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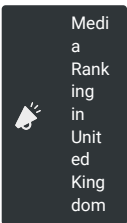
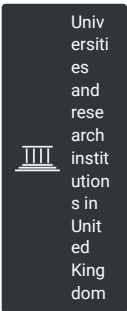
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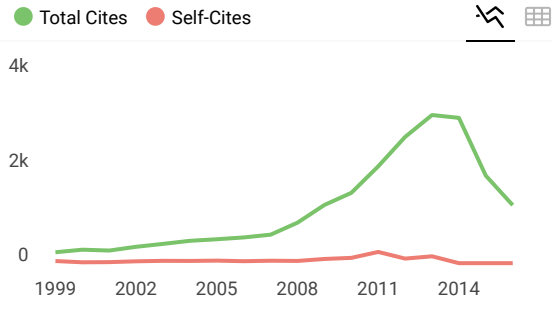
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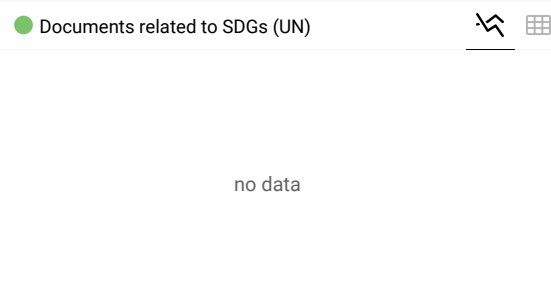
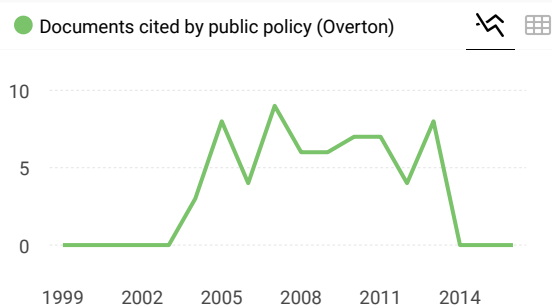
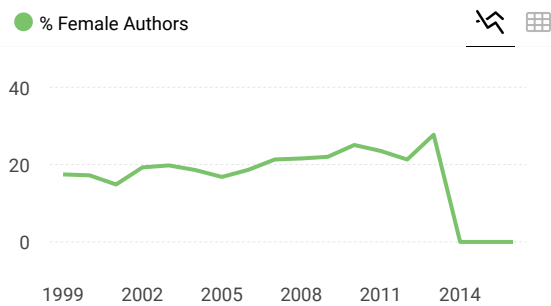
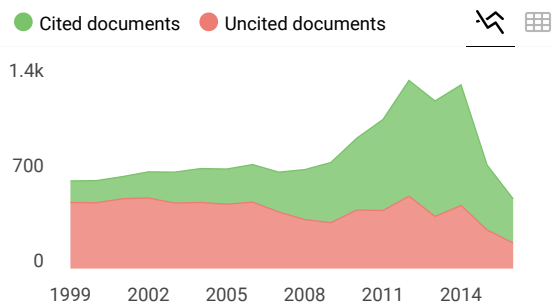
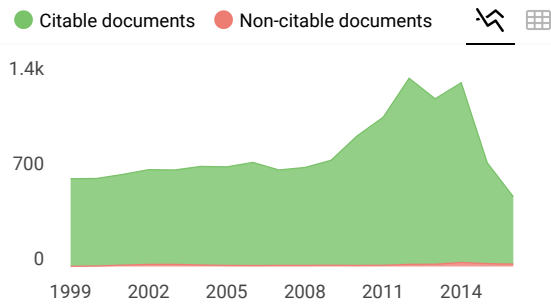
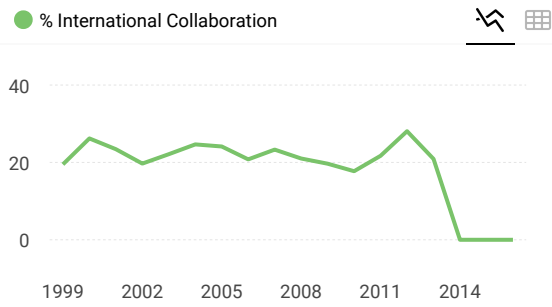
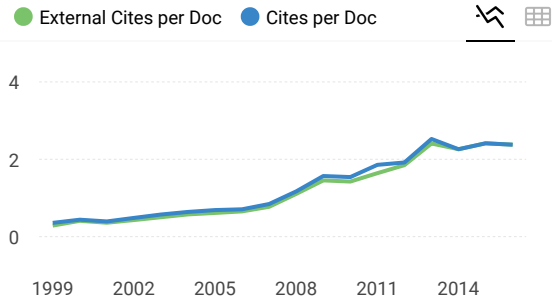
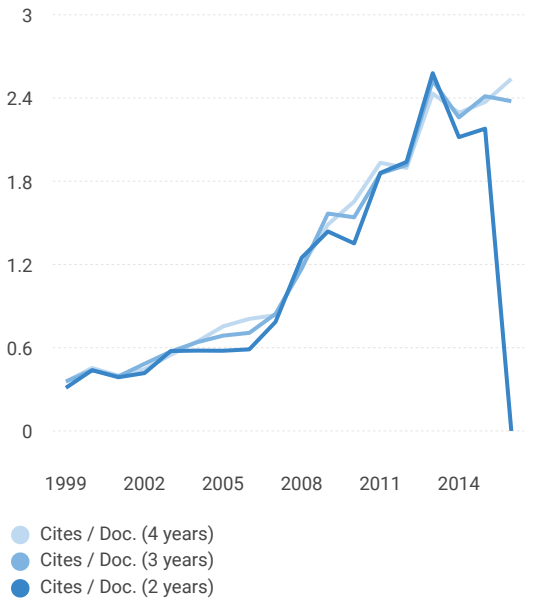
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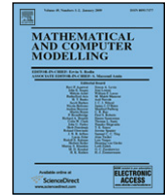
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## Economic order quantity model for deteriorating items with planned backorder level

