Economic order quantity model for deteriorating items and planned backorder level using cost-difference approach

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Abstract

In this study a deteriorating inventory problem with and without backorders is solved using cost-difference approach. From literature search, this study is one of the first attempts by researchers to solve a deteriorating inventory problem without using derivative. The optimal solutions are compared with the classical methods for solving deteriorating inventory model. The total cost of the simplified model almost similar as the original model for small values of deteriorating rate and replenishment time.

Keywords: deteriorating items, EOQ, cost-difference

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1. Introduction

The classical Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) have been developed by many researches for decades. Approaches without using derivative have attracted numerous attentions in recent years. Grubbström and Erdem [1] and Cárdenas-Barrón [2] were among the first researchers to derive EOQ and EPQ without derivatives, respectively. Later, Yang and Wee [3] and Cárdenas-Barrón *et al.* [4] developed a method without derivatives to solve an integrated vendor-buyer inventory system. EOQ problem with temporary sale price without derivative was developed by Wee *et al*. [5] Chang *et al*. [6] used algebraic approaches to solve EOQ and Economic Production Quantity (EPQ) model with shortage. Sphicas [7] solved EOQ and EPQ with linear and fixed backorder cost. Cárdenas-Barrón [8] solved the EPQ with rework process using the algebraic method. Minner [9] introduced cost-difference comparison method for solving the EOQ problem. Wee and Chung [10] extended the research by simplifying the solution procedure. Cárdenas-Barrón [11] applied non derivatives method to solve inventory problem in multi-stages multi customers supply chain. Teng [12] developed arithmetic-geometric-mean-inequality theorem to solve EOQ problem without derivatives. He applied the theorem to EOQ with/without backorder, and EPQ with backorder. The arithmetic-geometric-mean-inequality approach was used by Ouyang et al. [13] to solve an economic order quantity model with partially permissible delay in payments and defective items. Cárdenas-Barrón [14] developed the EOQ/EPQ models using the arithmetic-geometric mean and Cauchy-Bunyakovsky-Schwarz inequality. A complete review of different optimization approaches used in inventory field can be found in Cárdenas-Barrón[15] (In press).

Most items like vegetables, milks, and fruits deteriorate with time. Deterioration is defined as decay, evaporation, obsolescence, loss of quality or marginal value of a commodity that results in decreasing usefulness from the original condition. The longer the items are kept in inventory, the higher the deteriorating cost. The development of deteriorating inventory model has been done by many researchers. See, for example, Yang and Wee [16], Chung *et al*. [17], Ouyang *et al*. [18], Yang and Wee [19], Maity *et al.* [20], and Yang *et al.* [21]. Goyal and Giri [22] presented review of deteriorating inventory literatures since the early 1990s. Zipkin [23] solved deteriorating inventory problem with financing cost numerically using computer. None of the above researchers use algebraic approach to consider deteriorating item inventory.

In this paper, we proposed an EOQ model for deteriorating item problem. The solutions are derived using modified cost-different comparison used in Wee *et al*. [24] for none deteriorating items. Section 2 shows simple EOQ solution for deteriorating item. The deteriorating inventory problem with backorder is solved in Section 3 and some conclusions are derived in Section 4.

2. EOQ model and analysis

Notations:

A	setup cost (per order)
H	inventory holding cost (per unit and unit time)
C_b	backorder cost (per unit and unit time)
d	demand rate (per time)
θ	deterioration rate

r optimal fill rate

T replenishment time (unit)

T_1 non backorder time (unit), where $T_1 = r*T$

Q^* optimal order quantity (unit)

B^* Backorder level quantity (unit)

Assumptions:

1. The demand rate and deterioration rate are constant.
2. Lead time is negligible.
3. The cost of deteriorated units is constant and equal to unit cost.

