p+(C_r+C_{in})x\left(1-\theta b^2\right)-D_cx_1x/p_1+Sx_1x\left(1-\theta b^2\right)/p_1-2S_cb(p-d)\right)}{b^4}V_0$$
$$+\frac{2\left(K_s+K_r+Mb/2+S_cdb^2\right)V_0^2}{b^6}-\frac{\left(K_s+K_r+Mb/2+S_cdb^2\right)\left(-(d+Y_1)\left(x\theta\left(Y/d-2b(p-d)^2/d^2\right)\right)+2(p-d)^2/d^2\right)}{b^4}>0 \tag{A1}$$

where

$$V_0=\left(1+\frac{Y}{d}+\frac{x(1-\theta b^2)(d+Y_1)}{p_1d}-\frac{2b(p-d)}{d}\right)$$
$$Y=(p-d-x)$$
$$Y_1=(p_1-d-x_1)$$

if (A1) condition can be satisfied then the optimal the solution is convex when $T_{1a}=0$.

Simplifying second order of the total cost with respect to T_{1a} and set $T_{1a}=b$, where b is the upper bound of uniform distribution, one has:

$$\frac{\partial^2 TCT(T_{1a})}{\partial {T_{1a}}^2} = \frac{-2(C_r+C_{in})\left(x\theta(Y/d-2(-b(p-2d)/d)(p-d))-xb\theta(p-d)^2/d^2\right)+h_s(W_1)+h_r(W_2)+D_c\theta/d(Y+Y_1x^2/p_1^2)-2Sxx_1\left(W_2/b+b(p-d)^2/d^2p_1\right)+2S_c(p-d)^2/d}{W_3}$$

$$-2\left(\frac{(C_p+C_{in})p+(C_r+C_{in})\left(x(1-V_1)+xb\theta\left(Y/d-2\left(-b(p-2d)/d\right)(p-d)\right)\right)+h_s(W_4)+h_r(W_5)+D_c\left(\theta b^2/2d\left(Y+Y_1x^2/p_1^2\right)-bxx_1/p_1\right)-Sxx_1b(1-V_1)/p_1-S_cd\left(-b(p-2d)/d\right)(p-d)}{{W_3}^2}\right)W_6$$
$$+\frac{\left(1+Y/d+x(d+Y_1)(1-V_1-V_2)/dp_1-2b(p-2d)(p-d)/d^2\right)^2W_9}{W_3^3}$$
$$-\frac{\left(-2x\theta/dp_1(d+Y_1)(Y-2b(p-2d)(p-d))-2x\theta b(p-d)^2/d^2p_1+2(p-d)^2/d^2\right)W_9}{W_3^2}>0 \quad \text{(A2)}$$

where

$$W_1=Y+\frac{Y^2}{d}+\frac{Y_1x^2(d+Y_1)\left(((1-V_1)-V_2)^2-2((1-V_1)V_2)\right)}{dp_1^2}-\frac{2Y_1x^2b^2(d+Y_1)(1-V_1)\theta(p-d)^2}{d^3p_1^2}$$
$$W_2=2x\left(2+\frac{Y}{d}\right)+\frac{x^2\left(((1-V_1)-V_2)^2-2(1-V_1)V_2)\right)}{p_1}-\frac{2x^2b^2(1-V_1)\theta(p-d)^2}{d^2p_1}$$
$$W_3=b+\frac{Yb}{d}+\frac{xb(d+Y_1)(1-V_1)}{p_1}+\left(b-\frac{b(p-d)}{d}\right)^2$$
$$W_4=Yb+\frac{Y^2b}{d}+\frac{Y_1x^2b(d+Y_1)\left((1-V_1)^2-((1-V_1)V_2)\right)}{dp_1^2}$$
$$W_5=2xb\left(1+\frac{Y}{d}\right)+\frac{x^2b\left((1-V_1)^2-(1-V_1)V_2)\right)}{p_1}$$
$$W_6=1+\frac{Y}{d}+\frac{x(d+Y_1)((1-V_1)-V_2)}{dp_1}-\frac{2\left(b-b(p-d)/d\right)(p-d)}{d}$$
$$W_7=\frac{Yb^2}{2}\left(1+\frac{b}{d}\right)+\frac{Y_1x^2b^2(d+Y_1)(1-V_1)}{2dp_1^2}$$
$$W_8=xb\left(2+\frac{Y}{d}\right)+\frac{x^2b^2(1-V_1)^2}{2p_1}$$
$$W_9=2\left(\frac{K_s+K_r+Mb/2+(C_p+C_{in})pb+(C_r+C_{in})xb(1-V_1)+h_s(W_7)+h_r(W_8)+D_c\left(\theta b/d\left(Y+Y_1x^2/p_1^2\right)-xx_1/p_1\right)+2Sxx_1(1-V_1-V_2)/p_1+S_cd\left(b(p-2d)/d\right)^2}{{W_3}^2}\right)$$
$$V_1=\theta\left(\frac{Yb}{d}+\left(b-\frac{b(p-d)}{d}\right)^2\right)$$
$$V_2=\frac{b\theta}{d}\left(Y-2\left(b-\frac{b(p-d)}{d}\right)(p-d)\right)$$