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### ABSTRACT

In this study, a deteriorating inventory problem with and without backorders is developed. From the literature search, this study is one of the first attempts by researchers to solve a deteriorating inventory problem with a simplified approach. The optimal solutions are compared with the classical methods for solving deteriorating inventory model. The total cost of the simplified model is almost identical to the original model.

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### 1. Introduction

The classical Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) have been extended by many researchers for decades. Approaches without using derivative have attracted much attention in recent years. Grubbström and Erdem [1] and Cárdenas-Barrón [2] were among the first researchers to derive EOQ and EPQ with backorders without derivatives, respectively. Later, Yang and Wee [3] and Cárdenas-Barrón et al. [4] developed a method without derivatives to solve an integrated vendor–buyer inventory system. EOQ problem with temporary sale price without derivative was developed by Wee et al. [5]. Chang et al. [6] used algebraic approaches to solve EOQ and Economic Production Quantity (EPQ) model with shortage. Sphicas [7] solved EOQ and EPQ with linear and fixed backorder cost. Cárdenas-Barrón [8] solved the EPQ with rework process using the algebraic method. Minner [9] introduced the cost-difference comparison method for solving the EOQ problem. Wee and Chung [10] extended the research by simplifying the solution procedure. Cárdenas-Barrón [11] applied the nonderivatives method to solve inventory problem in multi-stages multi-customers supply chain. Teng [12] developed arithmetic–geometric–mean–inequality theorem to solve EOQ problem without derivatives. He applied the theorem to EOQ with/without backorder, and EPQ with backorder. The arithmetic–geometric–mean–inequality approach was used by Ouyang et al. [13] and Teng et al. [14]. Ouyang et al. [13] solved an economic order quantity model with partially permissible delay in payments and defective items, and Teng et al. [14] solved an integrated vendor–buyer inventory model considering backorders. Cárdenas-Barrón [15] developed the EOQ/EPQ models using the arithmetic–geometric mean and Cauchy–Bunyakovsky–Schwarz inequality. A complete review of several optimization approaches used in inventory can be found in [16].