The deteriorating inventory level is illustrated in Figure 1. The inventory level can be denoted by the following equation:

$$\frac{dI}{dt} + \theta I_t = -d, \qquad 0 \le t \le T \tag{1}$$

The inventory rate can be represented as (see Zipkin [23]):

$$\bar{I} = \frac{d(e^{\theta T} - \theta T - 1)}{T\theta^2} \tag{2}$$

The total inventory cost per unit time is equal to ordering cost plus holding cost:

$$TC(T) = \frac{A}{T} + \frac{hd(e^{\theta T} - \theta T - 1)}{T\theta^2} \tag{3}$$

Equation (3) is cumbersome to solve, some simplification is introduced. For $\theta T << 1$, the inventory depletion curve (Figure 1) can be treated as a straight line (see Misra [25] and Kang and Kim [26]. The simplified inventory system is shown in Figure 2.

A cost-difference method from Wee *et al*. [24] is used to derive the economic order quantity. In this method, the planning horizon is assumed finite with a fixed replenishment period *T*. The equal batch size can be represented as:

$$Q_n = \frac{dT}{n}, \quad where \ \ the \ \ batch \ \ number \ \ n \geq 1 \tag{4}$$

For consecutive batch numbers of *n-1, n* and *n+1*, the different unit-time total variable cost can be expressed as follows:

$$TC_u(i,T) = \frac{A(d+\theta)}{Q_i} + \frac{hQ_i}{2}, \ \ for \ i = n-1, \ \ n \ and \ n+1 \tag{5}$$

where TC_u(i,T) is equivalent to $TC_u(Q_i)$.

The optimal value of (5) can be derived when the cost-difference satisfies the following condition:

$$TC_u(n-1,T) - TC_u(n,T) \geq 0 \ \ and \ \ TC_u(n+1,T) - TC_u(n,T) \geq 0 \tag{6}$$

From (5) and (6), one has:

$$A(d+\theta)\left(\frac{1}{Q_{n-1}} - \frac{1}{Q_n}\right) + \frac{h}{2}\left(Q_{n-1} - Q_n\right) \geq 0 \ \ and$$

$$A(d+\theta)\left(\frac{1}{Q_{n+1}} - \frac{1}{Q_n}\right) + \frac{h}{2}\left(Q_{n+1} - Q_n\right) \geq 0 \tag{7}$$

Simplify (7), one has:

$$Q_{n-1}Q_n \geq \frac{2A(d+\theta)}{h} \quad and \quad Q_{n+1}Q_n \leq \frac{2A(d+\theta)}{h}$$

When the optimal batch numbers *n* and the planning horizon near to infinite, one has:

$$\lim_{n\to\infty} \frac{Q_{n-1}}{Q_n} = \lim_{n\to\infty} \frac{n}{n-1} = 1 \ \ and \ \ \lim_{n\to\infty} \frac{Q_{n+1}}{Q_n} = \lim_{n\to\infty} \frac{n}{n+1} = 1 \tag{8}$$

From (8), the ordering quantity can be written as $Q_{n-1} = Q_n = Q_{n+1}$ and the optimal deteriorating inventory order quantity is equal to:

$$Q^* = \sqrt{\frac{2A(d+\theta)}{h}} \tag{9}$$

When $\theta = 0$, (9) is similar to the traditional EOQ. We use two cases to analysis the result of simplified model with the original model. In the first case, we use different values of setup cost, inventory cost, demand rate and deteriorating rate. The comparisons of original and simplified deteriorating inventory model for each case are shown in Table 1. The original solution is achieved by solving (3) numerically. Since the solution procedure is out of paper scope, the detailed discussion is omitted here. The table shows that the total cost differences between the simplified model solutions with the original model solutions are less than 0.5%. In the second case, we use some data form Lin and Gong [27]. As Lin and Gong [27] example, different values of deteriorating rate are used. The results are shown in Table 2. The table shows that the different of the total costs of the original model and the simplified model are very small, even for a big value of deteriorating rate (0.75).