Calculating (A2) in two parts, one has the first part as

$$\frac{-2(C_r+C_{in})\left(x\theta\left(Y/d-2\left(-b(p-2d)/d\right)(p-d)\right)-xb\theta(p-d)^2/d^2\right)+h_s(W_1)+h_r(W_2)+D_c\theta/d\left(Y+Y_1x^2/p_1^2\right)-2Sxx_1\left(W_2/b+b(p-d)^2/d^2p_1\right)+2S_c(p-d)^2/d}{W_3}$$
$$>2\left(\frac{(C_p+C_{in})p+(C_r+C_{in})\left(x(1-V_1)+xb\theta\left(Y/d-2\left(-b(p-2d)/d\right)(p-d)\right)\right)+h_s(W_4)+h_r(W_5)+D_c\left(\theta b^2/2d\left(Y+Y_1x^2/p_1^2\right)-bxx_1/p_1\right)-Sxx_1b(1-V_1)/p_1-S_cd\left(-b(p-2d)/d\right)(p-d)}{{W_3}^2}\right)W_6 \quad \text{(A3)}$$

simplifying (A3), one has:

$$\left(-2(C_r+C_{in})\left(x\theta\left(\tfrac{Y}{d}-2\left(-\tfrac{b(p-2d)}{d}\right)(p-d)\right)-\tfrac{xb\theta(p-d)^2}{d^2}\right)+h_s(W_1)+h_r(W_2)+\tfrac{D_c\theta}{d}\left(Y+\tfrac{Y_1x^2}{p_1^2}\right)-2Sxx_1\left(\tfrac{W_2}{b}+\tfrac{b(p-d)^2}{d^2p_1}\right)+\tfrac{2S_c(p-d)^2}{d}\right)W_3$$
$$>2\left((C_p+C_{in})p+(C_r+C_{in})\left(x(1-V_1)+xb\theta\left(\frac{Y}{d}-2\left(-\frac{b(p-2d)}{d}\right)(p-d)\right)\right)+h_s(W_4)+h_r(W_5)+D_c\left(\frac{\theta b^2}{2d}\left(Y+\frac{Y_1x^2}{p_1^2}\right)-\frac{bxx_1}{p_1}\right)\right.$$
$$\left.-\frac{Sxx_1b(1-V_1)}{p_1}-S_cd\left(-\frac{b(p-2d)}{d}\right)(p-d)\right)W_6 \quad \text{(A4)}$$

The total cost per unit time is convex if (A4) is fulfilled. (A4) has bigger possibility to be fulfilled if C_p is not too big.

The second part of (A2) can be written as follows:

$$\frac{\left(1+Y/d+x(d+Y_1)(1-V_1-V_2)/dp_1-2b(p-2d)(p-d)/d^2\right)^2W_9}{W_3^3}$$
$$>\frac{\left(-2x\theta/dp_1(d+Y_1)(Y-2b(p-2d)(p-d))-2x\theta b(p-d)^2/d^2p_1+2(p-d)^2/d^2\right)W_9}{W_3^2} \quad \text{(A5)}$$

simplifying (A5), one has:

$$\left(1+\frac{Y}{d}+x\frac{(d+Y_1)(1-V_1-V_2)}{dp_1}-\frac{2b(p-2d)(p-d)}{d^2}\right)^2>\left(-\frac{2x\theta}{dp_1}(d+Y_1)(Y-2b(p-2d)(p-d))-\frac{2x\theta b(p-d)^2}{d^2p_1}+\frac{2(p-d)^2}{d^2}\right) \quad \text{(A6)}$$

when $p-d<d$, (A6) will be true.