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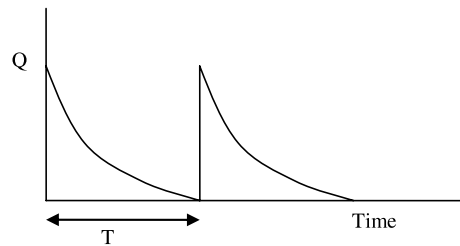


Fig. 1. Original deteriorating inventory level.

Some goods (i.e. milk, fruits, vegetables, among others) deteriorate with time. Deterioration means decay, evaporation, obsolescence, loss of quality or marginal value of a commodity. Obviously, deterioration decreases the usefulness of the good from its original condition. The longer the goods are kept in inventory, the higher the deteriorating cost. The development of deteriorating inventory model has been done by many researchers. See, for example, [17–25]. Most of research above cannot derive a close form solution since it is not possible to derive a close form solution for most deteriorating items inventory model. Goyal and Giri [26] presented a complete review of the deteriorating inventory literature since the early 1990s. Zipkin [27] solved deteriorating inventory problem with financing cost numerically using computer. None of the above researchers use algebraic approach to consider deteriorating item inventory. Our method can be used to develop in a simple way more complex inventory models considering imperfect production process (i.e. [28–30]) or imperfect quality products (i.e. [31,32]).

In this paper, we proposed an EOQ model for deteriorating item problem. The solutions are derived using modified cost-different comparison used in [33] for nondeteriorating items. Section 2 shows notations and the problem definition. Models development of simple EOQ solution for deteriorating item and the deteriorating inventory problem with backorder are shown in Section 3 and some conclusions are derived in Section 4.

## 2. Notations and problem definition

Notations:

$A$	setup cost (\$ per order)
$h$	inventory holding cost (\$ per unit and unit time)
$\hat{\pi}$	backorder cost (\$ per unit and unit time)
$D$	demand rate (units per time)
$\theta$	deterioration rate
$r$	optimal fill rate
$n$	batch number
$T$	replenishment time (units of time)
$T_1$	nonbackorder time (units of time), where $T_1 = r * T$
$Q^*$	optimal order quantity (units)
$B^*$	backorder level quantity (units)
$TC(T)$	total cost in $T$ replenishment time
$TC_u(i, T)$	total unit cost in $i$ batch and $T$ replenishment time
$TC(r, n, T)$	total unit cost in $r$ optimal fill rate, $n$ batch and $T$ replenishment time

The problem of deteriorating inventory model is similar as the basic inventory problem, except that inventory is assumed to deteriorate at a constant rate. Similar to the basic model, the demand rate in our model is assumed to be constant. When items are needed, they can be delivered instantaneously. The cost of deteriorated items is constant and equal to unit cost. It is also assumed that the deteriorated items are not repaired or replaced by a good unit. The number of units will be treated as a continuous variable. Similar to classic EOQ models, the batch size for each replenishment is the same.

## 3. Models development

In this paper, two EOQ models for deteriorating items without backorder and with backorder are developed and details of calculation methods are presented in the following sections.

### 3.1. EOQ without backorder

The deteriorating inventory level is illustrated in Fig. 1.

The inventory level can be denoted by the following equation:

$$\frac{dI}{dt} + \theta I_t = -D, \quad 0 \leq t \leq T. \quad (1)$$

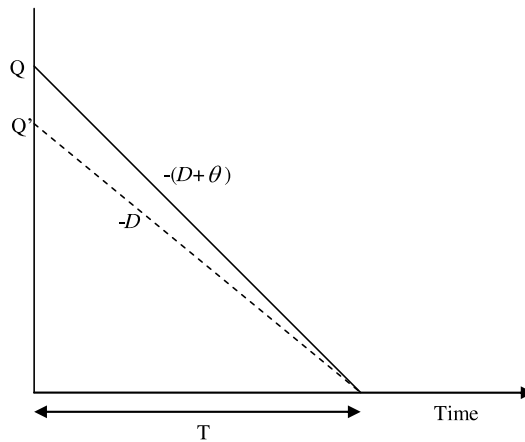


Fig. 2. Simplified inventory system for deteriorating inventor.