3. EOQ with backorder

The deteriorating inventory level with backorder problem is shown in Figure 3. From Figure 3 We can formulate the total cost of deteriorating inventory with backorder as follows:

$$TC(T, T_1) = \frac{A}{T} + \frac{hd(e^{\theta T} - \theta T - 1)}{T\theta^2} + \frac{C_b d(T - T_1)^2}{2T} \tag{10}$$

Equation (10) is cumbersome to solve. For a small value of θ and T, the deteriorating inventory level with backorder can be refigured as Figure 4. Since the items deteriorate with time, the inventory depletes at $d+\theta$ rate and the total time period (T) can be expressed as:

$$T = \frac{rQ}{d+\theta} + \frac{Q(1-r)}{d} \tag{11}$$

where r is the optimal fill rate.

The total cost of deteriorating inventory model with backorder can be modeled as follows:

$$TC = \frac{A}{T} + \frac{hr^2 Q_n}{2} + \frac{C_b (1-r)^2 Q_n}{2} \tag{12}$$

Substitute (11) to (12), one has:

$$TC = \frac{1}{(d+\theta(1-r))}\left(\frac{1}{Q_n}\left(Ad(d+\theta)\right) + Q_n (d+\theta(1-r))\left(\frac{hr^2 + C_b(1-r)^2}{2}\right)\right) \tag{13}$$

The cost function rate in (13) can be written as:

$$TC(r,n,t) = f_1(Q_n) + f_2(Q_n).g(r)$$

Using a similar condition as in (6), *g(r)* can be represented as:

$$\theta(1-r_{n-1}-r_n)\frac{h\left(r_{n-1}^2 - r_n^2\right)}{2}Q_n + \theta(1-r_{n-1}-r_n)\frac{C_b\left((1-r_{n-1})^2 - (1-r_n)^2\right)}{2}Q_n \geq 0 \quad and$$

$$\theta(1-r_{n+1}-r_n)\frac{h\left(r_{n+1}^2 - r_n^2\right)}{2}Q_n + \theta(1-r_{n+1}-r_n)\frac{C_b\left((1-r_{n+1})^2 - (1-r_n)^2\right)}{2}Q_n \geq 0 \tag{14}$$

Simplify (14), one has:

$$\frac{r_{n-1}+r_n}{2} \leq \frac{C_b}{h+C_b} \quad and \quad \frac{r_{n-1}+r_n}{2} \geq \frac{C_b}{h+C_b} \tag{15}$$

The optimal fill rate (r) is equal to:

$$r = \frac{C_b}{h + C_b} \tag{16}$$

since the optimal fill rate can be derived when $r_{n-1} = r = r_{n+1}$.

Using the condition as in (9) for the cost function rate, one has:

$$\left(\frac{1}{Q_{n-1}} - \frac{1}{Q_n}\right)\left(\frac{Ad(d+\theta)}{(d+\theta(1-r))}\right) + \left(Q_{n-1} - Q_n\right)\left(\frac{hr^2 + C_b(1-r)^2}{2}\right) \geq 0 \quad and$$

$$\left(\frac{1}{Q_{n+1}} - \frac{1}{Q_n}\right)\left(\frac{Ad(d+\theta)}{(d+\theta(1-r))}\right) + \left(Q_{n+1} - Q_n\right)\left(\frac{hr^2 + C_b(1-r)^2}{2}\right) \geq 0 \tag{17}$$

Simplifying (17), one has:

$$Q_{n-1}Q_n \geq \frac{2Ad(d+\theta)}{(hr^2 + C_b(1-r)^2)(d+\theta(1-r))} \quad and$$

$$Q_{n+1}Q_n \leq \frac{2Ad(d+\theta)}{(hr^2 + C_b(1-r)^2)(d+\theta(1-r))} \tag{18}$$

Since the optimal order quantity (Q) can be derived when $Q_{n-1} = Q = Q_{n+1}$, the optimal EOQ for deteriorating item with backorder is:

$$Q^* = \sqrt{\frac{2Ad(d+\theta)}{(hr^2 + C_b(1-r)^2)(d+\theta(1-r))}} \tag{19}$$

From Fig. 4 it is easy to see that the backorder level is $B^* = Q^* - rQ^* = Q^*(1-r)$

$$B^* = Q^*\left(1 - \frac{C_b}{h + C_b}\right)$$

$$B^* = \frac{h}{h + C_b} Q^* \tag{20}$$

Equation (20) is equal to the backorder level in the classical EOQ with backorder.