Since the second order of total cost per unit time respect to T_{1a} is a non-increasing equation, then the total cost per unit time is convex when $0\le T_{1a}\le b$, and conditions in (A1), (A4) and (A6) are fulfilled.

Convexity test of the exponential distribution maintenance time

The second order of the total cost per unit time for exponential distribution case respect to T_{1a} can be written as follows:

$$\frac{\partial^2 TCT(T_{1a})}{\partial {T_{1a}}^2}=\frac{-(C_r+C_{in})x\theta E_{x3}+h_sE_{x4}+h_rE_{x5}+D_c\theta/d(Y+Y_1x^2/p_1^2)-Sxx_1E_{x3}/p_1+S_c\lambda^2Y^2e^{-(\lambda YT_{1a}/d)/d}}{E_{x10}}$$
$$-\frac{2\left((C_p+C_{in})p+(C_r+C_{in})x(1-\theta T_{1a})(E_{x1}-T_{1a}E_{x2})+h_sE_{x6}+h_rE_{x7}+D_c\left(\theta YT_{1a}/d(Y+Y_1x^2/p_1^2)-x_1x/p_1\right)+Sxx_1/p_1((1-\theta T_{1a})E_{x1}-T_{1a}(1-0.5T_{1a})E_{x2})-S_c\lambda Ye^{-(\lambda YT_{1a/d})}\right)E_{x8}}{{E_{x10}}^2}$$
$$+\frac{\left(1+Y/d+x(d+Y_1)((1-\theta T_{1a})E_{x1}-T_{1a}(1-0.5\theta T_{1a})E_{x2})/p_1d-\lambda Ye^{-(\lambda YT_{1a}/d)/d}\right)^2E_{x9}}{{E_{x10}}^3}$$
$$-\frac{\left(x(d+Y_1)\left(\theta E_{x1}+(1-\theta T_{1a})E_{x2}/dp_1+T_{1a}(1-\theta T_{1a})\theta\lambda^2Y^2e^{-(\lambda YT_{1a}/d)}/d^2p_1\right)\right)E_{x9}}{{E_{x10}}^2}>0 \quad \text{(A7)}$$