The inventory rate can be represented as (see [27]):

$$\bar{I} = \frac{D(e^{\theta T} - \theta T - 1)}{T\theta^2}. \tag{2}$$

The total inventory cost per unit time is equal to ordering cost plus holding cost:

$$TC(T) = \frac{A}{T} + \frac{hD(e^{\theta T} - \theta T - 1)}{T\theta^2}. \tag{3}$$

Eq. (3) is cumbersome to solve, some simplification is introduced. For  $\theta T \ll 1$ , the inventory depletion curve (Fig. 1) can be treated as a straight line (see [34,35]). The simplified inventory system is shown in Fig. 2.

A cost-difference method from [33] is used to derive the economic order quantity. In this method, the planning horizon is assumed finite with a fixed replenishment period  $T$ . The equal batch size can be represented as:

$$Q_n = \frac{DT}{n}, \quad \text{where the batch number } n \geq 1. \tag{4}$$

For consecutive batch numbers of  $n - 1$ ,  $n$  and  $n + 1$ , the different unit time total variable cost can be expressed as follows:

$$TC_u(i, T) = \frac{A(D + \theta)}{Q_i} + \frac{hQ_i}{2}, \quad \text{for } i = n - 1, n \text{ and } n + 1 \tag{5}$$

where  $TC_u(i, T)$  is equivalent to  $TC_u(Q_i)$ .

The optimal value of (5) can be derived when the cost-difference satisfies the following condition:

$$TC_u(n - 1, T) - TC_u(n, T) \geq 0 \quad \text{and} \quad TC_u(n + 1, T) - TC_u(n, T) \geq 0. \tag{6}$$

From (5) and (6), one has:

$$\begin{aligned} A(D + \theta) \left( \frac{1}{Q_{n-1}} - \frac{1}{Q_n} \right) + \frac{h}{2}(Q_{n-1} - Q_n) &\geq 0 \quad \text{and} \\ A(D + \theta) \left( \frac{1}{Q_{n+1}} - \frac{1}{Q_n} \right) + \frac{h}{2}(Q_{n+1} - Q_n) &\geq 0. \end{aligned} \tag{7}$$

Simplifying (7), one has:

$$Q_{n-1}Q_n \geq \frac{2A(D + \theta)}{h} \quad \text{and} \quad Q_{n+1}Q_n \leq \frac{2A(D + \theta)}{h}.$$

When the optimal batch number is  $n$  and the planning horizon is near to infinite, one has:

$$\lim_{n \rightarrow \infty} \frac{Q_{n-1}}{Q_n} = \lim_{n \rightarrow \infty} \frac{n}{n-1} = 1 \quad \text{and} \quad \lim_{n \rightarrow \infty} \frac{Q_{n+1}}{Q_n} = \lim_{n \rightarrow \infty} \frac{n}{n+1} = 1. \tag{8}$$

From (8), the ordering quantity can be written as  $Q_{n-1} = Q_n = Q_{n+1}$  and the optimal deteriorating inventory order quantity is equal to:

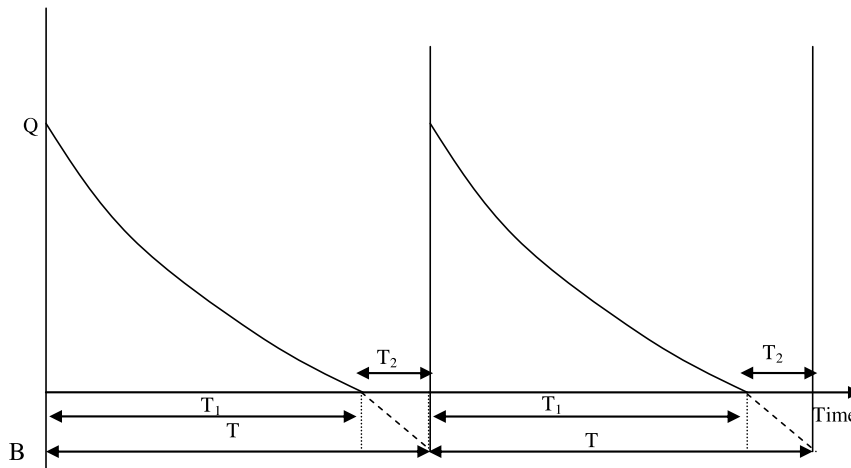
$$Q^* = \sqrt{\frac{2A(D + \theta)}{h}}. \tag{9}$$