For the case without deterioration, θ=0, one has:

$$Q^* = \sqrt{\frac{2Ad}{(hr^2 + C_b(1-r)^2)}} \tag{21}$$

Equation (21) is equal to the classical EOQ with backorder.

Table 3 shows the total cost solutions for the simplified model (20 and 21) and the original model (10) of deteriorating inventory problem with backorder. The total cost differences for the simplified model solutions and the original solutions are not more than 0.3 %. Table 4 shows similar result as Table 3, where there are no significant different of the total cost of the original model compare with the simplified model. The biggest difference cost is only 0.0345% for deteriorating rate equal to 0.75.

4. Conclusion

In this study, two EOQ models for deteriorating inventory problem have been developed. Since the deteriorating rate is usually small, we simplified the model by assuming $T\theta << 0$. The models solved without using derivatives are found to conform to the classical optimization method. The total cost comparison between the simplified model and original model show that the simplified model performance almost similar as the original model. The biggest difference between the simplified model and the original model in our example is not more than 0.5%. Since the simplified model is easier to solve than the original model, it can be used widely in practice.

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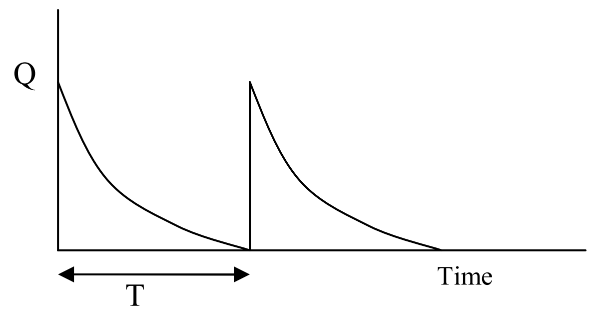