where

$$E_{x1}=1-\theta\left(\frac{YT_{1a}}{d}+e^{-(\lambda YT_{1a}/d)}\right)$$
$$E_{x2}=\frac{\theta}{d}\left(Y+\lambda Ye^{-(\lambda YT_{1a}/d)}\right)$$
$$E_{x3}=\left(E_{x1}+2(1-\theta T_{1a})E_{x2}+\frac{T_{1a}(1-0.5\theta T_{1a})\theta\lambda^2Y^2e^{-(\lambda YT_{1a}/d)}}{d^2}\right)$$
$$E_{x4}=Y(1+Y/d)+Y_1x^2(d+Y_1)\left(\frac{(1-\theta T_{1a})^2E_{x1}^2-4T_{1a}(1-0.5T_{1a})(1-\theta T_{1a})E_{x1}E_{x2}-(1-0.5T_{1a})\theta E_{x1}+{T_{1a}}^2(1-0.5T_{1a})^2E_{x2}^2}{dp_1^2}\right.$$
$$\left.-\frac{{T_{1a}}^2(1-0.5T_{1a})E_{x1}\theta\lambda^2Y^2e^{-(\lambda YT_{1a}/d)}}{d^3p_1^2}\right)$$
$$E_{x5}=2x(1+(1-1.5\theta T_{1a})(1+Y/d))+x^2\left(\frac{(1-\theta T_{1a})^2E_{x1}^2-4T_{1a}(1-0.5T_{1a})(1-\theta T_{1a})E_{x1}E_{x2}-(1-0.5T_{1a})\theta E_{x1}+{T_{1a}}^2(1-0.5T_{1a})^2E_{x2}^2}{p_1}\right.$$
$$\left.-\frac{{T_{1a}}^2E_{x1}(1-0.5T_{1a})\theta\lambda^2Y^2e^{-(\lambda YT_{1a}/d)}}{d^2p_1}\right)$$
$$E_{x6}=YT_{1a}(1+Y/d)+Y_1x^2(d+Y_1)\left(\frac{T_{1a}(1-0.5\theta T_{1a})(1-\theta T_{1a})E_{x1}^2-{T_{1a}}^2(1-0.5T_{1a})^2E_{x1}E_{x2}}{dp_1^2}\right)$$
$$E_{x7}=2xT_{1a}(1+(1-1.5\theta T_{1a})(1+Y/d))+x^2\left(\frac{(1-\theta T_{1a})(1-0.5T_{1a})E_{x1}^2-{T_{1a}}^2(1-0.5T_{1a})^2E_{x1}E_{x2}}{p_1}\right)$$
$$E_{x8}=(1+Y/d)+x(d+Y_1)\left(\frac{(1-\theta T_{1a})E_{x1}-T_{1a}(1-0.5T_{1a})E_{x2}}{dp_1}-\frac{\lambda Ye^{-(\lambda YT_{1a}/d)}}{d}\right)$$
$$E_{x9}=K_s+K_r+\frac{M}{\lambda}+(C_p+C_{in})pT_{1a}+(C_r+C_{in})xT_{1a}(1-0.5T_{1a})E_{x1}+h_s\left(\frac{YT_{1a}^2}{2}\left(1+\frac{Y}{d}\right)+\frac{(d+Y_1)\left(T_{1a}-0.5{T_{1a}}^2\right)^2{E_{x1}}^2}{p_1^2}\right)$$
$$+h_r\left(x^2T_{1a}^2(1+(1-0.5T_{1a})(1+Y/d))+\frac{x^2\left(T_{1a}-0.5{T_{1a}}^2\right)^2{E_{x1}}^2}{2}\right)+D_c\left(\frac{\theta{T_{1a}}^2}{2d}\left(Y+\frac{Y_1x^2}{p_1^2}\right)-\frac{x_1xT_{1a}}{p_1}\right)$$
$$+\frac{Sxx_1}{p_1}(T_{1a}(1-0.5\theta T_{1a})E_{x1})-S_cde^{-(\lambda YT_{1a}/d)}$$
$$E_{x10}=T_{1a}(1+Y/d)+x(d+Y_1)\left(\frac{T_{1a}(1-0.5\theta T_{1a})E_{x1}}{dp_1^2}\right)+e^{-(\lambda YT_{1a}/d)}$$

Eq. (A7) can be rewritten to two equation as (A8) and (A9):

$$\frac{2\left((C_p+C_{in})p+(C_r+C_{in})x(1-\theta T_{1a})(E_{x1}-T_{1a}E_{x2})+h_sE_{x6}+h_rE_{x7}+D_c\left(\theta YT_{1a}/d(Y+Y_1x^2/p_1^2)-(x_1x/p_1)\right)+(Sxx_1/p_1)((1-\theta T_{1a})E_{x1}-T_{1a}(1-0.5T_{1a})E_{x2})-S_c\lambda Ye^{-(\lambda YT_{1a/d})}\right)E_{x8}}{{E_{x10}}^2}$$
$$<\frac{-(C_r+C_{in})x\theta E_{x3}+h_sE_{x4}+h_rE_{x5}+\frac{D_c\theta}{d}(Y+Y_1x^2/p_1^2)-Sxx_1E_{x3}/p_1+S_c\lambda^2Y^2e^{-(\lambda YT_{1a}/d)/d}}{E_{x10}} \quad \text{(A8)}$$

$$\frac{\left(1+Y/d+x(d+Y_1)((1-\theta T_{1a})E_{x1}-T_{1a}(1-0.5\theta T_{1a})E_{x2})/p_1d-\lambda Ye^{-(\lambda YT_{1a/d})}/d\right)^2E_{x9}}{{E_{x10}}^3}$$

$$> \frac{\left(x(d+Y_1)\left(\theta E_{x1}+(1-\theta T_{1a})E_{x2}/dp_1+T_{1a}(1-\theta T_{1a})\theta\lambda^2 Y^2 e^{-(\lambda Y T_{1a/d})}/d^2 p_1\right)\right)E_{x9}}{E_{x10}{}^2} \tag{A9}$$