**Table 1**  
Comparison of the classical and the simplified model (case 1).

A	D	h	$\theta$	$\theta T$	T		TC		% difference
					Simplified	Original	Simplified	Original	
10	50	1	0.05	0.031294	0.63213954	0.625883926	31.7906	31.7890077	0.00500888
10	50	5	0.01	0.002826	0.28281443	0.282576391	70.745	70.7440036	0.00140846
10	50	5	0.05	0.014076	0.2827014	0.281517958	70.8778	70.8771491	0.00091834
10	200	5	0.05	0.007054	0.14140368	0.141089099	141.589	141.587925	0.00075933
10	50	5	0.1	0.028021	0.28256029	0.280210114	71.04578	71.0432316	0.00358705
10	50	5	0.2	0.055529	0.28227872	0.277643706	71.38431	71.3742486	0.01409674
100	50	5	0.05	0.044068	0.89398031	0.88136094	225.2906	225.267326	0.01033171
1000	50	5	0.1	0.259193	2.82560293	2.591925658	742.86322	739.708309	0.42650746

**Table 2**  
Comparison of the classical and the simplified model (case 2).

A	D	h	$\theta$	T		TC		% difference
				Simplified	Original	Simplified	Original	
50	7500	1	0.01	0.11546998	0.1154256	866.192118	866.192055	7.36684E–06
50	7500	1	0.05	0.11546967	0.1152484	866.859936	866.858337	0.000184455
50	7500	1	0.1	0.11546928	0.1150279	867.696871	867.690472	0.000737465
50	7500	1	0.25	0.11546813	0.1143734	870.222175	870.182123	0.004602665
50	7500	1	0.5	0.1154662	0.113305	874.479854	874.319248	0.018369195
50	7500	1	0.75	0.1154643	0.112263	878.799502	878.437214	0.041242427



**Fig. 3.** Deteriorating level for problem with backorder.

When  $\theta = 0$ , (9) is similar to the traditional EOQ. We use two cases to analyse the result of simplified model with the original model. In the first case, we use different values of setup cost, inventory cost, demand rate and deteriorating rate. The comparisons of original and simplified deteriorating inventory model for each case are shown in Table 1. The original solution is achieved by solving (3) numerically. Since the solution procedure is out of the scope of the paper, the detailed discussion is omitted here. The table shows that the total cost differences between the simplified model solutions with the original model solutions are less than 0.5%. In the second case, we use some data form [36]. As an example, different values of deteriorating rate are used by Lin and Gong [36]. The results are shown in Table 2. The table shows that the difference between the total costs of the original model and the simplified model are very small, even for a big value of deteriorating rate (0.75).

3.2. EOQ with backorder

The deteriorating inventory level with backorder problem is shown in Fig. 3.

From Fig. 3, we can formulate the total cost of deteriorating inventory with backorder as follows:

$$TC(T, T_1) = \frac{A}{T} + \frac{hD(e^{\theta T_1} - \theta T_1 - 1)}{T\theta^2} + \frac{\hat{\pi}D(T - T_1)^2}{2T}. \tag{10}$$

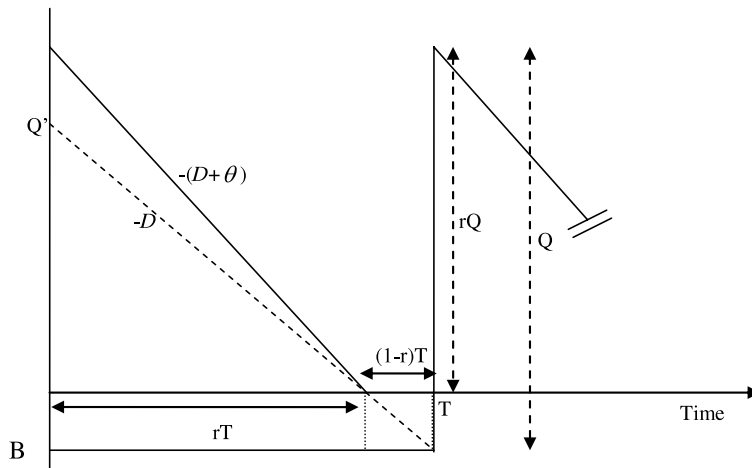


Fig. 4. Simplified inventory deteriorating level for problem with backorder.