Figure 1. Original deteriorating inventory level

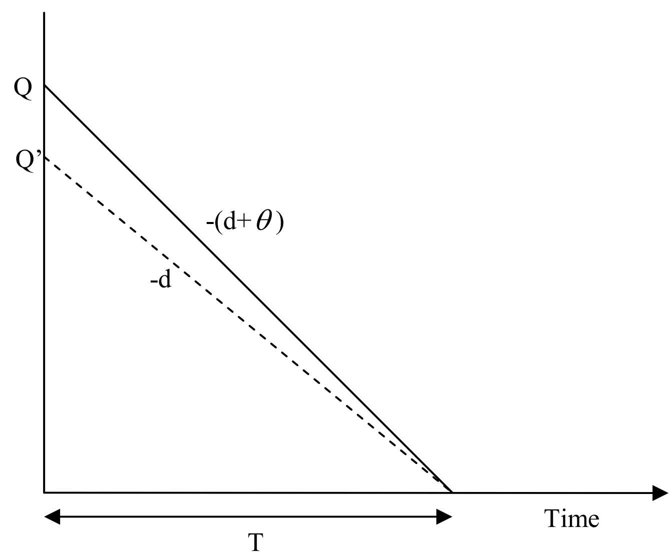

Figure 2. Simplified inventory system for deteriorating inventory

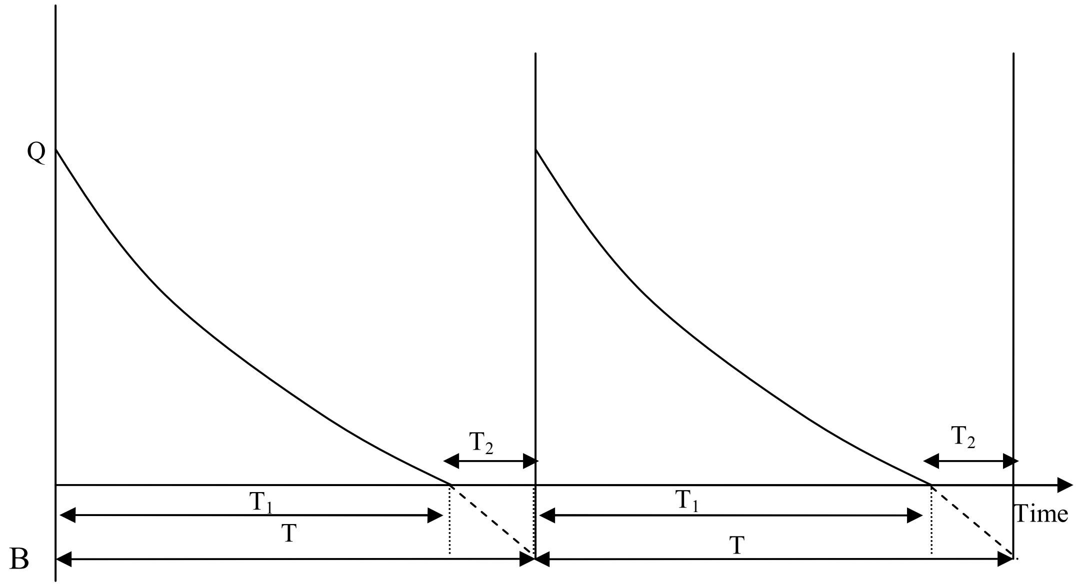

Figure 3. Deteriorating level for problem with backorder

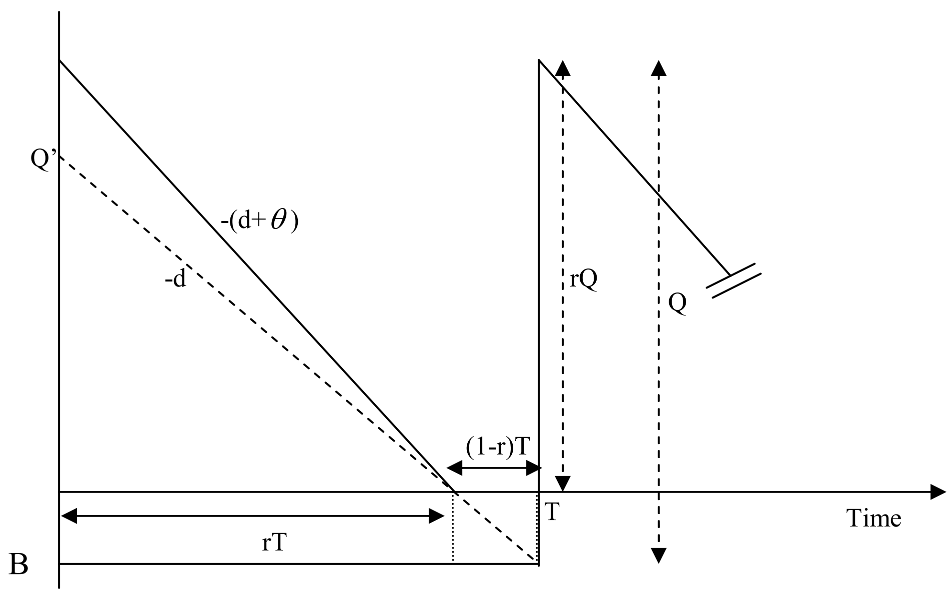

Figure 4. Simplified inventory deteriorating level for problem with backorder

Table 1. Comparison of the classical and the simplified model (case 1)