Eq. (A8) can be simplified as

$$2\left((C_p+C_{in})p+(C_r+C_{in})x(1-\theta T_{1a})(E_{x1}-T_{1a}E_{x2})+h_sE_{x6}+h_rE_{x7}+D_c\left(\frac{\theta Y T_{1a}}{d}\left(Y+\frac{Y_1x^2}{p_1^2}\right)-\frac{x_1x}{p_1}\right)\right.$$
$$\left.+\frac{Sxx_1}{p_1}((1-\theta T_{1a})E_{x1}-T_{1a}(1-0.5T_{1a})E_{x2})-S_c\lambda Y e^{-\left(\frac{\lambda Y T_{1a}}{d}\right)}\right)E_{x8}$$
$$<\left(-(C_r+C_{in})x\theta E_{x3}+h_sE_{x4}+h_rE_{x5}+\frac{D_c\theta}{d}\left(Y+\frac{Y_1x^2}{p_1^2}\right)-\frac{Sxx_1E_{x3}}{p_1}+\frac{S_c\lambda^2Y^2e^{-(\lambda Y T_{1a}/d)}}{d}\right)E_{x10} \tag{A10}$$

simplifying (A9), one has:

$$\left(1+\frac{Y}{d}+\frac{x(d+Y_1)((1-\theta T_{1a})E_{x1}-T_{1a}(1-0.5\theta T_{1a})E_{x2})}{p_1 d}-\frac{\lambda Y e^{-(\lambda Y T_{1a}/d)}}{d}\right)^2$$
$$>\left(x(d+Y_1)\left(\frac{\theta E_{x1}+(1-\theta T_{1a})E_{x2}}{dp_1}+\frac{T_{1a}(1-\theta T_{1a})\theta\lambda^2Y^2e^{-(\lambda Y T_{1a}/d)}}{d^2p_1}\right)\right)E_{x10} \tag{A11}$$

The total cost per unit time for exponential distribution case is convex when (A10) and (A11) can be fulfilled.