Eq. (10) is cumbersome to solve. For a small value of  $\theta$  and  $T$ , the deteriorating inventory level with backorder can be refigured as Fig. 4.

Since the items deteriorate with time, the inventory depletes at  $D + \theta$  rate and the total time period ( $T$ ) can be expressed as:

$$T = \frac{rQ}{D + \theta} + \frac{Q(1 - r)}{D} \tag{11}$$

where  $r$  is the optimal fill rate.

The total cost of deteriorating inventory model with backorder can be modeled as follows:

$$TC = \frac{A}{T} + \frac{hr^2Q_n}{2} + \frac{\hat{\pi}(1 - r)^2Q_n}{2}. \tag{12}$$

Substitute (11) to (12), one has:

$$TC = \frac{1}{(D + \theta(1 - r))} \left( \frac{1}{Q_n} (AD(D + \theta)) + Q_n(D + \theta(1 - r)) \left( \frac{hr^2 + \hat{\pi}(1 - r)^2}{2} \right) \right). \tag{13}$$

The cost function rate in (13) can be written as:

$$TC(r, n, T) = f_1(Q_n) + f_2(Q_n).g(r).$$

Using a similar condition as in (6),  $g(r)$  can be represented as:

$$\begin{aligned} \theta(1 - r_{n-1} - r_n) \frac{h(r_{n-1}^2 - r_n^2)}{2} Q_n + \theta(1 - r_{n-1} - r_n) \frac{\hat{\pi}((1 - r_{n-1})^2 - (1 - r_n)^2)}{2} Q_n &\geq 0 \quad \text{and} \\ \theta(1 - r_{n+1} - r_n) \frac{h(r_{n+1}^2 - r_n^2)}{2} Q_n + \theta(1 - r_{n+1} - r_n) \frac{\hat{\pi}((1 - r_{n+1})^2 - (1 - r_n)^2)}{2} Q_n &\geq 0. \end{aligned} \tag{14}$$

Simplifying (14), one has:

$$\frac{r_{n-1} + r_n}{2} \leq \frac{\hat{\pi}}{h + \hat{\pi}} \quad \text{and} \quad \frac{r_{n-1} + r_n}{2} \geq \frac{\hat{\pi}}{h + \hat{\pi}}. \tag{15}$$

The optimal fill rate ( $r$ ) is equal to:

$$r = \frac{\hat{\pi}}{h + \hat{\pi}} \tag{16}$$

since the optimal fill rate can be derived when  $r_{n-1} = r = r_{n+1}$ .

Using the condition as in (9) for the cost function rate, one has

$$\begin{aligned} \left( \frac{1}{Q_{n-1}} - \frac{1}{Q_n} \right) \left( \frac{AD(D + \theta)}{(D + \theta(1 - r))} \right) + (Q_{n-1} - Q_n) \left( \frac{hr^2 + \hat{\pi}(1 - r)^2}{2} \right) &\geq 0 \quad \text{and} \\ \left( \frac{1}{Q_{n+1}} - \frac{1}{Q_n} \right) \left( \frac{AD(D + \theta)}{(D + \theta(1 - r))} \right) + (Q_{n+1} - Q_n) \left( \frac{hr^2 + \hat{\pi}(1 - r)^2}{2} \right) &\geq 0. \end{aligned} \tag{17}$$