A	d	h	θ	θT	T simplified	T original	TC simplified	TC original	% different
10	50	1	0.05	0.031294	0.63213954	0.625883926	31.7906	31.7890077	0.00500888
10	50	5	0.01	0.002826	0.28281443	0.282576391	70.745	70.7440036	0.00140846
10	50	5	0.05	0.014076	0.2827014	0.281517958	70.8778	70.8771491	0.00091834
10	200	5	0.05	0.007054	0.14140368	0.141089099	141.589	141.587925	0.00075933
10	50	5	0.1	0.028021	0.28256029	0.280210114	71.04578	71.0432316	0.00358705
10	50	5	0.2	0.055529	0.28227872	0.277643706	71.38431	71.3742486	0.01409674
100	50	5	0.05	0.044068	0.89398031	0.88136094	225.2906	225.267326	0.01033171
1000	50	5	0.1	0.259193	2.82560293	2.591925658	742.86322	739.708309	0.42650746

Table 2. Comparison of the classical and the simplified model (case 2)

A	d	h	θ	T simplified	T original	TC simplified	TC original	% different
50	7500	1	0.01	0.11546998	0.1154256	866.192118	866.192055	7.36684E-06
50	7500	1	0.05	0.11546967	0.1152484	866.859936	866.858337	0.000184455
50	7500	1	0.1	0.11546928	0.1150279	867.696871	867.690472	0.000737465
50	7500	1	0.25	0.11546813	0.1143734	870.222175	870.182123	0.004602665
50	7500	1	0.5	0.1154662	0.113305	874.479854	874.319248	0.018369195
50	7500	1	0.75	0.1154643	0.112263	878.799502	878.437214	0.041242427

Table 3. Comparison of the original and the simplified model for backorder case (case 1)

A	d	h	Cb	θ	T_1 simplified	T_1 original	T simplified	T original	TC simplified	TC original	% different
10	50	1	5	0.05	0.5771099	0.571413806	0.69253188	0.68734476	28.9839994	28.9827473	0.00432
10	50	5	10	0.01	0.23092471	0.230732943	0.34638707	0.34621958	57.7498484	57.7498343	2.5E-05
10	50	5	10	0.05	0.23086319	0.229909113	0.34629479	0.34552684	57.809269	57.8089116	0.00062
10	100	5	10	0.05	0.16327211	0.162782938	0.24490817	0.24450685	81.72386	81.7235981	0.00032
10	200	5	10	0.05	0.11546043	0.115211552	0.17319065	0.1729841	115.544223	115.544033	0.00016
10	50	5	10	0.1	0.2307864	0.228890046	0.34617961	0.3446549	57.8838483	57.8824199	0.00247
10	50	5	10	0.2	0.23063321	0.226886775	0.34594982	0.34294342	58.0340267	58.0283227	0.00983
100	50	5	10	0.05	0.73005351	0.720114781	1.09508027	1.08673272	183.321248	183.30898	0.00669
1000	50	5	10	0.1	2.30786404	2.123503006	3.46179606	3.30640792	593.040093	591.452461	0.26843

Table 4. Comparison of the original and the simplified model for backorder case (case 2)

A	d	h	Cb	θ	T_1 simplified	T_1 original	T simplified	T original	TC simplified	TC original	% different
50	7500	1	5	0.01	0.105409	0.1053692	0.12649104	0.126454559	790.685186	790.685137	6.2063E-06
50	7500	1	5	0.05	0.105409	0.1052091	0.12649076	0.126306576	791.148879	791.147653	0.0001549
50	7500	1	5	0.1	0.105409	0.10501	0.1264904	0.126122734	791.729866	791.724963	0.00061924
50	7500	1	5	0.25	0.105408	0.1044187	0.12648935	0.125577381	793.482016	793.451359	0.00386376
50	7500	1	5	0.5	0.105406	0.1034524	0.12648759	0.124687336	796.433186	796.310449	0.01541329
50	7500	1	5	0.75	0.105405	0.1025094	0.12648584	0.123820012	799.423539	799.147113	0.03459013