References

[1] Schrady DA. A deterministic inventory model for repairable items. Naval Research Logistics 1967;48:484–95.
[2] Chung KJ, Hou KL. An optimal production runt time with imperfect production processes and allowable shortages. Computers and Operations Research 2003;30:486–90.
[3] Teunter R. Lot-sizing for inventory systems with product recovery. Computers & Industrial Engineering 2004;46:431–41.
[4] Widyadana GA, Wee HM. Revisiting lot sizing for an inventory system with product recovery. Computers and Mathematics with Applications 2010;59(8):2933–9.
[5] Buscher U, Lindner G. Optimizing a production system with rework and equal sized batch shipments. Computers & Operations Research 2007;34:515–35.
[6] Chiu SW, Wang SL, Chiu YSP. Determining the optimal run time for EPQ model with scrap, rework, stochastic breakdowns. European Journal of Operational Research 2007;180:664–76.
[7] Chiu SW, Ting CK, Chiu YS. Optimal production lot sizing with rework, scrap rate and service level constraint. Mathematical and Computer Modelling 2007;46:535–49.
[8] Liu N, Kim Y, Hwang H. An optimal operating policy for the production system with rework. Computers & Industrial Engineering 2009;56:874–87.
[9] Cardenas-Barron LE. Economic production quantity with rework process at a single-stage manufacturing system with planned backorders. Computers & Industrial Engineering 2009;57:1105–13.
[10] Sana SS. A production-inventory model in an imperfect production process. European Journal of Operational Research 2010;200:451–64.
[11] Chiu SW. Robust planning in optimization for production system subject to random machine breakdown and failure in rework. Computers & Operations Research 2010;37:899–908.
[12] Chiu YSP, Chen KK, Cheng FT, Wu MF. Optimization of the finite production rate with scrap, rework and stochastic machine breakdown. Computers and Mathematics with Applications 2010;59:919–32.
[13] Flapper SDP, Teunter RH. Logistic planning of rework with deteriorating work-in-process. International Journal of Production Economics 2004;51:51–9.
[14] Inderfuth K, Janiak A, Kovalyov MK, Werner F. Batching work and rework processes with limited deterioration of reworkables. Computers & Operations Research 2006;33:1595–605.
[15] Inderfuth K, Lindner G, Rachaniotis NP. Lot sizing in a production system with rework and product deterioration. International Journal of Production Research 2005;43(7):1355–74.
[16] Lin GC, Gong DC. On a production inventory system of deteriorating items subject to random machine breakdowns with a fixed repair time. Mathematical and Computer Modelling 2006;43:920–32.
[17] Wee HM, Yu J, Chen MC. Optimal inventory model for items with imperfect quality and shortage backordering. OMEGA 2007;35:7–11.
[18] Chang HC, Ho CH. Exact closed-form solutions for optimal inventory model for items with imperfect quality and shortage backordering. OMEGA 2010;38:233–7.
[19] Wee HM, Chung CJ. Optimizing replenishment policy for an integrated production inventory deteriorating model considering green component-value design and remanufacturing. International Journal of Production Research 2009;47:1343–68.
[20] Yang PC, Wee HM, Chung CL, Ho PC. Sequential and global optimization for a closed-loop deteriorating inventory supply chain. Mathematical and Computer Modelling 2010;52:161–76.
[21] Chung CJ, Wee HM. Green-product-design and information-technology investment on replenishment model with remanufacturing. International Journal of Computer Integrated Manufacturing 2010;23:466–85.
[22] Salameh MK, Jaber M. Optimal lot sizing with regular maintenance interruptions. Applied Mathematical Modelling 1997;21:85–90.
[23] Abboud NE, Jaber MY, Noueihed NA. Economic lot sizing with consideration of random machine unavailability time. Computers & Operations Research 2000;27(4):335–51.
[24] Chung CJ, Widyadana GA, Wee HM. Economic production quantity model for deteriorating inventory with random machine unavailability and shortage. International Journal of Production Research 2011;49:883–902.
[25] Sheu SH, Chen JA. Optimal lot-sizing problem with imperfect maintenance and imperfect production. International Journal of Systems Science 2004;35(1):69–77.
[26] Alfares HK, Khursheed SN, Noman SM. Integrating quality and maintenance decisions in a production-inventory model for deteriorating items. International Journal of Production Research 2005;43(5):899–911.
[27] Gharbi A, Kenne JP, Beit M. Optimal safety stocks and preventive maintenance period in unreliable manufacturing systems. International Journal of Production Economics 2007;107:422–34.
[28] Tsou JC, Chen WJ. The impact of preventive activities on the economics of production systems: modeling and application. Applied Mathematical Modelling 2008;32:1056–65.
[29] Chakraborty T, Giri BC, Chaudhuri KS. Production lot sizing with process deterioration and machine breakdown under inspection schedule. OMEGA 2009;37:257–71.
[30] Liao GL, Chen YH, Sheu SH. Optimal economic production quantity policy for imperfect process with imperfect repair and maintenance. European Journal of Operational Research 2009;195:348–57.
[31] Karamatsoukis CC, Kyriakidis EG. Optimal maintenance of a production-inventory system with idle period. European Journal of Operational Research 2009;196:744–51.
[32] Widyadana GA, Wee HM. An economic production quantity model for deteriorating items with preventive maintenance policy and random machine breakdown. International Journal of System Science 2012;43(10):1870–82.
[33] Berthaut F, Gharbi A, Dhouib K. Joint modified block replacement and production/inventory control policy for failure-prone manufacturing cell. OMEGA 2011;39(6):642–54.

[34] Wee HM, Widyadana GA. Economic production quantity models for deteriorating items with rework and stochastic preventive maintenance time. International Journal of Production Research 2012;50(11):2940–52.
[35] Lev B, Weiss HJ. Inventory models with cost changes. Operations Research 1990;38:53–63.
[36] Yang PC, Wee HM, Liu BS, Fong OK. Mitigating hi-tech products risks due to rapid technological innovation. OMEGA 2011;39:456–63.
[37] Wee HM, Wang WT. A supplement to the EPQ with partial backordering and phase-dependent backordering rate. OMEGA 2012;40(3):264–6.
[38] Yang PC, Wee HM. A collaborative inventory system with permissible delay in payment for deteriorating items. Mathematical and Computer Modelling 2006;43:209–21.
[39] Lo ST, Wee HM, Huang WC. An integrated production-inventory model with imperfect production processes and Weibull distribution deterioration under inflation. International Journal of Production Economics 2007;106(1):248–60.
[40] Yang PC, Pai S, Yang L, Wee HM, Liu YH. Mitigating production risks considering global recession with LIFO and FIFO backordering policies, working paper, Chung Yuan Christian University; 2012.