**Table 3**  
Comparison of the original and the simplified model for backorder case (case 1).

A	D	h	$\hat{\pi}$	$\theta$	$T_1$		T		TC		% different
					Simplified	Original	Simplified	Original	Simplified	Original	
10	50	1	5	0.05	0.5771099	0.571413806	0.69253188	0.68734476	28.9839994	28.9827473	0.00432
10	50	5	10	0.01	0.23092471	0.230732943	0.34638707	0.34621958	57.7498484	57.7498343	2.5E-05
10	50	5	10	0.05	0.23086319	0.229909113	0.34629479	0.34552684	57.809269	57.8089116	0.00062
10	100	5	10	0.05	0.16327211	0.162782938	0.24490817	0.24450685	81.72386	81.7235981	0.00032
10	200	5	10	0.05	0.11546043	0.115211552	0.17319065	0.1729841	115.544223	115.544033	0.00016
10	50	5	10	0.1	0.2307864	0.228890046	0.34617961	0.3446549	57.8838483	57.8824199	0.00247
10	50	5	10	0.2	0.23063321	0.226886775	0.34594982	0.34294342	58.0340267	58.0283227	0.00983
100	50	5	10	0.05	0.73005351	0.720114781	1.09508027	1.08673272	183.321248	183.30898	0.00669
1000	50	5	10	0.1	2.30786404	2.123503006	3.46179606	3.30640792	593.040093	591.452461	0.26843

**Table 4**  
Comparison of the original and the simplified model for backorder case (case 2).

A	D	h	$\hat{\pi}$	$\theta$	$T_1$		T		TC		% difference
					Simplified	Original	Simplified	Original	Simplified	Original	
50	7500	1	5	0.01	0.105409	0.1053692	0.12649104	0.126454559	790.685186	790.685137	6.2063E-06
50	7500	1	5	0.05	0.105409	0.1052091	0.12649076	0.126306576	791.148879	791.147653	0.0001549
50	7500	1	5	0.1	0.105409	0.10501	0.1264904	0.126122734	791.729866	791.724963	0.00061924
50	7500	1	5	0.25	0.105408	0.1044187	0.12648935	0.125577381	793.482016	793.451359	0.00386376
50	7500	1	5	0.5	0.105406	0.1034524	0.12648759	0.124687336	796.433186	796.310449	0.01541329
50	7500	1	5	0.75	0.105405	0.1025094	0.12648584	0.123820012	799.423539	799.147113	0.03459013

Simplifying (17), one has:

$$Q_{n-1}Q_n \geq \frac{2AD(D + \theta)}{(hr^2 + \hat{\pi}(1 - r)^2)(D + \theta(1 - r))} \quad \text{and}$$

$$Q_{n+1}Q_n \leq \frac{2AD(D + \theta)}{(hr^2 + \hat{\pi}(1 - r)^2)(D + \theta(1 - r))}. \tag{18}$$

Since the optimal order quantity (Q) can be derived when  $Q_{n-1} = Q = Q_{n+1}$ , the optimal EOQ for deteriorating item with backorder is:

$$Q^* = \sqrt{\frac{2AD(D + \theta)}{(hr^2 + \hat{\pi}(1 - r)^2)(D + \theta(1 - r))}}. \tag{19}$$

From Fig. 4, it is easy to see that the backorder level is  $B^* = Q^* - rQ^* = Q^*(1 - r)$ .

$$B^* = Q^* \left(1 - \frac{\hat{\pi}}{h + \hat{\pi}}\right)$$

$$B^* = \frac{h}{h + \hat{\pi}}Q^*. \tag{20}$$

Eq. (20) is equal to the backorder level in the classical EOQ with backorder.

For the case without deterioration,  $\theta=0$ , one has:

$$Q^* = \sqrt{\frac{2AD}{(hr^2 + \hat{\pi}(1 - r)^2)}}. \tag{21}$$

Eq. (21) is equal to the classical EOQ with backorder.

Table 3 shows the total cost solutions for the simplified model (20) and (21) and the original model (10) of deteriorating inventory problem with backorder. The total cost differences for the simplified model solutions and the original solutions are not more than 0.3%. Table 4 shows similar result as Table 3, where there is no significant difference in the total cost of the original model compared with the simplified model. The biggest cost-difference is only 0.0345% for deteriorating rate equal to 0.75.

**4. Conclusion**

In this study, two EOQ models for deteriorating inventory problem have been developed. Since the deteriorating rate is usually small, we simplified the model by assuming  $T\theta$  as a small value. The models solved without using derivatives are

found to conform to the classical optimization method. The total cost comparison between the simplified model and original model shows that the simplified model perform almost similar to the original model. The biggest difference between the simplified model and the original model in our example is not more than 0.5%. Since the simplified model is easier to solve than the original model, it can be used widely in practice.

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